Proceedings of the 18th International Symposium on OPERATIONAL RESEARCH in Slovenia

SOR '25

Bled, Slovenia

September 24-26, 2025

Edited by:
S. Drobne • L. Zadnik Stirn • M. Kljajić Borštnar • J. Povh • J. Žerovnik

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Slovenian Society INFORMATIKA (SDI)
Section for Operational Research (SOR)

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Proceedings of the 18th International Symposium on Operational Research in Slovenia, SOR'25, Bled, Slovenia, September 24-26, 2025.

Proceedings of the scientific symposium SOR'25 (https://sor.fov.um.si/). All contributions were peer-reviewed in a blinded manner.

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Revievers are listed below.

Keywords: operations research, mathematical programming & optimization, computational optimization, multicriteria decision-making, graph theory, game theory, econometrics & statistics, data science & artificial intelligence, logistics & transportation, environment & human resources, production & management, agricultural economics & policy, finance & investments, inventory & location research, Slovenian Society Informatika (SSI), Section for operational research (SOR)

Available at: https://www.drustvo-informatika.si/sekcije-drustva?stran=publikacije-sor

Organiser of SOR'25: Slovenian Society INFORMATIKA – Section for Operational Research, SI-1000 Ljubljana, Litostrojska cesta 54, Slovenia (https://www.drustvo-informatika.si/sekcije-drustva)

Co-organiser of SOR'25: University of Maribor, Faculty of Organizational Sciences, SI-4000 Kranj, Kidričeva cesta 55a, Slovenia (http://www.fov.um.si/)

Co-organiser of SOR'25: Rudolfovo – Science and Technology Centre Novo mesto, SI-8000 Novo mesto, Podbreznik 15, Slovenia (https://www.rudolfovo.eu/)

Publisher: Slovenian Society INFORMATIKA – Section for Operational Research, SI-1000 Ljubljana, Litostrojska cesta 54, Slovenia (https://www.drustvo-informatika.si/sekcije-drustva)

Ljubljana, 2025

First electronic version.

Kataložni zapis o publikaciji (CIP) pripravili v Narodni in univerzitetni knjižnici v Ljubljani
COBISS.SI-ID 248708867

ISBN 978-961-6165-64-8 (PDF)

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Proceedings of the 18th International Symposium on Operational Research in Slovenia (SOR'25) is cited in: ISI (Index to Scientific & Technical Proceedings on CD-ROM and ISI/ISTP&B online database), Current Mathematical Publications, Mathematical Review, MathSci, Zentralblatt für Mathematic / Mathematics Abstracts, MATH on STN International, CompactMath, INSPEC, Journal of Economic Literature

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Preface

This volume, Proceedings of the 18th International Symposium on Operational Research, called SOR'25, contains papers presented at SOR'25 (https://sor.fov.um.si/), organised by the Slovenian Society INFORMATIKA (SSI), the Section for Operational Research (SOR), and co-organized by the University of Maribor, Faculty of Organisational Sciences, Kranj, Slovenia (UM FOV), and Rudolfovo – Science and Technology Centre Novo mesto, Novo mesto, Slovenia, 24-26 September 2025 in Bled, Slovenia. The volume contains blind peer-reviewed papers or abstracts of papers presented at the symposium.

The opening at SOR'25 was moderated by Prof. Dr. Mirjana Kljajić Borštnar, Co-Chair of SOR'25 PC, and was given by Prof. Dr. Lidija Zadnik Stirn, President of SSI-SOR, Prof. Dr. Slavko Žitnik, President of SSI, Prof. Dr. Polona Šprajc, dean of UM FOV, Prof. Dr. Janez Povh, director of Rudolfovo – Science and Technology Centre Novo mesto, Dr. Sarah Fores, Manager of EURO, Prof. Dr. Tea Šestanović, President of the Croatian Operational Research Society (CRORS), Prof. Dr. Tomasz Wachowicz from the University of Economics in Katowice, Poland, and presidents/representatives of several other operational research societies from abroad.

SOR'25 is the scientific event in the field of Operational Research, another in the traditional series of biennial international OR conferences organised by SSI-SOR in Slovenia. It is a continuation of seventeen previous symposia. The main goal of SOR'25 is to promote knowledge, interest and education in the field of OR in Slovenia, Europe and worldwide to build the intellectual and social capital that is essential for maintaining the identity of OR, especially at the time when interdisciplinary cooperation is proclaimed as particularly important for solving problems in today's challenging times. By joining IFORS and EURO, SSI-SOR has also agreed to collaborate with different disciplines, i.e. to balance the depth of theoretical knowledge in OR and the understanding of theory, methods and problems in other fields within and outside OR. We believe that SOR'25 creates the benefit of these goals and contributes to the quality and reputation of OR by presenting and sharing new developments, opinions and experiences in the theory and practise of OR.

SOR'25 was highlighted by five distinguished keynote speakers. The first part of Proceedings SOR'25 contains invited abstracts, presented by five outstanding scientists: Prof. Tamás Terlaky, Alcoa Endowed Chair Professor, Quantum Computing Optimization Laboratory, Department of Industrial and Systems Engineering, Lehigh University, Bethlehem, PA, USA, Prof. Sanja Petrovic, Nottingham University Business School, Division of Operations Management and Information Systems, Jubilee Campus, Nottingham, UK, Prof. Josip Arnerić, University of Zagreb, Faculty of Economics and Business, Department of statistics, Zagreb, Croatia, Assist. Prof. Milica Maričić, University of Belgrade, Faculty of Organizational Sciences, Department of Operational Research and Statistics, Belgrade, Serbia, Assoc. Prof. Drago Bokal, University of Maribor, Faculty of Natural Sciences and Mathematics, Laboratory for Natural Science and Mathematical Applications and Research, Maribor, Slovenia.

The Proceedings includes 105 short papers or extended abstracts by 213 authors. Most of the authors of the contributed papers came from Slovenia (73), then Croatia (55), Czech Republic (13), Hungary (13), Poland (8), Slovak Republic (8), Serbia (6), Portugal (5), Spain (5), Bosnia and Herzegovina (4), Italy (4), P.R. China (4), USA (4), North Macedonia (3), Ireland (2), UK (2), Austria (1), Brazil (1), Israel (1), and Romania (1). The papers published in the Proceedings are divided into Plenary Lectures (5 abstracts), OR personalities (5 papers),

seven special sessions: Advances in Discrete Applied Mathematics and Graph Theory (5 papers), Agriculture and Agricultural Economics (10 papers and abstracts), Analysing the Role of Platform Economy and Organisations in Societal Change using OR and Statistical Method (6 papers), Doctoral Consortium (4 papers), High Performance Computing and Quantum Computing for Operations Research (6 papers), Industry & Society 5.0: Optimization in Human and Industrial Environments (6 papers), Research and Application of MCDM Methods and Decision Systems in Cybersecurity (3 papers), and nine regular sessions: Econometric Models and Statistics (15 papers), Human Resources (10 papers), Finance and Investments (4 papers), Location and Transport, Graphs and their Applications (5 papers), Mathematical Programming and Optimization (3 papers and abstracts), Multi-Criteria Decision-Making (6 papers), Mathematical Optimization and Production Planning (5 papers), Methodologies and Case Studies (3 papers) and Different Problems Approaching OR (4 papers and abstracts).

Proceedings of the previous seventeen International Symposia on Operational Research organised by the Slovenian Section on Operational Research, listed at https://www.drustvo-informatika.si/sekcije-drustva?stran=publikacije-sor, are indexed in the following secondary and tertiary publications: Current Mathematical Publications, Mathematical Review, Zentralblatt Mathematik / Mathematics Abstracts, MATH on STN International and CompactMath, INSPEC. It is expected that Proceedings SOR'25 will be covered by the same bibliographic databases.

The success of the scientific events of SOR'25 and the present proceedings is due to joint efforts. On behalf of the organisers, we would like to express our sincere thanks to all those who helped to prepare the event. Without the commitment and advice of the active members of the Slovenian Section for Operations Research, we would not have been able to attract so many high-calibre speakers from all over the world. Many thanks to them. We would also like to express our deepest gratitude to the prominent keynote speakers, the members of the Programme and Organising Committees, the reviewers who contributed to the quality of SOR'25 with their useful suggestions, the Section Chairs and all the numerous people - far too many to mention here individually - who helped in the organisation of the 18th International Symposium on Operations Research SOR'25 and the production of these proceedings. Finally, we would like to thank the authors for their efforts in preparing and presenting the papers that made the 18th Symposium on Operational Research SOR'25 a success.

Our special thanks go to MDPI AG, Mathematics, EURO – Association of European Operational Research Societies, Project EuroCC2 – National Competence Centre SLING and Project Excellerat P2 – The European Centre of Excellence for Engineering Applications for their financial support.

Ljubljana, Kranj and Novo mesto, September 8, 2025

Samo Drobne
Lidija Zadnik Stirn
Mirjana Kljajić Borštnar
Janez Povh
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(Editors)

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The 18th International Symposium on
Operational Research in Slovenia

SOR '25

Bled, SLOVENIA
September 24 - 26, 2025

Plenary Lectures

OVERCOMING TRADING IMPERFECTIONS IN MULTIVARIATE ASSET PRICES TO IMPROVE INTEGRATED COVARIANCE ESTIMATION

Josip Arnerić
University of Zagreb, Faculty of Economics and Business, Department of statistics
Zagreb, Croatia
E-mail: jarneric@net.efzg.hr

Abstract: In empirical finance, which covers portfolio selection, asset pricing, and option pricing, realized estimators for integrated covariance have been extensively used to summarize the time-varying relationships between underlying assets. Although the benefits of realized estimators of integrated covariance, computed using high-frequency or tick-by-tick data, are well-known, they come with several issues that must be addressed, particularly when dealing with real-life data. Eliminating trading imperfections, such as price jumps, microstructure noise, price discreteness, and asynchronicity, to obtain the most accurate integrated covariance estimation is complex and not straightforward. For example, jumps can lead to biases in covariance measurements, and several estimators have been proposed to separate these sources of variation in the multivariate case, such as bipower covariances, threshold covariances, and outlyingness-weighted covariances. Conversely, sparse sampling, multi-scale estimators, or pre-averaging techniques are recommended for reducing microstructure noise. By analyzing the theoretical and empirical properties of different realized estimators proposed in the literature, one can establish optimal criteria for selecting the appropriate elements needed to construct an estimator that combines robustness to jumps, noise, and asynchronicity.

Keywords: realized estimators, integrated covariance, trading imperfections

MAKING SENSE OF KNOWLEDGE IN THE ERA OF ARTIFICIAL INTELLIGENCE

Drago Bokal
University of Maribor, Faculty of Natural Sciences and Mathematics, Laboratory for Natural Science and Mathematical Applications and Research
Koroška cesta 160, 2000 Maribor, Slovenija
E-mail: drago.bokal@um.si

Abstract: In the era of Artificial Intelligence, knowledge is being commoditized. Given Bloom's taxonomy of educational goals, remembering has been superseded already by the first internet search engine Archie in 1990 and is dominated today by Google. Understanding has been superseded by web as corpus models in the 2000s and is today dominated by LLMs like ChatGPT, Bard, DeepSeek and others. Applying this knowledge is the next Bloom's taxon being addressed by the models, and there, operations research has a cornerstone position.

In the talk, we outline four models whose understanding has the potential to shape the sensemaking of knowledge in the near future, hoping that they all pass the paper-napkin test. Cynefin is Snowden's model of sensemaking in decision-demanding circumstances. It binds the other three models into a universal process of knowledge discovery, which is the second model. The third model addresses the availability of attention, a central modern active constraint. Finally, the Knowledge Exchange Game addresses the interests of individual agents in this process.

The world today is at a crossroads. Bulletin of Atomic Scientists, founded by Einstein and a team of Manhattan project scientists in 1945, has set the apocalyptic clock to 89 seconds till midnight. Has politics lost its compass in the chaos of interests? Have we abandoned Aristotle's "middle way is the golden way"? Are we objecting to Kant's "Don't do unto others what you don't wish to be done to yourself?" In modern days, Ricoeur has combined them into the categorical imperative of "To live a good life with others and for others in just institutions." And Trontelj has observed that declining institutions stop serving people and tend to favour preserving themselves, so he concluded that "The ethical imperative is to live a good life with others and for others in a just order."

As an inspiring mathematical scientific talk should have an open problem, in operations research preferably related to optimization, we conclude with one. In the context of Cynefin, intuitively, simple decision processes educate soldiers and bureaucrats, complicated decision processes educate experts, complex decision processes educate scientists, and chaotic decision processes educate saviours and dictators. Aspects of this claim have been investigated by various domains of science.

Hence, our optimisational open problem: Which is the smallest consistent set of scientific concepts and assumptions, in which the claim can be formulated within a relevant learning context, and be either confirmed or refuted?

The talk is based on joint work with Ernest Petrič, Anja Goričan, Špela Kaizer, Andreja Smole, Petra Fic, and Mitja Steinbacher.

Keywords: Agent-based models, Cynefin, Decision processes, Interface Theory of Perception, Learning spaces

THE ART AND SCIENCE OF COMPOSITE INDICATORS: WHERE OR MEETS STATISTICS

Milica Maričić
University of Belgrade, Faculty of Organizational Sciences, Department of Operational Research and Statistics
Belgrade, Serbia
E-mail: milica.maricic@fon.bg.ac.rs

Abstract: According to the Organisation for Economic Co-operation and Development (2004), composite indicators (CIs) are metrics formed "when individual indicators are compiled into a single index, on the basis of an underlying model of the multi-dimensional concept that is being measured". At the very outset, this definition provokes a number of questions, such as the selection of individual indicators as well as the choice of normalisation, aggregation and weighting method. These questions, coupled with the public and policymakers' desire for rankings, have led to a need for stable and methodologically sound composite indicators (Maričić et al., 2019). As a solution, methods of OR and multivariate statistics have been applied with the goal of creating robust, data-driven CIs.

The talk will address several aspects of CI creation and validation through the lens of OR and multivariate statistics. First, we will detangle the OECD methodological framework for CI creation and focus on the steps in which methods of OR and multivariate statistics can be applied. Second, we will overview the methodologies and frameworks of some of the methods used, paying special attention to those used in the step of assigning weights to individual indicators. Finally, we will explore real-world applications of these methods, demonstrating how OR and statistical methods have been used to create influential composite indicators in various fields. The talk will conclude with the challenges and limitations of current approaches, pointing towards future research directions in developing composite indicators through the application of OR and statistics.

Keywords: composite indicators, operations research, multivariate statistics, ranking methodology

WELL-BEING IN PERSONNEL SCHEDULING (THE EURO PLENARY)

Sanja Petrovic
Nottingham University Business School, Division of Operations Management and Information Systems,
Jubilee Campus, Wollaton Road, Nottingham NG8 1BB, UK
E-mail: sanja.petrovic@nottingham.ac.uk

Abstract: The research presented in this talk is motivated by the findings of employee well-being at work and especially of the effect of shift working and different shift patterns on employee well-being. These research results are mostly generated in the field of occupational medicine. The aim of our research is to bring the issue of employee well-being to the Operational Research rostering community. The presented research and insights provided from case studies are a joint work between OR academics and OR practitioners. An overview of well-being literature will be presented. This will be followed by a description of the well-being measures that we propose to be included in the objective function while generating a roster. These measures include work-life balance measures, Fatigue and Risk indices, and compliance with Health and Safety Executive (HSE) guidelines. As an experimental environment, we used three case studies of personnel scheduling, which are of very different nature: scheduling of fixed shift patterns, nurse rostering, and cyclic scheduling of flexible shifts in a call-centre service industry. The aim was to investigate to what extent employee well-being could be improved while maintaining high performance of rosters.

Keywords: personnel scheduling, well-being measures, fatigue, risk, case studies

QUANTUM INTERIOR POINT METHODS (QIPMS) WITH ITERATIVE REFINEMENT FOR LINEAR AND SEMIDEFINITE OPTIMIZATION

Tamás Terlaky
Lehigh University, Department of Industrial and Systems Engineering, Quantum Computing Optimization Laboratory, Bethlehem, PA, USA
E-mail: terlaky@lehigh.edu

Abstract: QIPMs build on classic polynomial time IPMs. In QIPMs we apply Quantum Linear System Algorithms (QLSAs) to solve Newton systems within IPMs to gain quantum speedup in solving Linear Optimization (LO) and Semidefinite Optimization (SDO) problems. Due to their inexact nature, QLSAs mandate the development of inexact variants of IPMs which by default are inexact infeasible methods. We also discuss "quantum inspired" Inexact-Feasible IPMs (IF-IPMs) for LO and SDO problems, using novel Newton systems to generate inexact but feasible steps. We show that IF-QIPMs enjoys the to-date best iteration complexity. Further, we explore how QLSAs can be used efficiently in iterative refinement (IR) schemes to find optimal solutions without excessive calls to QLSAs. Notably, the IR-IPM scheme enjoys quadratic convergence of the optimality gap.

Keywords: interior point methods, linear optimization, semidefinite optimization, newton systems, quantum computing

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Operational Research in Slovenia

SOR '25

Bled, SLOVENIA
September 24 - 26, 2025

OR personalities

REVEILING THE (UN)KNOWN – JOSIP ARNERIĆ

Tea Šestanović
University of Split, Faculty of Economics, Business and Tourism, Department of Quantitative Methods
Cvite Fiskovića 5, 21000 Split, Croatia
E-mail: <tea.sestanovic@efst.hr>

Abstract: In this short note, we will briefly reveal the life of Josip Arnerić, his studies, teaching and scientific work. We will present as well his impact on the Croatian, but also worldwide students, through his selfless efforts to help. Additionally, we will present his contribution to scientific work in Croatia and beyond, as well as his institutional contributions.

Keywords: operational research, statistics, multivariate GARCH models

1 CAREER OVERVIEW

Josip Arnerić was born on December 5, 1980, in Split. He graduated in 2003 from the Faculty of Economics, University of Split, and immediately after graduation was employed as a teaching assistant in the subjects of Statistics and Business Statistics at the Department of Quantitative Methods, Faculty of Economics, University of Split. In September 2004, he was employed as research assistant on the same University. As an external associate, he participated in teaching Statistics at the Faculty of Electrical Engineering, Mechanical Engineering, and Naval Architecture in Split. Upon employment, he enrolled in postgraduate studies in Operations Research at the Faculty of Economics, University of Zagreb, and in 2007 successfully defended his scientific master's thesis "*Econometric Modeling of Risk in the Croatian Capital Market*".



Figure 1: Josip Arnerić

He successfully defended his doctoral dissertation "*Univariate and Multivariate GARCH Models in the Analysis of Integration of the Croatian Capital Market*" in 2010 at the Faculty of Economics, University of Split. In April 2010, Josip Arnerić was employed as senior assistant. After obtaining the title of research associate in May 2011, in October 2011, he obtained the position as assistant professor at the Department of Statistics, Faculty of Economics, University of Zagreb. In January 2017, he was elected as associate professor and in March 2022 he became full professor.

2 TEACHING AND MENTORING

At his home Faculty, he teaches Statistics, Business Statistics, Statistical Methods for Economic Analysis, Econometrics, Financial Econometrics, as well as Methods of Time Series Analysis and Forecasting and Stochastic Processes (courses in the postgraduate specialist program "Statistical Methods for Economic Analysis and Forecasting").

Since 2014, as a visiting lecturer, he has participated in teaching the course Econometrics at the Shanghai University of International Business and Economics. Within the QFRG and DSLab seminar series at the Faculty of Economic Sciences, University of Warsaw, he delivered a seminar titled: "Recurrent neural architectures for nonlinear volatility modelling" as well as the course "Financial Econometrics using R" within the invited lecture series of the Spatial

Warsaw initiative. He is also involved in teaching at the doctoral programs of the Faculty of Economics in Rijeka and the Faculty of Economics in Mostar.

Josip Arnerić has continued his collaboration with the Faculty of Economics in Split, where he introduced a new course at the graduate program, Time Series and Panel Data Analysis, as well as at the postgraduate specialist program Analysis of Financial Time Series.

At the postgraduate specialist program of the Faculty of Economics in Rijeka, together with full professor Blanka Škarbić Perić, he co-introduced a new course, Econometric Methods in Finance, while at the doctoral program of the Faculty of Economics in Rijeka he co-teaches two courses: Quantitative and Qualitative Methods in Scientific Research and Advanced Econometrics.

To date, he has successfully supervised three doctoral students, including Mario Matković with the topic: *Analysis of realized volatility estimators of European stock indices*, Tea Šestanović with the topic: *Neural network structure identification in inflation forecasting,* and Marija Čuljak (co-menthor with full professor Saša Žiković) with the topic: *Analysis and robustness of the return distribution estimators, volatility and covolatility of stock markets by using high frequency data.*

His collaboration with other universities in teaching statistics, probability, multivariate statistics, econometrics and time-series analysis at graduate or postgraduate level (Faculty of Electrical Engineering, Mechanical Engineering, and Naval Architecture in Split, Faculty of Economics in Split, Rijeka and Mostar as well as Shanghai University of International Business and Economics in China, Ruhr University Bochum in Germany and University of Warsaw – Faculty of Economic Sciences) makes him appreciated as a teacher.

3 SCIENTIFIC CONTRIBUTION

His fields of interest are financial econometrics, time-series analysis, stochastic processes and neural networks. Related to these fields of interest Josip Arnerić was research associate on projects funded by the Ministry of Science and Education: *Mathematical models in financial management* (0055011) and *Mathematical models in analysis of the Croatian financial market* (055-0000000-1435), as well as the project manager of research funded by the Croatian Science Foundation: *Volatility measurement, modelling, and forecasting* (UIP-2013-11-5199) and project manager of research funded by the University of Zagreb: *Analysis of financial market volatility by using high-frequency data* (2019-05). For three years, he was a member of the Croatian Science Foundation's project evaluation panel.

So far he is a (co)author of more than 75 published papers in prestigious journals, conference proceedings, books and book chapters. Notable Publications in period 2019-2025 are given in references.

Josip Arnerić was Editor-in-Chief and a section editor of the journal *Croatian Operational Research Review (CRORR)* for years. He is an associate editor for *Central European Journal of Operations Research (CEJOR).* He is a member of the editorial board of *Proceedings of the Faculty of Economics in Rijeka: Journal of Economics and Business*, and is currently Editor-in-Chief of the journal *Croatian Review of Economic, Business and Social Statistics (CREBSS).* He was also the Editor-in-Chief of *CRORS News Magazine.*

His involvement and leadership in numerous projects, along with his extensive publication record in esteemed journals, have earned him significant recognition in academia. His reputation extends beyond national borders, as he serves as a valued editor and associate/section editor for several journals.



Figure 2: The team members of project Volatility measurement, modelling, and forecasting (UIP-2013-11-5199) (from left: Zdravka Aljinović, Josip Arnerić and Tea Šestanović) participated in a training program at Cass Business School, London, on the topic "Dynamic Models for Volatility and Heavy Tails", which was held from December 15 to 16, 2014. The lectures were delivered by Prof. Andrew Harvey (on the right), University of Cambridge.

3 INSTITUTIONAL CONTRIBUTIONS

For four years, he was vice president of the Croatian Operational Research Society (HDOI). Currently, he is the president of Croatian Statistical Association (CSA). He is also a member of the American Statistical Association (ASA) and the Croatian Biometric Society (HBMD). He is the head of the R Club at the Croatian Statistical Association.

He was a program committee and/or organizing committee member of International Statistical Conferences in Croatia (ISCCRO), International Conferences on Operational Research (KOI) and International Symposium's on Operations Research (SOR).

Josip Arnerić was invited speaker on international conference "The 3rd International Conference in Croatia – ISCCRO'20", held 15-16.10.2020 in Zagreb with plenary lecture on *"Volatility estimation using high–frequency data: Challenges and issues"*.



Figure 3: Josip Arnerić was invited to participate in the workshop "Summer Research Workshop: Quantitative Methods in Economics" organized by Brandenburg University of Technology (BUT), Cottbus. He accepted the invitation and recognized the opportunity to present part of the results of volatility analysis using high-frequency data to eminent professors and experts in the field: Wolfgang Karl Härdle (Humboldt University of Berlin), Cathy Chen (Chung Hua University, Taiwan), Ostap Okhrin (Technische Universität Dresden), and Andrija Mihoci (Brandenburg University of Technology).

He has attended more than 30 international conferences as well as numerous workshops covering quantitative finance, advances in time-series analysis, machine learning and neural

networks, data envelopment analysis, spatial statistics, spatial econometrics and structural equation modelling. He has reviewed more than 100 articles for prestigious publications. He pursued further professional development abroad in the field of quantitative methods in economics at Brandenburg University of Technology, Cottbus-Senftenberg, Germany, and CASS Business School, London, UK. He has also has collaborated with Croatian Bureau of Statistics, Republic of Croatia, in terms of involvement in practices and economy.

It is noteworthy that Josip Arnerić is only 44 years old, yet his teaching, research, and institutional contributions to the field of operational research—both internationally and in Croatia—speak for themselves.

Acknowledgement

I would like to thank Josip Arnerić for his unconditional and selfless help to me when I started my work as a teaching assistant, for his supervision through my doctoral studies as he was my PhD mentor, for his advices in my latter professional path, as well as for his friendship.

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GIVING MEANING TO KNOWLEDGE – A REFLECTION ON AN INTERVIEW WITH DRAGO BOKAL

Lidija Zadnik Stirn[1], Drago Bokal[2,3,4]
[1]University of Ljubljana, Biotechnical Faculty, Slovenia
[2]University of Maribor, Faculty of Natural Sciences and Mathematics, Slovenia
[3]DataBitLab d.o.o., Slovenia
[4]Institute of Mathematics, Physics, and Mechanics, Slovenia
lidija.zadnik@bf.uni-lj.si; drago.bokal@um.si

Abstract: In this abridged reflection following an interview, we highlight the people and influences that shaped Drago Bokal's pursuit of knowledge, its exploration, acquisition, and discovery, and the impact it had on him as a student, researcher, professional, and citizen.

Keywords: operational research, crossing numbers, maturity models, learning spaces, Cynefin

1 INTRODUCTION

Drago Bokal is an associate professor at the University of Maribor, adjunct researcher at the Institute of Mathematics, Physics and Mechanics at the University of Ljubljana, and founder/co-founder/manager of several start-ups including DataBitLab, where he is currently employed. He holds a PhD in mathematics from the University of Ljubljana and has expanded his expertise in operations research, graph theory, analytics, management, social sciences, and computer science. This article is not about his extensive teaching, research, and public engagement, but - based on a conversation with Drago - about his thoughts on knowledge acquisition and, above all, about the people who have most influenced his career, and to whom gratitude is due, although their essential contributions often go unmentioned. As an invited speaker at SOR25, his short CV with photo can be viewed on the conference website (SOR, 2025). His publications, mentorships, presentations, and research reports can be found at (Cobiss, 2025). References to the works can be found in the full interview (Zadnik Stirn, 2026).

2 INFLUENCES AND STRUCTURE

When asked about his career, Drago recalls the Takao Tanabe exhibition he attended in Vancouver with his friends Dr Matt DeVoss, Dr Deborah Kent, and Dr Andrej Vodopivec, where the artist's path unfolded much like Snowden's Cynefin framework. Early paintings such as A Landscape with Trees (1956) reflect the chaotic realm of raw, youthful insight; Untitled #5 (1968) provides the strict order of the simple realm detached from reality; in The Land 20/75 (1975), reality returned, transforming the geometric order into an intricate interplay of curves and depth; and with Inside Passage 5/98 (1998), Tanabe achieved an almost photographic realism, consistent with the complex realm where clarity emerges through patterns that reveal themselves over time. An important influence that inspires him is Edvard Kocbek's Request, from which he often quotes: "It is likely that none of us fully understands the whole game, but courage itself is absolute and all-knowing". This quote has inspired his pursuit of knowledge since his PhD on "Structural Approach to Crossing Numbers of Graphs", for which he is grateful to Dr Bojan Mohar, a profound and knowledgeable mentor.

He cites three TED Talks as influences. Steindl-Rast's Want to be happy? Be grateful inspires him to focus on gratitude towards people rather than their own works. Urban's Inside

the mind of a master procrastinator raised the question of how to fill life's 876,000 hours; he wants the overview to emphasise inspiration and its results, so that readers tackle their own challenges rather than admire past authorities. Finally, Waldinger has shown in What Makes a Good Life that happiness is not based on the quantity, but on the quality of close relationships.

To summarise, in the hope of encouraging readers to reflect on their own 876,000-hour knapsack problem, Drago asked in this bio to focus on experiences rather than outcomes, on people rather than papers, and to divide them into the four stages of development - the simple, formative youth; the complicated, expertise-building research experience; the complex, outcome-generating professional experience; and the messy, hope-demanding and challenge-revealing future experience.

3 SIMPLE KNOWLEDGE ACQUISITION: EARLY CAREER

Drago once asked his students, “How many teachers have you met who were really interested in what you were doing, who worked with you and inspired you to seek knowledge, and not just looked at what you were doing after listening to them and giving you grades?” Only one in twenty named a single teacher. That used to be different - or was Drago just lucky? His own list starts with his mother, Ljudmila Bokal, a lexicologist, who introduced him to the love of language, and his father, mag. Drago Bokal, who challenged him with nature, maths, and later with applications in power grid projects. His grandparents Franc and Marija Stanonik, Ludvik and Angela Bokal, as well as his uncles and aunts Janez Stanonik, DDr Marija Stanonik, Ciril Stanonik, Breda Karner, Angela Vidić, Vida Neugebauer-Bokal, who helped him experience joy of the seeing results of a hard day’s work.

At school, Jože Kavčič and Francka Hribernik encouraged his passion for mathematics, Jože Pečovnik introduced him to programming, and Martina Polajnar, together with Dr Boris Žemva, supervised a chemistry project, which he later presented at a symposium on chemistry education thanks to Dr Aleksandra Kornhauser Frazer. In the upper school, Matjaž Filo and Dr Jurij Šilc supervised a graphics editor project with his friend Gorazd Breskvar; Metka Škornik supervised his simulation of DNA replication and protein synthesis and later an artificial ecosystem of assembler programs, for which he built a compiler and debugger, while Breskvar developed the interpreter. Marjeta Hočevar challenged him to write essays that exceeded the criteria for exam length but provided interesting reading.

The most influential was Vilko Domajnko, whose mentorship led to a GUI programme for testing group axioms, several leisure papers on game theory and - together with Dr Neža Mramor-Kosta - a research project in which over 200 pages of findings on solving mathematical problems were compiled, which he wrote together with his friend Dr Sanja Fidler.

4 COMPLICATED KNOWLEDGE ACQUISITION: STUDY AND RESEARCH

Dr Izidor Hafner introduced Drago early on to Dr Tomaž Pisanski, with whom he worked on polyhedral cages and their relationship to fullerenes. At a conference in Braunschweig, Pisanski introduced him to Dr Gunnar Brunkmann, who invited him to Bielefeld for his diploma thesis on edge-critical graphs. He graduated cum laude in 2001, a day after 11 September 2001 and continued his research on the crossing numbers of graphs under the supervision of Dr Bojan Mohar. At a conference in the High Tatras, Dr Marian Klešč posed a problem that Drago solved on the train home in a concise four-page paper, thus answering a 20-year-old conjecture. His graph-theoretic research focuses on c-crossing critical graphs, which he began in his PhD thesis and continued in 2006/07 as a postdoc at the University of

Waterloo with Dr Bruce Richter and Dr Gelasio Salazar. In this field, limited number c of crossings can be considered scarce resource. For c=1, there are only the two Kuratowski graphs; for c=2, infinitely many but tightly bounded graphs composed of 42 pieces; for c=3, a new structure of Hliněný emerges. Up to c=12, these structures become richer but without anything essentially new; for c≥13, the abundance enables a new structure, namely graphs with arbitrarily frequent arbitrarily high degrees. Drago raises an open problem: does this reflect societies, in which scarcity breeds authoritarian management, but abundance allows democracy to flourish?

5 COMPLEX KNOWLEDGE ACQUISITION: PROFESSIONAL EXPERIENCE

Parallel to research, Drago developed his professional skills. Under mag. Branko Mihorko, he learnt about Oracle systems and tinkered with Forrest&Trees triggers to simulate if clauses, which are not available in SQL. Together with Klemen Žagar, and under the guidance of Mark Morgenthaler and Dr Kurt Konolige, he proposed and developed a statistical algorithm to automatically configure the disparity of a small vision module.

Together with his friends Jure Pompe, Miha Čebulj, Daniel Pipan, Rok Žižek, Matevž Kovačič, Dr Tea Tušar, and Dejan Tušar, he co-founded the "Virtual University" project, the first platform for online distribution of teaching materials at the University of Ljubljana. Drago built the prototype of the server and client, and the team expanded it, with the support of Dr Matjaž Omladič and Matija Lokar. From this project emerged XLab, which Drago co-founded, but left to focus on his PhD. He continued his work as a postdoc with Dr Bruce Richter. His mentors encouraged him to work on applied challenges in the areas of optimisation, timetabling, and power grids.

At Cosylab under Dr Mark Pleško, he formed a research group that led several projects that resulted in a joint venture startup for solar microinverters. Subsequently, together with Anja Goričan, Tadej Žerak, Amadeja Bratuša, and Branko Mihorko, he founded DataBitLab, where he continues to develop tailored analysis tools that companies can use to introduce scientific methods into their market research and process optimisation.

6 CHAOTIC KNOWLEDGE ACQUISITION: TECHNOLOGICAL VALLEY OF DEATH

In 2008, while observing a strike at Mura company, Drago asked himself: “What do I, a mathematics professor, give to this nation, this economy, this society, that allows me the privilege of the system to collect taxes from tens of people working for a minimum wage and convert them into my salary?” The answer emerged slowly over the years that followed: “To them, as a professor, I cannot offer anything anymore. Maybe as a consultant I can help them, and their employees assess and manage the risks their company is facing. They grew up in a system that gave them joy and purpose through a valuable work to which they devoted their life. But when the system found a way cheaper provider for the output of their work, it left them in the lurch. However, as a professor I can help their children, so that if they trust me with a few years of their time, I empower them with the knowledge they need to avoid these pitfalls of the system.”

For investigating this vision, he thanks Dr Markus Chimani and Alen Vegi Kalamar, with whom they designed a workshop model testing these challenges, where students identify concepts, test them, and often write their first research papers. Dr Sandi Klavžar, Dr Vesna Iršič Chenoweth, and Dr John Baptist Gauci share credit for testing and deepening the model. Several mentees succeeded under Drago's mentorship undergoing such processes, and all but one secured employment before finishing their theses. That one Drago employed at the

company he worked with to bridge the gap between former employee's promises and future co-owner of a successful company that valued her work. Drago is most proud of Tadej Kolmanič, who rose from supporting an unemployed mother to managing overseas operations in Cosylab, a living answer to the above opening question. But, the overall approach is most concisely described in the inspiring words of Vilko Domajnko: "The secret is simple. You just must like them."

Formational discussions included Dr Jože Trontelj (ethics in the academic processes), Dr Anton Stres (late Wittgenstein's language games versus early Wittgenstein's truth tables favoured by Russel and adopted by modern mathematics), and Dr Ernest Petrič (emerging regulation favouring one or the other with focus on individual's responsibility). Contributions from students-turned-colleagues shaped the empowerment approach: Andreja Smole (incentives and motivation), Anja Šketa (knowledge transfer and the "valley of death"), Špela Tertinek and Mitja Steinbacher (Csikszentmihalyi's flow as optimal experience), and Petra Fic Žagar (valley of death as an emergent phenomenon), many published at SOR.

Recent work connects these to maturity models and learning spaces. For this, Drago credits Dr Janja Jerebic with Špela Kajzer (representations of progress in maturity models), Anja Šketa, Nikola Čokor, and Andraž Čevka (applications), and, most personally, Gregor Bokal and Davor Bokal, whose mentorship through fatherhood is a most rewarding experience.

Acknowledgement

Lidija thanks Drago for the interview, which has moulded the above words into a reflection to inspire readers. Drago thanks Lidija for the opportunity and the challenges that triggered the reflection on gratitude, knowledge, and past events. But above all, he is grateful to all those mentioned and some for whom the space was too constrained for their formative role.

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INVITED SPEAKER MILICA MARIČIĆ, PHD

Veljko Jeremić
University of Belgrade – Faculty of Organizational Sciences, Department of Operational Research and Statistics
Jove Ilića 154, Belgrade, 11000, Serbia
E-mail: veljko.jeremic@fon.bg.ac.rs

Abstract: At the 18th International Symposium on Operations Research in Slovenia (SOR'25), associate professor of the University of Belgrade, Faculty of Organizational Sciences, Milica Maričić has been chosen as the invited speaker. As someone who has had the privilege of mentoring Milica throughout her academic journey, first as a BSc thesis supervisor, then as a committee member on all three of her MSc theses, a co-mentor during her PhD studies, and now as a colleague in the same Department (Department of Operational Research and Statistics) and Laboratory (Laboratory of Statistics), I have had a comprehensive view of her development, work and aspirations over the past decade. It is with great pleasure that I will introduce Milica and her contributions to the fields of OR and computational statistics.

1 INTRODUCTION

Milica Maričić is an associate professor at the University of Belgrade (UB) – Faculty of Organizational Sciences (FOS). She holds a PhD in Information Systems and Quantitative Management from the University of Belgrade, with a dissertation focused on the development of methodologies for constructing and evaluating composite indicators. She earned multiple master's degrees in Business Analytics, Management and Organization, and Social Sciences and Computing, providing a strong foundation in statistics, data analysis, and quantitative methods. Since 2015, she has been actively working at the UB-FOS, contributing to teaching and research in statistics, computational analytics, and econometric methods at the Department of Operational Research and Statistics.

2 TEACHING, MENTORING AND LEADERSHIP

For over a decade, she has been actively engaged in teaching statistics, data analysis, econometric methods, and statistical modelling at the BSc, MSc, and PhD levels. Her teaching experience spans both at FOS and at international universities through ERASMUS mobility programs in Sweden, Norway, Germany and Italy. Since starting her teaching career, Milica has consistently received excellent student evaluations. She was recognised as one of the best teaching associates in 2016, and her course Econometric Methods has repeatedly been ranked among the best. Students particularly value her patience, clarity in explaining complex concepts, and the practical relevance of her courses. She has a remarkable ability to convey statistical concepts in an accessible and engaging way, making her not only an accomplished lecturer but also an inspiring educator. Besides teaching, she actively mentors students' theses on all levels of study, promoting the field of business data science among future professionals.

Milica has demonstrated strong educational leadership through her coordination of several courses at UB-FOS, including Econometric Methods (BSc), Business Statistics (MSc), and Statistics in Management (PhD). She has actively contributed to curriculum development both for the courses she coordinates and for other courses offered by the Laboratory for Statistics. In addition, Milica has served as a member of the Faculty Council for Bachelor Studies, further supporting academic governance and program development.

Beyond her teaching contributions, Milica has demonstrated exceptional organisational and leadership skills. She served as the President of the Organising Committee for the IC-SHARE

conference (May 2024, Belgrade) and as a member of the Organising Committee for the SMS Extension Conference (October 2024, Belgrade).

Milica has made significant contributions to students' extracurricular activities and case-based learning. She authored a case for the Business Case Innovation Challenge 2021 and served on the evaluation team for the 2022 high school case competition. She is an active member of the academic advisory board and case writer for the Belgrade Business International Case Competition (2022, 2024, 2025). In addition, she has delivered technical tutorials at international AI and data science conferences, covering topics such as structural equation modelling, biclustering algorithms, regularisation methods in R, and SEM modelling approaches.

3 CONTRIBUTIONS TO THE RESEARCH FIELD OF COMPUTATIONAL STATISTICS

Milica Maričić is a researcher with a strong focus on advanced statistical methods, composite indicators, and their applications across various domains, particularly in the emerging field of sharing economy and marketing. So far, she has written more than 20 Web of Science-indexed papers and has been cited more than 580 times in the Google Scholar database. Her work demonstrates a consistent trajectory from foundational research in composite indicators to cutting-edge applications in contemporary business models and management-related issues. As her research spans several fields, in the paragraphs below, I will outline several contributions.

Machine learning and Composite Indicators: Her doctoral dissertation, "Development of methodology for formation and evaluation of composite indicators" [1], laid the groundwork for her subsequent research. She has made significant contributions to the development and refinement of composite indicator methodologies based on data-driven machine learning algorithms, as evidenced by her cited paper on a hybrid enhanced Scatter Search—Composite I-Distance Indicator optimisation approach for determining weights within composite indicators [2]. Her expertise in this area is further demonstrated through works such as "Shedding light on the Doing Business Index: A machine learning approach" [3] and "Assessing the quality of life in the European Union: The European Index of Life Satisfaction (EILS)" [4]. The research on the EILS has been internationally recognised. Namely, she was awarded First Place in the Young Scholars for Better Statistics Award by the OECD and IAOS at the joint conference “Better Statistics for Better Lives” in 2018 [5]. These publications showcase her ability to apply advanced statistical methods to evaluate and improve existing indices across various fields. Her most recent work focused on determining unsupervised constraints to the Benefit-of-the-Doubt (BoD) model using Ivanović distance [6].

Sharing economy: In recent years, within her work at the PANACEA project, Milica Maričić has turned her attention to the sharing economy, applying her expertise in composite indicators and statistical analysis to this emerging field. A notable recent publication is "Confluence of open innovation and sharing economy: lessons learned and roadmap for future research" [7]. This work indicates a growing focus on understanding consumer behaviour, generational differences, and the intersection of the sharing economy with other business concepts. Two papers in the field should be mentioned as well, one related to trends and patterns in the usage of shared accommodation in Europe and Serbia [8] and investigating the relationship between environmental consciousness and sharing economy attitudes among youth in Serbia [9] using SEM analysis. She also worked as the editor for the Proceedings of the first International Conference on Sharing Economy and Contemporary Business Models: Theory and Practice [10], demonstrating her leadership in this area. Additionally, at international conferences, SOR, KOI and ISI WSC, together with Prof. Veljko Jeremić, she organised special sessions on the topic of SE with great success.

Marketing and marketing analytics applications: As one of her completed master's studies was marketing engineering and communication, throughout her career, she closely collaborated with colleagues from the Marketing department at FOS. Besides being a committee member of a defended PhD thesis on the topic of marketing in creative industries and being engaged on a PhD course related to Marketing communications, she has papers published in the field of marketing. The paper which stands out is the "See it and believe it? Conceptual model for exploring the recall and recognition of embedded advertisements of sponsors" [11], where she proposed a conceptual model to discover the factors which impact the sponsorship recall and recognition. Currently, she is working on a conceptual model for recall and recognition in OCR (Obstacle course racing) competitions, which takes into account the impact of the presence of influencers and digital marketing activities.

Interdisciplinary Applications: Her research spans multiple disciplines, showcasing the versatility of her methodological expertise. Her work includes applications in Higher Education with publications on the perceived usefulness of ChatGPT in educational purposes [12], Sports Analytics with contributions to sports performance analysis [13], as well as Management with studies on various aspects of business operations, including the exploration of the effects of the procedural justice of downsizing on survivors' behaviour [14].

Through her publications, she has demonstrated not only methodological innovation but also a strong ability to apply advanced statistical methods to real-world business and economic challenges. Her work reflects a balance of theoretical depth and practical relevance, positioning her as a valuable contributor to both academic and applied research communities.

4 INTERNATIONAL COLLABORATIONS AND PROJECT ACTIVITIES

Since 2016, Milica has actively participated in several international research projects. She was a researcher in the ERASMUS+ project Accelerating the Transition towards Edu 4.0 in HEIs (2020–2023), the Croatian Science Fund project Intelligent System for Automatic Selection of Machine Learning Algorithms in Social Sciences (2021–2024), and the Serbian Science Fund projects PANACEA (2022–2025) and POPENVIROS (2023–2026). Most recently, she has served as Principal Investigator from the FOS side in the DAAD COIL project with HWR Berlin on Data Analytics in Marketing and Finance (2024–2025).

Besides being part of national and international project teams, Milica is an active member of the international statistical community. Since June 2024, she has been an elected member of the International Statistical Institute (ISI), where she also serves as the Country Representative on the ISI Committee on Women in Statistics. In addition, she is engaged in the Young ISI division, which promotes statistics to young enthusiasts, and is a member of the ISI Special Interest Group on Sports Statistics.

Milica also holds editorial roles in reputable international journals. She is a member of the editorial committee of the Journal of Sports Analytics and of the Croatian Review of Economic, Business and Social Statistics (CREBSS).

5 CONCLUDING REMARKS

In conclusion, Milica Maričić has established herself as a promising scholar at the intersection of computational statistics, operations research, and business analytics. Her research has advanced methodological developments in composite indicators, machine learning, and statistical modelling, while also finding impactful applications in areas such as the sharing economy, marketing analytics, and sports performance. She has been actively engaged in international collaborations, editorial work, and professional associations, which highlight her recognition within the global academic community. With a track record of high-quality

publications, leadership in projects, and contributions to both theory and practice, Milica is at the beginning of a brilliant academic career that promises to shape and influence the fields of statistics and operations research for years to come.

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EURO PLENARY SPEAKER: PROFESSOR SANJA PETROVIC

Sarah Fores
Manager of EURO
manager@euro-online.org

Abstract: At the 18th International Symposium on Operations Research in Slovenia (SOR'25), the President of the UK OR Society, Professor Sanja Petrovic, has been chosen as the invited EURO Plenary speaker. As I have had the pleasure of knowing Sanja over a number of years, as a colleague and friend, I am extremely pleased to acknowledge Sanja's OR journey and achievements (to date) and to recognise her contributions to optimisation and scheduling.

1 INTRODUCTION

Sanja Petrovic is a Professor of Operational Research (OR) in Nottingham University Business School, UK. Her research areas include optimisation methods and meta-heuristics, case-based reasoning, multicriteria decision making, data-mining, and fuzzy sets and fuzzy logic. Sanja has been active in the field of Operational Research for a number of years and I met her first, with her twin sister Dobrila, at a UK OR Society conference; and in 2015, as Secretary of the Yorkshire and Humberside regional OR group, I had the pleasure of inviting Sanja to give a talk in Leeds on her research into Radiotherapy Treatment Planning for Brain Tumours. She was Chair of the UK OR Society conference – OR53, held in Nottingham in 2011 and was Vice-president of the UK OR Society in two terms (2013-2018). Being appointed President of the UK OR Society for 2025-2026 is well deserved indeed and she has a great ideas on how to navigate the modern landscape of OR and its relationship with topics such as AI and complex decision-making. It is not surprising that she also wants to nurture new talent and early career researchers, and will continue to advocate for the Level 7 OR Specialist Apprenticeships, to help attract and support the next generation of Operational Researchers Sanja is well established in Nottingham, not far from where I grew up (!), and we have been in contact over the years through friendship and our passion for OR both within the UK and EURO, and I was most recently grateful for her support of my most recent activities within the UK OR Society.

2 CONTRIBUTIONS TO OPERATIONAL RESEARCH

Sanja has published around 60 papers in international scientific journals and 30 book chapters and has supervised 19 PhD students to completion. Sanja was included in the prestigious Stanford University Top 2% Scientists Ranking in 2023. This clearly demonstrates her excellence and dedication to research and supervision, and she is extremely successful in gaining and completing projects mostly funded by Engineering and Physical Sciences Research Council (EPSRC) and Technology Strategy Board. Her reach is, however, not just within the UK community as she is a Member of the International Advisory Board of the IMA Journal of Management Mathematics, Associate Editor of the Journal of Scheduling and a member of the Editorial Board of the Yugoslav Journal of Operations Research (YUJOR). Her work is focused on optimisation and scheduling but the methods she adopts range widely and show her ability to find the right model for the job! Indeed the differing real-world projects that she tackles require the full range of OR soft and hard skills that benefit the end-users but also any students that she teaches as she makes sure they are up-to-date with relevant techniques.

3 CONTRIBUTIONS TO EURO

EURO – The Association of European Operational Research Societies – is currently celebrating its 50th Anniversary. It was the first regional grouping of IFORS – The International Federation of Operational Research Societies, founded in 1959. One of the main reasons for establishing regional groupings was the need to organise more frequent meetings among European Operational Research workers between international conferences, and one of the main early 'instruments' of EURO was the Working Groups which allowed researchers and practitioners with particular interests to collaborate as smaller Special Interest groups. One of these Working Groups, established in 1996, was dedicated to Automated Scheduling (formerly WATT and latterly PATAT). It remains one of the most successful Working Groups within EURO, with thanks in no small part to Sanja's commitment as one of the coordinators since 2006, but also as a regular contributor to the conferences since 2000. Sanja's research concentrates on real world optimisation and scheduling problems including "scheduling of patients, employee timetabling, vehicle routing problems, hub locations and routing, radiotherapy planning, production scheduling, and university timetabling". There is no doubt about the impact of her research in this field and a testament to her success that she is able to continue to work with such a range of practitioners in tackling such complex problems.



Clearly EURO was also formed to support member societies predominantly within the region of Europe, and the UK OR Society was one of the 10 societies which 'gave birth' to EURO. Sanja joined the UK OR Society on 20th July 2006 and has been an active member since then, culminating in her well deserved appointment to be the President in 2025. Not only was she the President of the hosting society (held in Leeds after a successful bid by the UK OR Society) for the 50th anniversary EURO conference but she also gave an excellent Keynote presentation there "Efficient algorithms for online scheduling of patients in an ophthalmology clinic" as part of the Healthcare stream.

Sanja is clearly deserving of the honour of being a EURO Plenary speaker and, I am sure, her research and leadership will continue to strengthen and inspire for many years to come.

THE UNKNOWN TAMÁS TERLAKY: YEARS FROM 1955 UNTIL 1989

Tibor Illés
Corvinus University of Budapest, Corvinus Institute for Advanced Studies,
Corvinus Centre for Operations Research
1093 Budapest, Fővám tér 8., Hungary

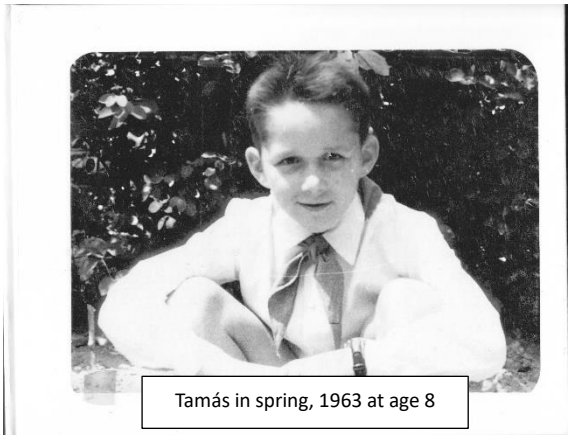
E-mail: tibor.illes@uni-corvinus.hu

Abstract: In this short note, we will briefly review Tamás Terlaky's early years, his studies, and later his teaching duties at Eötvös Loránd University, Budapest, Hungary, and research topics during the period of 1979-1989. Due to the strict page limit of this note, we only mention three of his important publications (duality theory of *lp* programming, that contains main results of his PhD degree (1981); and two publications related to his famous criss-cross algorithm for linear programming and oriented matroid programming, that form the ground of his second degree, the CSc obtained from Hungarian Academy of Sciences in 1985) from this period of his life.
We summarise the awards Professor Terlaky received in Hungary.

Keywords: mathematics, operations research, linear and nonlinear programming, optimisation algorithms

1 LET US START THE STORY ...

Tamás Terlaky was born on January 10, 1955, in Kaposvár, Hungary. His father was József, and his mother Edit. During his primary school years from September 1, 1961, until the end of June 1969 he attended Béla Bartók Primary School in Kaposvár. Secondary school years he spent in Mihály Táncsics Gymnasium as a pupil of a special mathematics and physics class. His math professor was Józsefné Gál, who had a great influence on the development of Tamás' mathematical interest and knowledge. During these years Tamás participated in the KöMaL maths problem solving competition. As it was usual, at that time in Hungary, after graduating from secondary school, he served his one-year compulsory military service (Hódmező-vásárhely, Hungary, September 1973 – August 1974).



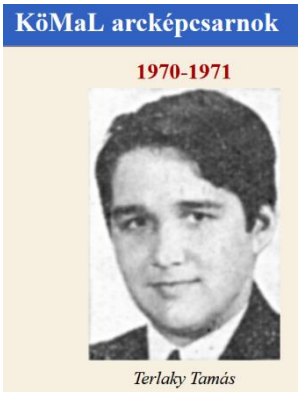
Tamás in spring, 1963 at age 8

2 MEMBER OF THE KöMaL CLUB

KöMaL, the first monthly periodical for high school students in Europe, has been founded by Dániel Arany, a high school maths and physics teacher from Győr, in the school year of 1893/94. First issue was published on January 1, 1894. Generations of would-be scientists, mathematicians, and interested students have had their first mathematical experience by tackling the problems posed every month in the journal's yearly problem-solving contest.

From that time several generations of mathematicians and scientists developed their problem-solving skills through KöMaL competition. The best solutions with the names of the 14-18-

year-old authors are printed in the periodical. Furthermore, a photo of the 20 best problem solvers, together with their names, was published each year.



It has been the intellectual cradle for mathematicians like von Neumann, Paul Erdős, Paul Turán, János Surányi, Emil Klafszky (Tamás' PhD supervisor), György Sonnevend (known for the definition of the central path), Miklós Simonovits, Imre Bárány, József Pelikán, János Pintér, László Szeidl, László Lovász and many others.

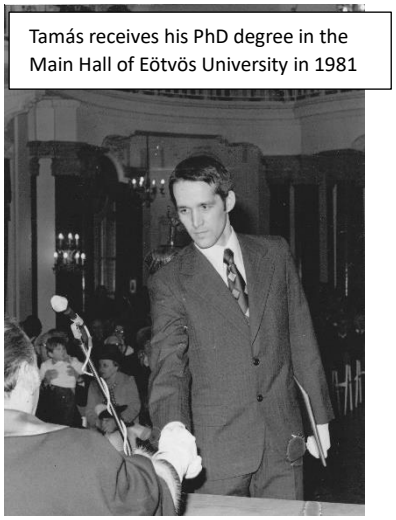
I could even mention further names from KöMaL competitors like János Harsányi (recipient of the Nobel Memorial Prize in Economic Sciences in 1994) or Ferenc Krausz (recipient of the Nobel Prize in Physics in 2023) who were among the best math problem solvers of their time during their secondary education.



Tamás Terlaky is a member of the KöMaL club! Tamás received the 3rd prize during the school year 1972/73 and his award was 100 Ft. This was his first income from mathematics. (At that time, Tamás could buy 50 scoops of ice cream from his award.)

3 HIGHER EDUCATION, SCIENTIFIC DEGREES AND AWARDS FROM HUNGARY

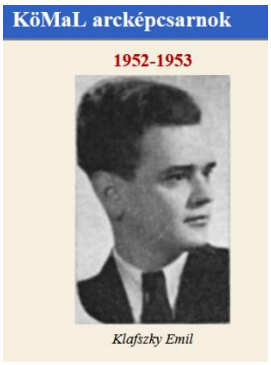
Tamás studied mathematics (1974 - 1979) with specialisation in operations research at Loránd Eötvös University (commonly known as ELTE), Faculty of Sciences, in Budapest, Hungary. He graduated with MSc degree in mathematics, in 1979 under the supervision of Professor E. Klafszky.


Tamás receives his PhD degree in the Main Hall of Eötvös University in 1981

Interestingly enough, Tamás received all his scientific degrees in Hungary: Dr. of Natural Sciences (PhD) at ELTE, Faculty of Sciences, in 1981 (supervisor E. Klafszky); Candidate of Mathematical Sciences in OR (CSc), Hungarian Academy of Sciences (HAS), Budapest, Hungary, 1985, (supervisor E. Klafszky); Doctor of Science (DSc), HAS, Budapest, Hungary, 2006.

Tamás' favourite professors during the undergraduate part (1974-1977) of his studies were Pál (Paul) Turán (number theory), József Mogyoródi (probability theory), Miklós Simonovits (mathematical analysis - tutorial). For the last two years (master part, 1977-1979) of his education, he specialised in *operations research*. The operations research specialisation at the Eötvös University was initiated and developed by A. Prékopa in the 1960's.

During his OR studies at ELTE (1977-1979) Tamás was fascinated by the lectures of Emil Klafszky (nonlinear programming) and enjoyed a lot the courses delivered by Péter Kas (network flows), and Gábor Náhlik (engineering economics and systems). For Tamás, it was a natural choice to write his MSc thesis under the supervision of Professor E. Klafszky. Klafszky's large knowledge, interesting and accurate research style had a deep impact on Tamás' research during the years he spent in Hungary.



Klafszky Emil

In 1984, Tamás received the Gy. Farkas Prize from the János Bolyai Mathematical Society in Budapest, Hungary. This prize has been founded by A. Prékopa in the first part of 1970's to recognize the research achievements of the most talented young, applied mathematicians.

Tamás Terlaky, with the President of the Hungarian OR Society (HORS), Professor L. Szeidl initiated the J. Egerváry Award of the HORS, for life-long scientific contributions in optimization, operations research or applied mathematics, and for the services of the OR community in Hungary, in 2003. The first recipient of the J. Egerváry Award was A. Prékopa in 2003, while the second Egerváry Award was given to E. Klafszky, the supervisor of Tamás Terlaky, in 2004.


L. Szeidl, president of HORS, the wife of Klafszky, E. Klafszky recipient of the J. Egerváry Award, T. Terlaky at the conference dinner of VOCAL Conference, Veszprém in December 2004


T. Terlaky and G. Galambos on November 24, 2017, in Szeged at SWORDS Conference

Knowing Terlaky's research and professional public activities, services to the Hungarian OR community, it was only a matter of time before he would receive the J. Egerváry Award of HORS, too. President of HORS, Professor G. Galambos rewarded him the Egerváry Award of HORS at the SWORDS Conference in Szeged, on November 24, 2017.

The latest recognition from the Hungarian OR community to Professor Terlaky arrived from the Corvinus Centre for Operations Research founded among others by his former PhD student (the author of this article), Tibor Illés and his former PhD students, M. E.-Nagy and P.R. Rigó, as Tamás used to say, *his research grandchildren*, by inviting him as the *Invited Distinguished Research Fellow* of the Corvinus University of Budapest in 2022.

4 TEACHING AND RESEARCH ACTIVITIES FROM 1979 UNTIL 1989 AT EÖTVÖS UNIVERSITY

After working for two years at the Computer Centre of the Oil Refinery Industry in Százhalombatta, Hungary (September 1979 – August 1981), Tamás Terlaky joined the

Department of Numerical Analysis and Computer Science at ELTE as a lecturer (September 1981 – August 1982). In 1982, Tamás received a three-year research fellowship from HAS to complete his CSc degree. Since September 1985 he had been working at the Department of Operations Research, ELTE as assistant professor, and was promoted for associate professor in the spring semester of 1989.

Tamás' teaching portfolio contained Numerical Analysis, Operations Research I and II, courses for computer science students specialised in applied maths at ELTE. Furthermore, he got opportunity to teach the course of Nonlinear Programming I ad II for mathematics students specialised in OR at ELTE, instead of his former supervisor E. Klafszky during the years of 1981 – 1989. (… and life repeats itself only with different characters …) T. Illés felt that the most interesting course of his studies as OR specialized student of mathematics, was the NLP course of Terlaky, and in September 1986 asked him for a master thesis topic … and now is typing this article about early years of Professor Terlaky. T. Illés became the first, and the only PhD student of T. Terlaky, who defended his PhD thesis at ELTE (1989), in Budapest, Hungary.

Although, during the years of 1980's Tamás was the only member of the Department of Operations Research, ELTE who was working on linear programming and published his results in leading international journals of OR, he got right to teach the Linear Programming undergraduate course for students of mathematics quite lately, at the end of 1980's.

Tamás' research topics during the years from 1979 until 1989 were quite broad and covered many different areas of linear and nonlinear programming, as well as structured convex optimisation problems, duality theory [2]; applications of nonlinear programming; pivot algorithms, criss-cross algorithms for linear programming [3], oriented matroids [4], monotone linear complementarity problems, ellipsoid method, interior point methods.

The first result that made him world known in linear programming was the development of a very simple, one phase algorithm for linear programming, the criss-cross algorithm.

I mentioned only few of his papers in this note. However, if someone is interested in research topics and most famous, most influential results of Professor Terlaky and international awards he received during his long and very impressive research carrier, see the recent paper of T. Illés et al. [1].

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The 18th International Symposium on
Operational Research in Slovenia

SOR ’25

Bled, SLOVENIA
September 24 - 26, 2025

Special Session 1:

Advances in Discrete Applied Mathematics and Graph Theory

Towards a randomized algorithm for estimating the chromatic number of a graph

Omkar Bihani[1], **Janez Povh** [1,2], **and Janez Žerovnik**[1,2]

[1] Rudolfovo - Science and Technology Centre Novo Mesto, Podbreznik 15, 8000 Novo mesto, Slovenia.

[2] University of Ljubljana, Faculty of Mechanical Engineering, Aškerčeva cesta 6, 1000 Ljubljana, Slovenia;

Abstract: We consider the classical graph coloring problem and aim to estimate the chromatic number of a given graph using a heuristic approach. Building upon the algorithm of Petford and Welsh for the k-coloring decision problem, we propose a dynamic variation that does not require a pre-specified number of colors. Experiments on DIMACS benchmark graphs demonstrate the efficacy of our method in approximating the chromatic number.

Keywords: Graph coloring, randomized algorithm, chromatic number, heuristic algorithm

1 Introduction

The graph coloring problem is one of the fundamental challenges in combinatorial optimization. Given a graph $G = (V, E)$, the task is to assign colors to its vertices such that no two adjacent vertices share the same color, using the smallest possible number of colors. The smallest number of colors needed for such a proper coloring is known as the *chromatic number*, denoted by $\chi(G)$.

Heuristic algorithms, especially those involving randomization and local search, have shown promise in providing approximate solutions in practice. One notable example is the algorithm of Petford and Welsh [1], which introduces a stochastic iterative process for solving the k-coloring decision problem. Although effective, this method requires k to be known a priori.

In this paper, we propose a dynamic extension to the Petford-Welsh algorithm that allows us to estimate the chromatic number without explicitly providing k. The algorithm initializes with a small number of colors and incrementally adjusts the coloring, guided by a probabilistic recoloring strategy influenced by local conflict information. The temperature-like parameter allows the algorithm to escape local minima, inspired by ideas from simulated annealing[2] and Boltzmann machines [3].

Our experiments on standard graph coloring benchmarks from the DIMACS dataset indicate that the proposed dynamic method can yield competitive estimates of the chromatic number while maintaining flexibility and general applicability.

2 The Petford-Welsh Algorithm for k-Coloring

Given a coloring c, define the cost function $E(c)$ as the number of *bad* edges—edges whose endpoints share the same color. A valid coloring satisfies $E(c) = 0$.

The Petford-Welsh algorithm [1, 4] begins with a random k-coloring and iteratively recolors vertices involved in conflicts. At each step, a bad vertex is selected uniformly at random and assigned a new color based on a probability distribution that favors colors less frequent in its neighborhood.

Algorithm 1 Petford-Welsh algorithm

color vertices randomly with colors $1, 2, \ldots, k$
while not stopping condition **do**
    select a bad vertex v (randomly)
    assign a new color to v
end while

The probability p_i of assigning color i to vertex v is given by

$$p_i \propto \exp(-S_i/T), \tag{1}$$

where S_i is the number of neighbors of v colored i, and T is a temperature parameter. Originally, 4^{-S_i} was used, corresponding to $T \approx 0.72$.

Higher T values increase randomness, aiding exploration; lower T encourages rapid descent to local optima. The algorithm stops upon finding a valid coloring or reaching a time limit. If unsuccessful, it returns the best found coloring with minimal $E(c)$.

For a recent report on experiments with Petford-Welsh algorithm,including the study of critical regions where phase transition seems to occur, we refer to [5]. A brief discussion of relation to the quantum computing is also given in [5].

3 Dynamic algorithm for estimating $\chi(G)$

We generalize Petford-Welsh algorithm to dynamically adjust the number of colors, following and extending the ideas of [6]. The algorithm 2 above incrementally tries to find a proper coloring of the graph. It begins with an initial coloring using a small number of colors (e.g., two), and iteratively improves the coloring by randomly selecting a bad vertex and assigning it a new color based on neighborhood color frequencies. A temporary color expansion is allowed via a switch mechanism, enabling the algorithm to escape local conflicts as in algorithm 3.

Once a valid coloring is found, the algorithm stores the solution and attempts to reduce the number of colors by decrementing `max_color` and recoloring weakly bad vertices (vertices colored with `max_color`). This process continues until the iteration limit is reached. This dynamic approach adapts the number of colors during execution and can be used to approximate the chromatic number of the graph.

Algorithm 2 Dynamic k-Coloring Algorithm

$G_{init} \leftarrow$ InitializeColoring
Set `max_color` $\leftarrow$ `max_color`(G_{init}), `switch` $\leftarrow 1$
while not valid coloring **or** iteration limit not reached **do**
    $v \leftarrow$ Randomly select bad vertex
    Recolor v using ChooseColor(v, `switch` $= 1$)
end while
`switch` $\leftarrow 0$
while iteration limit not reached **do**
    if coloring is valid **then**
        Save current solution
        `max_color` $\leftarrow$ `max_color` $- 1$
        for all weakly bad vertices **do**
            Recolor vertex using ChooseColor(vertex, switch=0)
        end for
    else
        $v \leftarrow$ Randomly select bad vertex
        Recolor v using ChooseColor(v, `switch` $= 0$)
    end if
end while

Algorithm 3 Procedure: ChooseColor

function ChooseColor(node, switch)
    $C \leftarrow \{1, \ldots, \texttt{max_color} + \texttt{switch}\}$
    Compute neighborhood color frequencies S_i
    $w_i \leftarrow \texttt{base}^{-S_i}$ for all $i \in C$
    return random color from C using weights w_i
end function

4 Experiments

We tested our method on several graphs from the DIMACS dataset. We varied the `base` $\in \{4, 10, 16, 20\}$ and used three initialization strategies:

- **Random2:** Random coloring with 2 colors.
- **Greedy:** Greedy proper coloring with smallest available color.
- **GreedyProp2:** Local greedy propagation with 2 colors to minimize conflicts.

Each configuration was run 10 times; the best chromatic estimate is reported. The maximum iterations were set to $500n$, where n is the number of vertices. The results for graphs with number of nodes between 300 and 600, are shown in Table 1.

5 Discussions and Conclusions

We presented a dynamic extension of the Petford and Welsh algorithm aimed at estimating the chromatic number of a graph without requiring the number of colors k as input. The key idea is to begin with a minimal coloring and iteratively adjust the number of colors based on the quality of the current solution. A temporary increase in available colors facilitates exploration, while a controlled reduction drives the algorithm towards minimal valid colorings.

Our results confirm that both the initialization strategy and the choice of base (analogous to temperature) play significant roles in the success of the method. In particular, greedy-based initialization generally yields better convergence and final solutions than purely random ones. Lower base values tend to promote local refinement, while higher base values improve exploration.

This dynamic framework offers a promising direction for heuristically estimating chromatic numbers, especially for large or irregular graphs where exact methods are computationally prohibitive. In future work, we aim to:

- Explore the choice of base and consider several methods for initializing the coloring to obtain better convergence.
- Investigate distributed or parallel implementations to improve scalability.
- Compare the dynamic method against recent advances in deep learning-based and quantum heuristic solvers.

6 Acknowledgements

The research was co-funded by the Slovenian Research and Innovation Agency (ARIS) through the annual work program of Rudolfovo. The authors acknowledge that the project Quantum Solver for Hard Binary Quadratic Problems (QBIQ), ID J7-50186, was financially supported by the Slovenian Research and Innovation Agency.

Graph	$\|n\|$	$\|e\|$	True k	Init. Fn.	Best k (base = 4)	Best k (base = 10)	Best k (base = 16)	best k (base = 20)
fpsol2.i.2.col	363	8691	30	Greedy	**30**	**30**	**30**	**30**
				GreedyProp2	34	36	39	40
				Random2	32	37	41	39
fpsol2.i.3.col	363	8688	30	Greedy	**30**	**30**	**30**	**30**
				GreedyProp2	31	38	36	40
				Random2	34	36	38	41
inithx.i.1.col	519	18707	54	Greedy	**54**	**54**	**54**	**54**
				GreedyProp2	**54**	56	57	57
				Random2	**54**	55	56	58
inithx.i.3.col	559	13969	31	Greedy	**31**	**31**	**31**	**31**
				GreedyProp2	37	41	42	46
				Random2	37	42	43	44
le450_15b.col	450	8169	15	Greedy	17	17	16	16
				GreedyProp2	19	17	16	**15**
				Random2	19	17	16	**15**
le450_15c.col	450	16680	15	Greedy	26	16	17	20
				GreedyProp2	28	16	18	21
				Random2	28	16	20	20
le450_15d.col	450	16750	15	Greedy	25	17	21	21
				GreedyProp2	28	16	17	20
				Random2	29	16	17	21
le450_25a.col	450	8260	25	Greedy	**25**	**25**	**25**	**25**
				GreedyProp2	28	**25**	**25**	**25**
				Random2	28	**25**	**25**	**25**
le450_25b.col	450	8263	25	Greedy	**25**	**25**	**25**	**25**
				GreedyProp2	27	**25**	**25**	**25**
				Random2	27	**25**	**25**	**25**
le450_25c.col	450	17343	25	Greedy	30	29	28	27
				GreedyProp2	36	30	28	27
				Random2	36	30	28	27
le450_25d.col	450	17425	25	Greedy	30	29	28	27
				GreedyProp2	36	30	28	27
				Random2	36	30	28	27
le450_5a.col	450	5714	5	Greedy	6	6	7	7
				GreedyProp2	**5**	6	6	6
				Random2	6	6	6	6
le450_5b.col	450	5734	5	Greedy	6	6	6	6
				GreedyProp2	6	6	6	7
				Random2	6	6	6	6
le450_5c.col	450	9803	5	Greedy	6	6	6	6
				GreedyProp2	6	6	6	7
				Random2	6	7	7	6
le450_5d.col	450	9757	5	Greedy	6	6	7	7
				GreedyProp2	6	6	7	6
				Random2	6	6	6	6

Table 1: Estimated chromatic number from algorithm 2.

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ON THE APPLICATION OF UNIFORMLY DEPLOYED SETS TO THE MATRIX PERMUTATION PROBLEM

Lýdia Gábrišová
University of Žilina, Faculty of Management Science and Informatics
Univerzitná 8215/1 Žilina, Slovakia
E-mail: lydia.gabrisova@fri.uniza.sk

Peter Czimmermann
University of Žilina, Faculty of Management Science and Informatics
Univerzitná 8215/1 Žilina, Slovakia
E-mail: peter.czimmermann@fri.uniza.sk

Andrej Fašánok
University of Žilina, Faculty of Management Science and Informatics
Univerzitná 8215/1 Žilina, Slovakia
E-mail: fasanok@stud.uniza.sk

Abstract: The matrix permutation problem involves permuting the columns of a matrix to minimize a measure of irregularity in the row sums. This problem has been applied in practice to create balanced schedules (for example, when assigning garbage truck crews to specific districts). It is known that this problem is NP-hard [8]. In the context of a new research project at our department, which focuses on regional equality in access to public rescue services provided from service centers, we have decided to revisit this problem. In our contribution, we study it using genetic algorithms and compare the computational efficiency of randomly generated initial populations with that of initial populations obtained through uniformly distributed sets.

Keywords: Matrix permutation problem, genetic algorithms, uniformly deployed sets.

1 INTRODUCTION

In [7], the problem of assigning garbage truck crews to service regions was studied. The objective was to achieve the most balanced distribution of workload among vehicle crews. The problem was formulated as a **Matrix Permutation Problem (MPP)**.

Definition. Given a real matrix $A_{m\times n}$ $(m, n \geq 2)$, the goal is to permute the elements within each column such that, for the resulting matrix $B_{m\times n} = \left(b_{i,j}\right)_{m\times n}$, and associated row sums

$$x_i = \sum_{j=1}^{n} b_{i,j} \text{ (for } i = 1, \cdots, m),$$

the function

$$f(x_1, \cdots, x_m) = \frac{1}{m}\sum_{i=1}^{m}|x_i - \bar{x}|,$$

is minimized. Here, $\bar{x}$ is the average of the values x_1,⋯,x_m.

The function $f(x_1, \cdots, x_m)$ is called an irregularity measure and reaches zero value when all row sums are equal, (i.e., $x_1 = \cdots = x_m$).

In [8], it was proven that the MPP is NP-hard for matrices with $m \geq 2$ rows or $n \geq 3$ columns. The problem is solvable in polynomial time only in the special case where $n = 2$.

Any solution to the MPP for an $m \times n$ matrix can be represented as a vector of permutations $\pi = (\pi_1, \cdots, \pi_n)$, where each π_i is a permutation of the m elements in i-th column. Conversely, every such vector of n permutations defines a valid solution to the problem. This representation makes the MPP a suitable candidate for solving via genetic algorithms.

The concept of **uniformly deployed sets** was introduced in [4] and [6] for use in solving location problems via heuristic algorithms. Let S be a set of binary words of length n. The set S is said to be uniformly deployed if every pair of binary words in S has Hamming distance at least k (where k is a suitably chosen threshold). In the works [2] and [5], the set S was further constrained to contain only binary words with exactly p ones, which complicates the construction of such sets. If we omit this restriction, we can use results from coding theory. There exist well-established methods for constructing sets of codewords with minimum Hamming distance k using matrix-based techniques [1].

2 TWO-ROW CASE

In the specific two-row case, the permutation of elements in each column can be represented by a binary value: 1 indicates that the two elements in the column are swapped, while 0 indicates that they remain in their original positions. Thus, the vector $\vec{\pi}$ of permutations can be encoded as a binary string of length n. This binary representation simplifies the application of genetic algorithms to the two-row MPP, because standard notions, such as initial population, crossover, and mutation, can be defined in a natural manner.

Our testing of genetic algorithms was conducted on matrices with between 100 and 300 columns. The matrix values were randomly generated natural numbers from the interval $\langle a, b\rangle$, where $\min a = 10$, $\max a = 50$, $\min b = 50$, and $\max b = 200$. For each interval type and each given number of columns, we used 50 different inputs and computed the average value of the objective function.

For the creation of the initial population, we compared two different approaches:

- The initial population is a randomly generated set (RGS) of *n*-bit binary words (the number of such words is predefined).
- The initial population is a uniformly deployed set (UDS) with prescribed parameters.

To construct the UDS, we used the method proposed in [3]. This method enables the construction of uniformly deployed sets in which all words contain exactly p ones. Thus, in this approach, the initial population consisted of elements represented by words of weight p. In subsequent iterations, however, this weight was not preserved.

We present several examples from the test results that are typical representatives of the observed phenomena. Figure 1 shows a comparison of the convergence of the objective function to the optimum with respect to the population size for RGS and UDS for $n = 100$, $p = 5$ and values from interval $\langle 10{,}50\rangle$. It can be observed that uniformly deployed sets facilitate faster convergence to the optimal solution in 90% of cases. Figure 2 shows the dependence of the speed of convergence on the parameter p of the generated uniformly deployed sets, with $n = 300$, 30 iterations, values from the interval $\langle 20{,}80\rangle$ and an initial population size of 25.

3 THREE-ROW CASE

In the three-row case, we must adopt a different method for encoding permutations. We can use the number of inversions in the permutations. An inversion in a permutation $\pi\colon\{1,\cdots,m\} \to \{1,\cdots,m\}$ is defined as a situation where for a pair of elements $i, j \in \{1,\cdots,m\}$ such that $i < j$, it holds $\pi(i) > \pi(j)$.

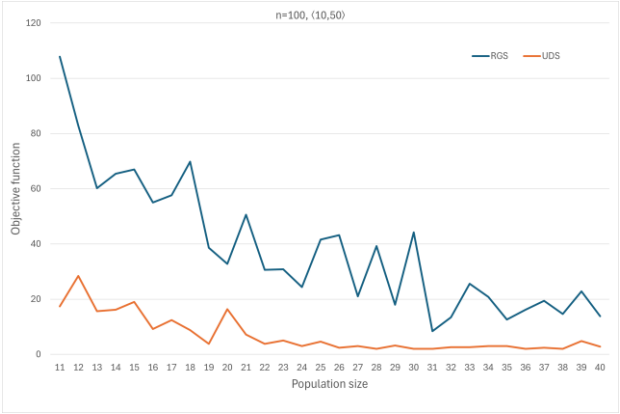


*Figure 1:*The dependence of the objective function on the population size. (Two rows.)

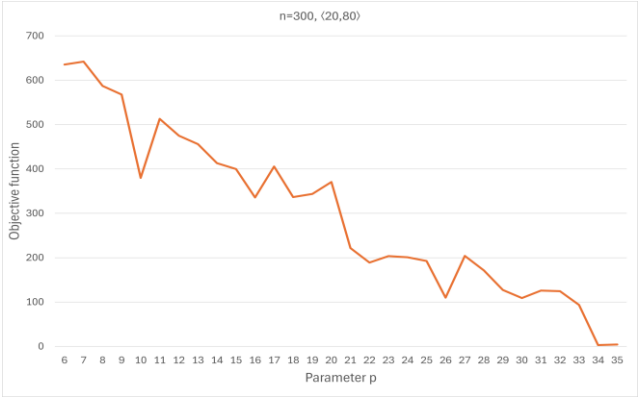


Figure 2: The dependence of the objective function on the parameter p. (Two rows.)

Permutations of a three-element set can have 0, 1, 2, or 3 inversions. Each permutation can be assigned a two-bit binary code corresponding to its number of inversions, as shown in Table 1.

Table 1

No. of inv.	Permutations	Binary code
0	123	00
1	132,213	01
2	231,312	10
3	321	11

A vector of such permutations $\vec{\pi} = (\pi_1, \cdots, \pi_n)$ can be represented by a $2n$-bit binary word, where the pair of bits at positions $2i-1$ and $2i$ encodes the number of inversions in permutation π_i. Conversely, if a $2n$-bit binary word is given and the pair of bits at positions $2i-1$ and $2i$ is 01 or 10, then the permutation π_i is randomly selected from among those with the corresponding number of inversions. This approach does not ensure that all permutation vectors appear with equal probability when generating UDS consisting of $2n$-bit words. However, testing has shown that an initial population generated in this manner using UDS yields better computational results than a randomly generated initial population. Figure 3 shows similar comparison for three-row case than Figure 1. We worked with 3×100 matrices and values were selected from interval $\langle 20,80 \rangle$.

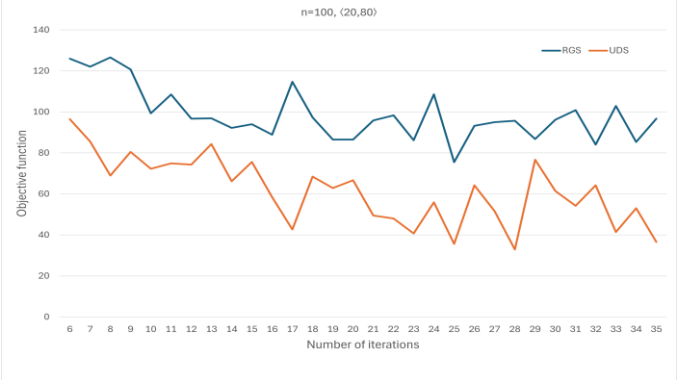


Figure 3: The dependence of the objective function on the number of iterations. (Three rows.)

4 CONCLUSIONS

This paper serves as an introduction to our new research project, which focuses on the regional equality in access to public service centres. Within this context, it seems to be useful to study the Matrix Permutation Problem as a basic computational problem, which we can further modify and adapt. In our study, we tested an approach to the MPP based on genetic algorithms and, for specific cases, used the UDS as the initial population. Based on our results, we can conclude that genetic algorithms are suitable for solving the MPP, and that using UDS as the initial population yields better results in 90% of tested two-row and three-row cases. Unfortunately, the limited space does not allow us to provide a deeper analysis of the test results.

Acknowledgement

This work was supported by the research grant VEGA 1/0220/25 „Regional equality in access to public rescue services provided from service centers on the transport network “.

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DERIVATIVE-FREE OPTIMIZATION OF A MAGNETIC LEVITATION SYSTEM

Reznichenko Igor
University of Ljubljana, Faculty of Mechanical Engineering
Aškerčeva cesta 6, 1000 Ljubljana, Slovenia
ir4016@student.uni-lj.si
Primož Podržaj
University of Ljubljana, Faculty of Mechanical Engineering
Aškerčeva cesta 6, 1000 Ljubljana, Slovenia
primoz.podrzaj@fs.uni-lj.si
Aljoša Peperko
University of Ljubljana, Faculty of Mechanical Engineering
Aškerčeva cesta 6, 1000 Ljubljana, Slovenia
Institute of Mathematics, Physics and Mechanics, Jadranska 19, 1000 Ljubljana, Slovenia
aljosa.peperko@fs.uni-lj.si

Abstract: This research is concerned with design optimization of a PID controlled magnetic levitation system. Dimensions of a magnetic coil as well as controller parameters form the optimization variables. The chosen objective depends on step response characteristics. Numerical tests using two non-gradient search algorithms were compared. Overall, genetic algorithm was able to find a more favorable solution, however, pattern search yields a more compact coil.

Keywords: control theory, derivative-free optimization, magnetic levitation, PID control

1 INTRODUCTION

Non-gradient optimization algorithms do not rely on estimating the derivatives of the objective function. Although a global minimum is not guaranteed, these methods are applicable to more tasks than classic search methods. Optimization often leads to non-intuitive solutions. Heuristic algorithms have the ability to search the variable space in areas that on previous steps were considered irrelevant to the search.

Pattern search is a method of searching the variable space in cross patterns [1]. The algorithm begins with selecting a random point in the search space. Function evaluations along the sides of the cross from this point are conducted along all sides. From the smallest poll point, the cross doubles its size and three new points are added. If the search runs out of the boundaries the cross is compressed. This process continues until the minimum is found or the maximum number of attempts is exceeded. This method has been proven to converge to a local minimum, however it is slower than gradient-based solvers.

Genetic algorithm is a heuristic search inspired by generation evolutions [2]. It deals with populations, rather than single evaluation points. The starting population is usually random. It is also a non-gradient method, however, in contrast to pattern search, it has stochastic iterates. This method can be adapted to run in parallel, while also allowing integer constraints.

There exist various applications of magnetic fields. Among them are medical imaging techniques [3]. For example, they are used to non-invasively modulate and read activity of targeted brain networks. Magnetic levitation is a well-known phenomenon often used in control system studies. Due to the advent of magnetic suspension it has become more important in industrial applications. One of the best known examples are high-speed train transportation [4]. Another notable example are active magnetic bearings for which a linearized model is often adopted for determining controller settings. However, the model parameters vary depending on equilibrium points. Therefore, robustness of closed-loop controlled active magnetic bearings is possible, if the controller parameters are calculated over the entire operating range [5].

2 METHODOLOGY

Consider an engineering system under design. Let vector $\vec{\chi} = (\chi_1, \chi_2, \ldots, \chi_n)$, $\chi_i \in X$, be the parameters of the designed system and vector $\vec{\eta} = (\eta_1, \eta_2, \ldots, \eta_p)$, $\eta_k \in H$, represent the output characteristics, where X and $H \subseteq \mathbb{R}$ are feasible sets. The optimization problem is to find min $f_{obj.}(\vec{\eta})$ with a chosen minimum search method. The choice of such algorithms is a field of mathematical research [6].

The goal of a magnetic levitation system is to maintain or move a permanent magnet at a certain height. A solenoid may be approximated with a ring current. The magnetic field component B_z of a single ring of radius R and current i is [7]

$$B_z = \frac{\mu_0}{2\pi} i \frac{\pi R^2}{(z^2 + R^2)^{3/2}}, \quad \frac{\partial B_z}{\partial z} = -\frac{3}{2} \cdot \frac{\mu_0 i R^2 z}{(z^2 + R^2)^{5/2}}. \tag{1}$$

A permanent magnet can be modeled as a magnetic dipole. If a magnetic dipole moment μ has only a vertical component μ_z, the force F_z acting on a magnetic dipole in an external magnetic field B_z is

$$F_z = \mu_z \frac{\partial B_z}{\partial z} = -\frac{3\mu_z}{2} \cdot \frac{\mu_0 i R^2 z}{(z^2 + R^2)^{5/2}}, \tag{2}$$

where $\mu_0 = 4\pi \cdot 10^{-7}$ H / m is the magnetic permeability of vacuum and μ_z is a magnetic dipole moment of the permanent magnet. We will assume that the permanent magnet has a form of a cylinder.

The model of the coil is given by the equation:

$$U_c = L_c \frac{di}{dt} + R_c i \quad \Rightarrow \quad \frac{di}{dt} = \frac{1}{L_c}(U_c - R_c i), \tag{3}$$

where R_c and L_c are the resistance and inductance of the coil. The input of the coil is the voltage $U_c(t)$ while the output is the current $i(t)$. First, we make the substitute

$$i(t) = i_0 + \hat{i}(t), \quad U_c(t) = U_{c0} + \hat{U_c}(t), \tag{4}$$

where i_0 and U_{c0} are the initial current and voltage while the $\hat{i}(t)$ and the $\hat{U_c}(t)$ are the deviations. Then, the transfer function of this part is

$$G_{elec.}(s) = \frac{\hat{I}(s)}{\hat{U_c}(s)} = \frac{1}{Ls + R_c}. \tag{5}$$

If we neglect the viscous resistive forces the movement along the Z-axis is described by

$$m \frac{d^2 z}{dt^2} = F_z - mg, \tag{6}$$

where m is the mass of the magnet and g is the gravitational constant. Here we make a similar substitute as before $z(t) = z_0 + \hat{z}(t)$, where z_0 is the equilibrium state. One can calculate the initial current i_0 from expression $F_z = mg$, while the initial voltage is $U_{c0} = i_0 R_c$. We linearize the force F_z at the point z_0

$$m \frac{d^2 \hat{z}}{dt^2} = a\hat{z} + b\hat{i}, \quad a = \frac{1}{m} \frac{\partial F_z}{\partial z}(i_0, z_0), \quad b = \frac{1}{m} \frac{\partial F_z}{\partial i}(i_0, z_0) \tag{7}$$

The Laplace transform of this equation is $s^2 \hat{Z}(s) = a\hat{Z}(s) + b\hat{I}(s)$. The resulting transfer function of the magnetic levitation system is

$$G_o(s) = \frac{1}{Ls + R_c} \cdot \frac{b}{s^2 - a}. \tag{8}$$

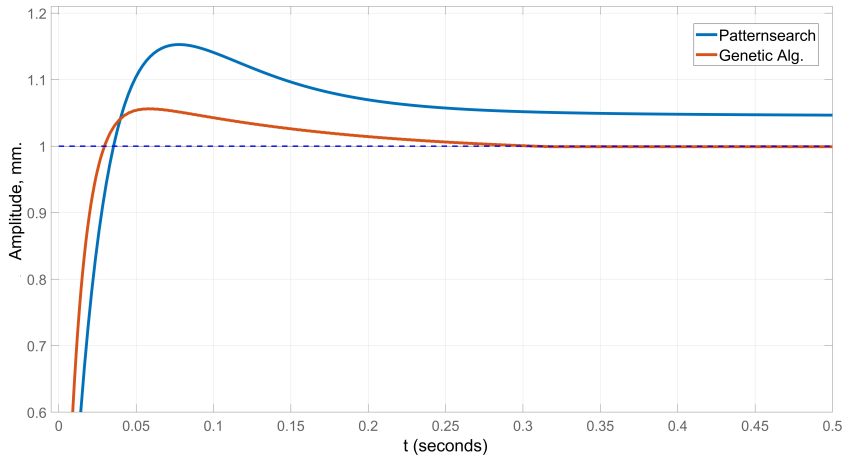


Figure 1: A comparison of step responses with lowest value of $f_{obj.}(\vec{\eta})$ for maximum of $N = 235$ objective function evaluations.

The transfer function of the PID controller in a parallel form is $G_c(s) = K_p + K_i/s + K_d \cdot s$, where K_p, K_i and K_d are positive real values. For a unity feedback loop, the transfer function of the closed-loop system is

$$T(s) = \frac{Z(s)}{Z_d(s)} = \frac{G_c(s) \cdot G_o(s)}{1 + G_c(s) \cdot G_o(s)} = \frac{A(K_d s^2 + K_p s + K_i)}{a_3 s^4 + a_2 s^3 + (a_1 + AK_d)s^2 + (a_0 + AK_p)s + AK_i}, \quad (9)$$

where $Z_d(s)$ and $Z(s)$ are the Laplace images of the desired and actual positions of the permanent magnet. We choose the vector of step-response characteristics to consist of percentage overshoot and settling time $\vec{\eta} = (PO,\ T_s)$. The input parameters vector $\vec{\chi}$ consists of four variables $\vec{\chi} = (R,\ K_p,\ K_i,\ K_d)$. The objective function

$$f_{obj.}(\vec{\eta}) = \frac{PO}{100} + \frac{T_s}{10}, \quad (10)$$

takes into account the mentioned step response characteristics.

3 RESULTS

The aim of numerical tests is to compare pattern search and genetic optimization algorithms. We assumed the coil's resistance and inductance to be $R_c = 16.1$ Ohm and $L_c = 52.1$ mH, while the mass of the magnet is $m = 3$ grams. We begin by setting up lower and upper bounds. The radius $R \in [0.75, 7.5]$ in centimeters, while the PID controller parameters are $K_p \in [25, 250]$, $K_i \in [20, 200]$ and $K_d \in [1, 10]$. Testing of the search algorithms was conducted using Matlab's Global Optimization Toolbox [8]. Table 1 has the results of the tests while Figure 1 presents the optimal step responses.

Table 1: Comparison of performance of pattern search for maximum of $N = 235$ objective function evaluations.

Alg. type	R, cm	K_p	K_i	K_d	PO, %	T_s, sec.	$f(x)$
Pattern search	1.8	131	23.1	10	15.3	2.47	0.39
Genetic Alg.	4.1	69.8	90.5	9.33	5.63	0.17	0.07

4 CONCLUSION

Table 1 and Figure 1 show that the two non-gradient search methods have found rather different solutions. The solution found with genetic algorithm is characterized by a much lower overshoot and way shorter settling time than the one acquired with pattern search. On the other hand, the coil's mean radius found with pattern search is approximately two times lower. This may play a key factor when considering real world applications since the size of the coil compartment may be limited.

5 Acknowledgements

This research was co-funded by ARIS—Slovenian Research and Innovation Agency, grants P2-0270 and P1-0222.

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Decay Stability in Graph Products and Trees: New Insights

Darja Rupnik Poklukar[1] and Janez Žerovnik[1,2]

[1] Faculty of Mechanical Engineering, University of Ljubljana

[2] Rudolfovo—Scientific and Technological Center, Novo mesto

darja.rupnik-poklukar@fs.uni-lj.si

Abstract

Generalized closeness centrality and decay stability have emerged as useful tools in the analysis of network structures. This contribution briefly reports our recent results that are disproving two conjectures on the decay stability of Cartesian and strong product graphs. In the study summarized, a full characterization of decay-stable trees is provided. We also highlight key structural properties, confirm one conjecture and refute another, and support our claims with theoretical results and illustrative examples.

1 Introduction

Closeness centrality is a classical measure for node importance in graphs. However, its sensitivity to distant nodes has led to the development of *decay-based* variants. One such measure is the *generalized closeness*:

$$\mathcal{C}_\alpha(v_i, G) = \sum_{j \neq i} \alpha^{d(v_i, v_j)}, \qquad 0 < \alpha < 1,$$

with corresponding graph-level centrality

$$\mathcal{C}_\alpha(G) = \sum_i \mathcal{C}_\alpha(v_i, G).$$

Decay stability examines whether the ranking of vertices by generalized closeness remains invariant as the decay factor α varies. For more details and references to related work, we refer to [2]. The paper [2] explores decay stability in graph products. More recently, decay stability of trees was studied in [3]. Our results include both disproofs of earlier conjectures and asymptotic analysis of decay behavior in trees.

In this extendend abstract, after summarizing the main results of [2, 3], we also provide a result of simulation on a sample of random trees showing clear evidence that the proportion of stable trees among all trees vanishes with growing n, the number of vertices (see Fig. 1).

2 Decay Stability and Node Ordering

The results of [2, 3] are in part based on the following observation that was elaborated in [2], and may be of independent interest. For example, for extending the research considering decay stability of some other families of graphs.

The generalized closeness of a vertex v can be written as a polynomial, where the coefficients correspond to the vertex's *distance-degree sequence* (DDS), $(d_1, \ldots, d_k)$, with d_i being the number of vertices at distance i from v. Thus,

$$\mathcal{C}_\alpha(v) = \sum_{i=1}^{k} d_i \alpha^i.$$

Real roots of pairwise differences $\mathcal{C}_\alpha(v_i) - \mathcal{C}_\alpha(v_j)$ in the interval $(0, 1)$ are termed *decay thresholds*. A graph is *decay-stable* if its vertex ranking remains unchanged for all $\alpha \in (0, 1)$.

In [2], we define a partial order on the vertex set based on their DDS:

Definition 2.1. *Let $u, v \in V(G)$, and let $k = \min\{e(u), e(v)\}$. Then $u \succeq v$ if and only if*

$$\sum_{i=1}^{j} x_i \geq \sum_{i=1}^{j} y_i \quad \text{for all } j = 1, \ldots, k,$$

where $DDS_G(u) = (x_1, \ldots, x_k)$, $DDS_G(v) = (y_1, \ldots, y_k)$.

This relation defines a partial order. If it is total, we have:

Theorem 2.2. *[2] A graph G is decay-stable if and only if $\succeq$ defines a linear (total) order on $V(G)$.*

Additionally we prove:

Theorem 2.3. *[2] If there exist $u, v \in V(G)$ such that $e(u) < e(v)$ and $\deg(u) < \deg(v)$, then G is decay-unstable.*

3 Decay Stability in Graph Products

In [1], the following two conjectures were proposed regarding decay stability of product graphs:

Conjecture 3.1 ([1])**.** *For all $l, k_1, \ldots, k_l \in \mathbb{N}$, the grid $P_{k_1} \square \cdots \square P_{k_l}$ is decay-stable.*

Conjecture 3.2 ([1])**.** *The strong product $P_k \boxtimes P_l$ is decay-stable for all $k, l \in \mathbb{N}$.*

We provide counterexamples to both, see [2].

Example 3.3. *In the Cartesian product $P_8 \square P_3$, take $u = (2, 2)$, $v = (3, 3)$. Then:*

$$DDS(u) = (4, 5, 3, 3, 3, 3, 2), \quad DDS(v) = (3, 5, 5, 4, 3, 2, 1).$$

The partial sums violate the relation $u \succeq v$ and $v \succeq u$, hence the graph is decay-unstable.

Example 3.4. *In $P_6 \square P_3$, vertices $u = (2, 2)$, $v = (3, 3)$ yield:*

$$DDS(u) = (4, 5, 3, 3, 2), \quad DDS(v) = (3, 5, 5, 3, 1),$$

showing again a violation of the ordering, confirming decay instability.

In [2], we also proved that

Proposition 3.5. *The Cartesian product $P_k \square P_l$, for $k > l > 2$ and $k + l \geq 8$, is decay-unstable.*

Proposition 3.6. *The strong product $P_k \boxtimes P_l$, for $k > l > 2$ and $k + l \geq 8$, is decay-unstable.*

4 Decay Stability in Trees

In contrast to grid-like graphs, trees exhibit different stability behavior. In [1], the following was conjectured:

Conjecture 4.1 ([1])**.** *Almost every tree is decay-unstable.*

We provide a full proof of this statement and give a constructive characterization of decay-stable trees.

Proposition 4.2. *[3] Let T be a tree and let c be its central vertex. If $\deg(c) < \deg(v)$ for some $v \in V(T) \setminus \{c\}$, then T is decay-unstable.*

4.1 Characterization via Construction

In [3] we propose a systematic construction of rooted trees with given eccentricity a and order b. The tree is built level-by-level from the root, embedding a fixed path of length a. A Prüfer sequence-like method ensures all trees with the required properties are generated.

We introduce a classification based on local monotonicity of degrees, see [3]:

Definition 4.3. *A rooted tree from this construction is* non-monotonic *if there exists a vertex whose degree exceeds that of its parent. Otherwise, it is* monotonic.

Let $T(a, b)$ be the number of trees constructed with eccentricity a and b vertices, and $T_{\text{non}}(a, b)$ the number of non-monotonic ones.

Proposition 4.4. *[3]*

$$\lim_{b\to\infty} \frac{T_{\text{non}}(a, b)}{T(a, b)} = 0.$$

That is, almost all trees with fixed eccentricity are monotonic.

This supports our final asymptotic result:

Theorem 4.5. *[3] Let $\mathcal{T}_n$ be the number of unrooted trees with n vertices, and $\mathcal{T}_n^{\text{stab}}$ the number of decay-stable ones. Then*

$$\lim_{n\to\infty} \frac{\mathcal{T}_n^{\text{stab}}}{\mathcal{T}_n} = 0.$$

This result is further supported by simulation (Figure 1), which visualizes the proportion of stable trees under the construction.

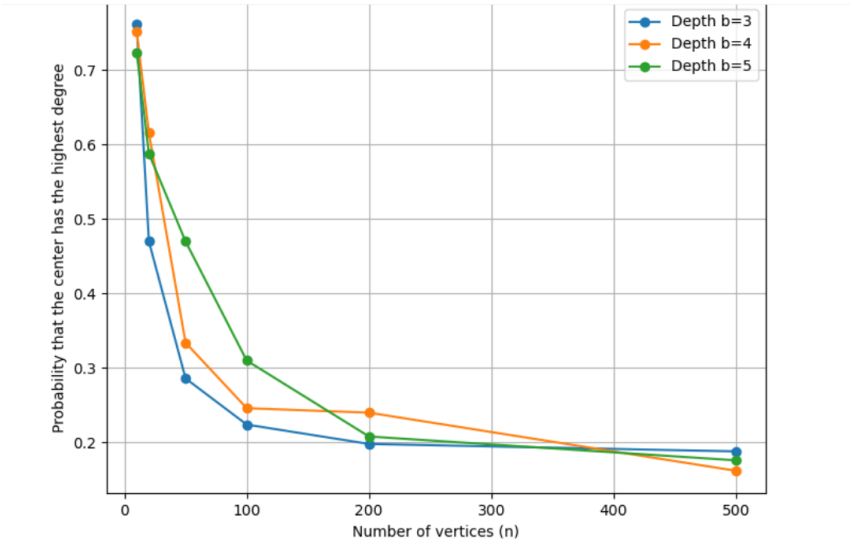


Figure 1: Simulation of decay stability in rooted trees with fixed eccentricity. Most constructed trees exhibit decay instability, in line with theoretical predictions.

5 Conclusion

We have disproved two conjectures on the decay stability of graph products and proved that almost all trees are decay-unstable. Our analysis combines algebraic tools such as distance-degree sequences with asymptotic and constructive methods. Further work will address enumeration and recognition of decay-stable trees of small order and applications in hierarchical network modeling.

The authors acknowledge financial support from the Slovenian Research Agency (ARIS) through research programme P2-0248 and projects L1-60136 , N1-0278, and J1-4031. The second author (J.Ž.) was also partially supported by ARIS through the annual work program of Rudolfovo.

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On 2-domination and 2-rainbow domination of cylindrical graphs

Janez Žerovnik[1,2]

[1] Rudolfovo - Science and Technology Centre Novo Mesto, Podbreznik 15, 8000 Novo mesto, Slovenia.

[2] University of Ljubljana, Faculty of Mechanical Engineering, Aškerčeva cesta 6, 1000 Ljubljana, Slovenia;

Abstract: Cylindrical graphs and torus grid graphs are naturally constructed from subgraphs of the infinite grid by certain identifications of boundary vertices. Considering various domination type problems, it is usually possible to find an optimal solution on the infinite grid. To the contrary, exact values of invariants for the cylindrical and torus grid graphs are typically only known for special subfamilies, and are in general hard to compute. The 2-domination and 2-rainbow domination of cylindrical graphs is studied, and some new formulae and improved bounds are reported. We also consider weak 2-domination and singleton rainbow domination.

Keywords: 2-domination, weak 2-domination, singleton 2-rainbow domination, cylindrical graphs

1 Introduction

Domination of graphs is a popular topic in graph theory [10]. There are many variations of the basic domination, see e.g. [11] Here we are interested in 2-domination and in rainbow domination, more precisely in 2-rainbow, singleton 2-rainbow domination, and weak 2-domination. Two among them are well-known and extensively studied in the literature. The k-rainbow domination problem was initially explored in [2], for a recent survey see [1]. The 2-domination is a special case of k-domination, a variation that has been studied in the literature for a long time [5].

As it is well known that the domination problems mentioned above are NP-hard in general [1, 13], it interesting to study classes of graphs for which efficient algorithms may exist. In some cases, it is even possible to find exact formulae for infinite families of graphs. An example of such family are the cylindrical graphs $P_m \Box C_n$ that have recently enjoyed significant interest. Recent work regarding the domination of cylindrical graphs includes the following. Improved lower bounds for basic domination problem were provided recently in [8]. The best so far lower bound for the 2-domination number of cylindrical graphs is proven in [13]. The computation of the 2-domination number of cylindrical graphs for arbitrary paths and small cylinders was addressed in [7], and for cylinders with small paths and arbitrary cycles in [6].

In this exteended abstract of the presentation at the SOR 25 conference, we are recalling the results that are mainly proved in [16].

2 The results

Recently, Martinez et al. [13] have provided a lower bound for 2-domination of the Cartesian product of a path P_m, $m \geq 16$, and a cycle C_n, $n \geq 16$:

$$\gamma_2(P_m \Box C_n) \geq \frac{(m+2)n}{3} \tag{1}$$

In the same paper, they note that for $m \geq 8$ and $n \geq 3$, $n \equiv 0 (\mathrm{mod} 3)$, it holds

$$\gamma_2(P_m \Box C_n) = \frac{(m+2)n}{3} \tag{2}$$

The paper [16] provides an alternative proof of (1) that at the same time generalizes the result. The is valid for smaller m, i.e. $m \geq 4$, and it also holds for some other closely related variations of the 2-domination. We will use the following inequalities

$$\gamma_{w2}(G) \leq \gamma_{r2}(G) \leq \tilde{\gamma}_{r2}(G) \qquad \text{and} \qquad \gamma_{w2}(G) \leq \gamma_2(G) \leq \tilde{\gamma}_{r2}(G) \tag{3}$$

where $\gamma_{w2}(G)$, $\gamma_2(G)$, $\gamma_{r2}(G)$, and $\tilde{\gamma}_{r2}(G)$ denote the weak 2-domination number, 2-domination number, 2-rainbow domination number, and singleton 2-rainbow domination number, respectively. The r-rainbow domination and weak 2-domination were first studied in [2], singleton domination was defined in [4], while the k-domination appears in the literature much earlier [5]. For definitions see Section 3.

Also, [16] gives a constructive proof of the upper bound for singleton 2-rainbow domination

$$\tilde{\gamma}_{r2}(P_m \Box C_n) \leq (m+2)\left\lfloor \frac{n}{3} \right\rfloor + (n^2 \bmod 3)m\,. \tag{4}$$

On the other hand, a lower bound (5) for weak 2-domination is shown to be

$$\gamma_{w2}(P_m \Box C_n) \geq \frac{(m+2)n}{3}\,. \tag{5}$$

These two bounds, together with the inequalities among the domination varieties (3) give us improved upper and lower bounds for cylindrical graphs. In some cases, exact values are obtained (see Theorem 2.1).

Theorem 2.1 *For $m \geq 4$ and $n \equiv 0 \pmod 3$,*

$$\gamma_{w2}(P_m \Box C_n) = \gamma_2(P_m \Box C_n) = \gamma_{r2}(P_m \Box C_n) = \tilde{\gamma}_{r2}(P_m \Box C_n) = \frac{(m+2)n}{3}\,. \tag{6}$$

The bounds are summarized in the next theorems. We write them as separate assertions to emphasize the possible independent importance of each of them.

Theorem 2.2 *For $m \geq 4$ and $n \geq 3$. Then for the weak 2-domination mumber γ_{w2} the 2-domination mumber γ_2, the 2-rainbow domination mumber γ_{r2}, and the singleton 2-rainbow domination mumber $\tilde{\gamma}_2$ of cylindrical graph $P_m \Box C_n$) we have*

$$\frac{(m+2)n}{3} \leq \gamma_{w2}(P_m \Box C_n) \leq \begin{array}{c} \gamma_2(P_m \Box C_n) \\ \gamma_{r2}(P_m \Box C_n) \end{array} \leq \tilde{\gamma}_2(P_m \Box C_n) \leq (m+2)\left\lfloor \frac{n}{3} \right\rfloor + \begin{cases} 0 & \text{if } n \bmod 3 = 0 \\ m & \text{if } n \bmod 3 \neq 0 \end{cases}$$

3 Definitions

A finite, simple and undirected graph $G = (V(G), E(G))$ is given by a set of vertices $V(G)$ and a set of edges $E(G)$. Edges are pairs of vertices. As usual, the edge $\{i, j\} \in E(G)$ is shortly denoted by ij.

The Cartesian product of two graphs, $G \Box H$, is the graph with vertex set $V(G) \times V(H)$, in which two vertices are adjacent if and only if they are equal in one coordinate and adjacent in the other. The Cartesian product of graphs is one of the standard graph products [9]. The Cartesian product is commutative. In other words: $G \Box H$ is isomorphic to $H \Box G$. In Cartesian product, copies of each factor appear as subgraphs and are called layers. In $G \Box H$, there are $|V(G)|$ isomorphic copies of H and $|V(H)|$ isomorphic copies of G.

The path on m vertices P_m is usually defined as the graph on vertices $V(P_m) = \{i \mid 0 \leq i < m\}$ and the set of edges $E(P_m) = \{ij \mid 0 \leq i < m-1, j = i+1\}$. The cycle on n vertices C_n is usually defined as the graph on vertices $V(C_n) = \{i \mid 0 \leq i < n\}$ and the set of edges

$E(C_n) = \{ij \mid 0 \leq i < n, j \equiv i+1 \pmod{n}\}$. Here we are mainly interested in *cylindrical graphs* that are Cartesian products of a path and a cycle, $P_m \Box C_n$.

A set of vertices S is a *dominating set* of G if every vertex in the complement $V(G) \setminus S$ is adjacent to a vertex in S. The minimum cardinality of a dominating set of G is called the *domination number*, $\gamma(G)$. There are many variations of the basic domination, see e.g. [11]. Here we are interested in 2-domination and in rainbow domination, more precisely in 2-rainbow, singleton 2-rainbow domination and weak 2-domination.

3.1 Variations of domination

A *2-dominating set* D is a set of vertices such that each vertex not in D has at least two neighbors in D. The *2-domination number* $\gamma_2(G)$ is the number of vertices in a smallest 2-dominating set for G.

A *k-rainbow dominating function* (kRDF) f of G assigns subsets of $\{1, 2, \ldots, k\}$ to vertices, such that for vertex v with $f(v) = \emptyset$, $\bigcup_{u \in N(v)} f(u) = \{1, 2, \ldots, k\}$. The *weight* $w(f)$ of kRDF f is $w(f) = \sum_{v \in V(G)} |f(v)|$. The minimum weight of a kRDF of G is the *k-rainbow domination number* denoted by $\gamma_{rk}(G)$.

An interesting special case are kRD functions that assign only singletons and empty sets. Such functions are called *singleton kRD functions* (SkRDF) and the minimal weight obtained when considering only SkRD functions is the *singleton k-rainbow domination number*, $\tilde{\gamma}_{rt}$ [4]. Clearly, $\gamma_{rk} \leq \tilde{\gamma}_{rk}$.

Weak 2-domination was introduced in [2] as an auxiliary notion in the study of 2-rainbow domination on trees. A function $g : V(G) \to \{0, 1, 2\}$ is called a *weak 2-domination* function (W2DF) if it has the following property. For any vertex with $g(v) = 0$ it holds $\sum_{u \in N(v)} g(u) \geq 2$. The *weight* of g is $w(g) = \sum_{v \in V} g(v)$ and the *weak 2-domination number* of G, $\gamma_{w2}(G)$, is the minimum weight over all W2DF of G. Obviously, any 2-rainbow domination function f gives rise to a weak 2-domination function g defined as $g(v) = |f(v)|$. In words, $g(v)$ is the cardinality of the set of colors assigned by f. Thus clearly $\gamma_{w2} \leq \gamma_{r2}$.

Furthermore, it is straightforward to see that weak 2-domination is also a relaxation of 2-domination as any 2-domination function is also a weak 2-domination function. On the other hand, a weak 2-domination function is a 2-domination function only if $f(v) \leq 1$ for all $v \in V$. Consequently, $\gamma_{w2} \leq \gamma_2$.

Similarly, any singleton 2-rainbow domination function is a 2-domination function, while the opposite is not true in general, so we have $\gamma_2 \leq \tilde{\gamma}_{r2}$ [4].

4 Conclusions

As already mentioned, we do not see a straightforward avenue that would give an improvement of the construction that leads to the general upper bound in our theorems. We conjecture that, at least in some cases, the upper bound is indeed the exact value of at least some of the variations of domination studied here. The challenge which may be an interesting research question is to prove this conjecture. At least for small enough m, the algebraic approach [12] (see also [13, 6]) may be the tool of choice. For small n, the algebraic approach may be even more useful as it is not neccessary to store the whole auxiliary matrix in the memory, since it is enough to work with only one row of the matrix, as observed in [14, 17].

Another interesting research task may be to handle the twisted products, i.e. the graph bundles [15] that are obtained from the infinite grid by nontrivial identifications. When generalizing the cylindrical graphs to bundles, the only nontrivial case is to take the path as the fibre, and use the only nontrivial automorphism of the path, that is reflection, to obtain bands that are discretizations of the Moebius band. It is obvious that the lower bound proven here

applies, and we conjecture that the lower bound will be strict in this case, similarly as it holds for graph bundles of cycles over cycles [3].

5 Acknowledgements

The research was co-funded by the Slovenian Research and Innovation Agency (ARIS) through the annual work program of Rudolfovo. The author also acknowledges partial support of ARIS research grants P2-0248 and J1-4031.

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The 18th International Symposium on
Operational Research in Slovenia

SOR '25

Bled, SLOVENIA
September 24 - 26, 2025

Special Session 2:

Agriculture and Agricultural Economics

THE VALUE OF ENDOGENOUS ABOVE EXOGENOUS INFORMATION IN IRREVERSIBLE ENVIRONMENTAL DECISIONS

Giuseppe Attanasi
Sapienza University of Rome & Corvinus University of Budapest
E-mail: giuseppe.attanasi@uniroma1.it

Aldo Montesano
Bocconi University Milan & Accademia dei Lincei Rome
E-mail: aldo.montesano@unibocconi.it

Abstract: This paper introduces the concept of the Testing Value into the analysis of environmental decisions under uncertainty and irreversibility. It is defined as the value of endogenous above exogenous information. We start from a situation where information concerning future economic benefits and costs of resource preservation is exogenous. We show that if information can also depend on the level of development carried out (i.e., it may be acquired also endogenously) the Testing Value could push a risk-neutral decision maker to preserve more in the present and eventually in the future. Although its existence stems from endogenous information, surprisingly enough, the Testing Value can be positively related to the probability of acquiring information exogenously.

Keywords: Environmental Preservation, Irreversibility, Exogenous and Endogenous Information, Value of Information, Testing Value.

A HYBRID MODELLING APPROACH TO WINEGROWING UNDER CLIMATE STRESS: INTEGRATING MACHINE LEARNING AND SYSTEM DYNAMICS

Maja Borlinič Gačnik
University of Maribor, Faculty of Tourism
Cesta prvih borcev 36, 8250 Brežice, Slovenia
E-mail: maja.borlinic@um.si

Črtomir Rozman
University of Maribor, Faculty of Agriculture and Life Sciences
Pivola 10, 2311 Hoče, Slovenia
E-mail: crt.rozman@um.si

Andrej Škraba
University of Maribor, Faculty of Organizational Sciences
Kidričeva cesta 55a, 4000 Kranj, Slovenia
E-mail: andrej.skraba@um.si

Boris Prevolšek
University of Maribor, Faculty of Tourism
Cesta prvih borcev 36, 8250 Brežice, Slovenia
E-mail: boris.prevolsek@um.si

Martina Senekovič
University of Maribor, Faculty of Agriculture and Life Sciences
Pivola 10, 2311 Hoče, Slovenia
E-mail: martina.senekovic@student.um.si

Abstract: This paper presents a hybrid modelling approach combining machine learning and system dynamics to analyze climate impacts on viticulture in Slovenia. Based on long-term weather data and over 70 000 wine ratings, we explore the influence of temperature on wine quality and vine biomass. Results show peak growth at 25 °C, with decline under heat stress (>35 °C). Regional factors strongly affect wine quality. The integrated framework enhances both predictive accuracy and explanatory insight, supporting adaptive vineyard management under climate stress.

Keywords: climate change, viticulture, wine quality, machine learning, system dynamics, Slovenia

1 INTRODUCTION

Viticulture is among the agricultural sectors most sensitive to climate change, given the strong dependence of grapevine physiology and wine quality on temperature, precipitation, and seasonal dynamics (van Leeuwen et al., 2019; Baltazar et al., 2025). Global warming, manifested through rising average temperatures and an increasing frequency of extreme heat events, has already begun to alter traditional winegrowing patterns in Europe (Lima et al., 2023). These shifts affect the phenological stages of the vine — such as budburst, flowering, and ripening — which in turn influence grape composition and, ultimately, wine quality (Rienth et al., 2021).

In Slovenia, a country with three distinct wine regions and a centuries-long winemaking tradition, the impact of climate variability has become increasingly evident. The diversity of microclimates, topographies, and varietal structures makes it a suitable setting for studying complex agroecological dynamics. Recent studies indicate that optimal temperatures for vine growth and wine production are being exceeded more frequently in several subregions, threatening both yield stability and quality standards (Vršič et al., 2024; de Rosas et al., 2022).

Traditional analytical approaches have made significant progress in identifying correlations between weather variables and wine quality. However, they often struggle to capture systemic interactions, feedback structures, and threshold behaviors typical of ecological systems (Naveed, 2024). To address these limitations, this paper adopts a hybrid modelling approach combining machine learning (ML) and system dynamics (SD). While ML techniques allow for pattern recognition and prediction based on historical data, SD modelling provides an explanatory framework for exploring causal relationships and dynamic processes within the viticultural system (Rouf et al., 2021; Borlinič Gačnik et al., 2023).

The main objective of this study is to investigate how rising temperatures affect two critical outcomes in viticulture: wine quality and vineyard biomass development. The novelty of this work lies in the integration of predictive and systemic tools to support evidence-based and context-sensitive climate adaptation strategies in viticulture. Through the use of long-term weather data, wine rating records, and simulation models, we aim to offer insights that are both empirically grounded and practically relevant for vineyard management.

2 DATA AND METHODS

This study integrates multiple data sources and modelling techniques to assess the effects of climate variability on viticulture. The dataset comprises over 70 000 expert wine ratings and daily meteorological records from Slovenian winegrowing regions between 2011 and 2021. The wine quality data were aggregated annually and linked to corresponding seasonal temperature trends.

Machine learning models, including Random Forest and Support Vector Machines, were employed to identify the most influential predictors of wine quality. Key variables included average seasonal temperatures, number of hot days, and regional origin. To complement predictive modelling, a system dynamics model was developed to simulate vineyard biomass development under varying temperature regimes. The model incorporates temperature thresholds, feedback loops, and growth limitations based on empirical findings.

This combined approach enables both accurate prediction and dynamic interpretation of how viticultural systems respond to climatic stressors.

3 RESULTS

The results from machine learning models confirm the strong influence of geographical origin and temperature variables on wine quality. Among all predictors, the regional classification emerged as the most important factor, followed by summer average temperature and the number of days exceeding 30 °C. Both Random Forest and SVM models achieved high predictive accuracy ($R^2 > 0.80$), indicating that weather variables, when combined with regional identifiers, provide reliable estimates of wine quality ratings. Figure 1and Table 1 illustrates the importance ranking of key predictors, highlighting the dominance of regional origin and seasonal temperatures.

Table 1: Scores - sampling type - Stratified 10-fold Cross validation

Model	***MSE***	***RMSE***	***MAE***	***MAPE***	***R2***
Random Forest	0.023	0.153	0.119	0.007	0.779
Tree	0.032	0.178	0.133	0.008	0.701
SVM	0.029	0.171	0.134	0.008	0.725

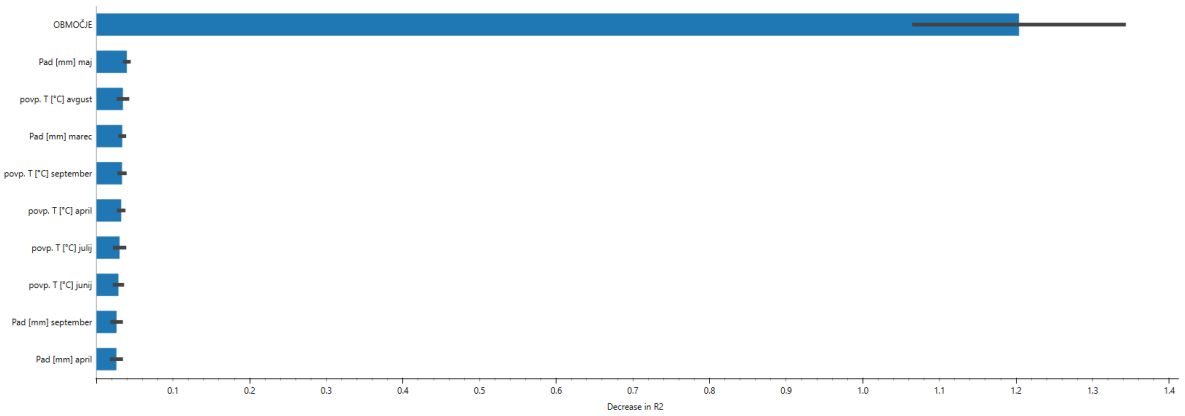


Figure 1: Importance of predictors in the Random Forest model

The analysis further showed that temperature variations in April, August, and September had a statistically significant influence on wine quality, pointing to the sensitivity of early vegetative and late ripening stages. Precipitation patterns had a weaker but still observable effect, particularly in the spring months, consistent with the importance of water availability during the flowering period.

System dynamics simulations provided a complementary perspective, focusing on vineyard biomass responses to temperature changes (see Figure 2). The model reveals a clear non-linear relationship: biomass increases with temperature up to an optimal threshold near 25 °C, beyond which growth begins to decline. In scenarios with prolonged heatwaves (>35 °C), biomass dropped significantly, simulating conditions of stress-induced growth inhibition. These findings align with recent physiological studies on vine temperature tolerance and validate the relevance of dynamic modelling for scenario analysis. Figure 2 presents a simplified causal loop diagram that visualizes the feedback structure governing vine biomass development.

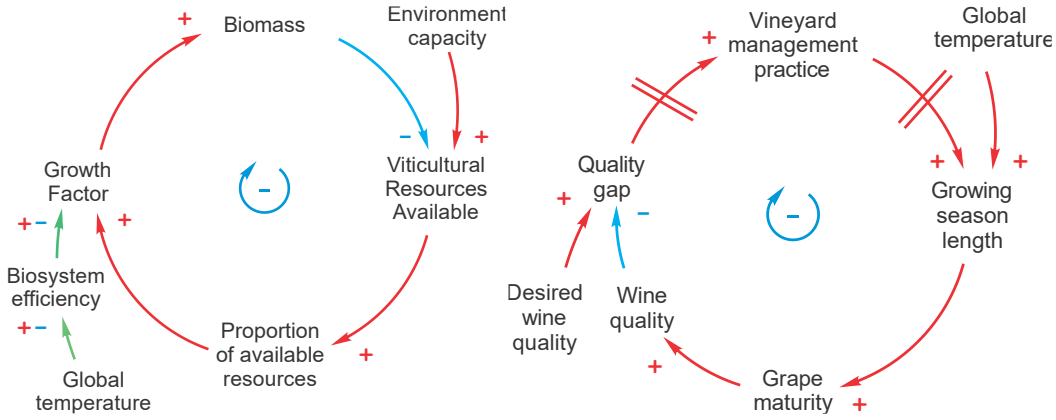


Figure 2: System dynamics model (CLD models) of vineyard biomass under heat stress

Together, the two modelling approaches offer both predictive and explanatory insights. While ML identifies key drivers of wine quality, SD explains how cumulative and delayed effects of heat impact biomass. This combination improves the robustness of climate impact assessments and enables more nuanced, region-specific recommendations for vineyard management.

4 DISCUSSION AND CONCLUSIONS

This study demonstrates the value of hybrid modelling in addressing complex agricultural challenges related to climate change. By integrating machine learning and system dynamics,

we provide both predictive accuracy and system-level understanding of how viticulture responds to rising temperatures.

One of the main advantages of this approach lies in its complementarity. While ML excels at identifying significant predictors and forecasting outcomes, SD captures time-dependent processes and systemic feedbacks that are often overlooked in static analyses. Together, they offer a more comprehensive toolkit for adaptive management in viticulture.

Importantly, the integration of regional variables into predictive models reveals the embedded influence of microclimatic, pedological, and agronomic factors not captured through meteorological data alone. This finding supports the call for more locally grounded adaptation strategies in the face of global climate trends.

However, some limitations remain. The system dynamics model does not yet account for soil moisture variability, pest pressures, or socio-economic factors influencing vineyard decisions. Additionally, the wine quality data, although extensive, may contain subjective elements and regional aggregation biases.

Despite these limitations, the proposed framework offers practical utility. It supports scenario-based planning for vineyard adaptation, identifies critical temperature thresholds for vine health, and enables communication of climate risks to stakeholders in an accessible way. Such tools are essential for designing resilient agricultural systems in regions already experiencing climatic stress.

Future research could expand the model to include more ecological variables (e.g., evapotranspiration, drought indices) and socio-economic dimensions (e.g., labor availability, cost structures), and test its applicability across different wine-producing countries. There is also potential to integrate stakeholder input through participatory modelling approaches to co-create adaptation pathways.

Ultimately, this interdisciplinary approach aligns with the increasing need for transparent, data-driven tools to support sustainable transitions in agriculture under climate pressure.

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OPTIMIZING CATTLE FEED RATIONS UNDER FARM-LEVEL CONSTRAINTS: A LP APPROACH USING THE SITFARM MODEL

Jure Brečko
Agricultural institute of Slovenia, Department of Agricultural economics
Hacquetova 17, 1000 Ljubljana, Slovenia
E-mail: jure.brecko@kis.si

Jaka Žgajnar
University of Ljubljana, Biotechnical faculty, Department of Animal Science
Groblje 3, 1230 Domzale, Slovenia
E-mail: jaka.zgajnar@bf.uni-lj.si

Abstract: This study addresses the optimization of feed rations using the SiTFarm bio-economic model for a typical Slovenian cattle fattening farm. The model integrates linear programming to minimize feed costs while satisfying the animals' nutritional requirements and simultaneously calculating key farm-level economic indicators. Three scenarios were analysed: (i) cost minimisation without additonal constraints, (ii) optimisation under farm production constraints, and (iii) adaptation to yield losses resulting from production risks. The results indicate gross margins of 16.4 €/h in the baseline scenario, 14.2 €/h under farm production constraints, and 6.5 €/h in the risk scenario, underscoring the farm system's vulnerability to feed availability and price volatility. The model employs two-stage optimization (ration cost-minimization followed by farm profit-maximization) to formulate cattle feed rations that balance nutritional requirements, homegrown forage availability, and market volatility. The linear programming approach demonstrates how mathematical optimization can enhance feeding decisions under production constraints, supporting economically viable livestock management amid climatic and price uncertainty.

Keywords: farm model, linear programming, economic indicators, feed ration optimization

1 INTRODUCTION

Feed costs are an important determinant of profitability in dairy and beef production systems, accounting for up to 70 % of total expenses [5]. Therefore, formulating rations at the lowest feasible cost–while still meeting animals' nutritional requirements–offers significant potential to improve farm profitability, often surpassing the impact of other cost-saving measures. Enhancing feed efficiency not only increases economic returns but may also stimulate further investment in production systems [3]. However, cost reduction strategies must prioritize the fulfilment of dietary needs, as inadequate nutrition can adversely affect productivity and health. In cattle fattening, this is particularly critical, as rations must provide a carefully balanced supply of protein and energy to support optimal growth and feed conversion [4]. To support the development of nutritionally adequate and economically efficient rations, modeling approaches are recommended. Such models quantify the relationships between nutrient content and dietary composition, framing feed formulation as an optimization problem. Solving this problem yields cost-effective and precise ration formulations that maximize animal performance while minimizing feed cost [6]. Economic optimization models for agro-ecological systems–referred to as bio-economic farm models (BEFMs)–are commonly used to simulate such decision-making environments. These models integrate biological resource use with economic decision rules through mathematical programming techniques [2]. One of the BEFMs applied in this analysis is the SiTFarm model, which was developed and specifically adapted to the production conditions of Slovenia [8]. The applied approach in the model enables the formulation of feed rations that simultaneously account for the daily nutritional constraints and recommendations specific to each fattening phase. It also incorporates the

annual availability of home-grown forages, which form the basis of actual on-farm feeding practices.

2 FARM MODEL

For our research, SiTFarm model [7] based on model calculations (MC) prepared by Agricultural Institute of Slovenia is used [1]. SiTFarm utilises mathematical programming techniques and features a modular structure, with distinct components dedicated to sequential analytical stages. The optimization framework within SiTFarm is based on two-stage linear programming (LP). In the first stage, feed rations are optimised to minimize costs subject to nutritional constraints. In the second stage, the model maximizes the farm's overall gross margin by allocating resources across all production activities, while respecting the constraints defined in the first stage and other farm-specific limitations.

This single-objective approach reflects practical agricultural decision-making by prioritizing economic returns, subject to the biological and resource constraints inherent in farming systems. The production possibility matrix, derived from this process, illustrates how optimal production plans are formulated through maximization of the objective function–gross margin–based on farm-specific characteristics and operational boundaries. The sequential design of the model ensures that cost-effective, nutritionally adequate rations are established before evaluating whole-farm profitability, thereby capturing the nested and hierarchical nature of decision-making in our analysis.

LP solutions exhibit high parameter sensitivity, where price fluctuations drastically alter optimal rations. However, feed availability depends fundamentally on land-use ratios (arable vs. grassland) and agronomic capacity rather than market economics. Thus, ration feasibility is constrained more by physical production limits than by economic optimization potential.
The linear programming model used in this study is formulated as follows:

$$\max EGM = \sum_{j=1}^{n} c_j x_j \tag{1}$$

$$\sum_{j=1;f=1}^{n;r} a_{ij} x_j \leq b_i \qquad \text{for all } i = 1 \text{ to } m \tag{2}$$

$$x_j \geq 0 \qquad \text{for all } j \tag{3}$$

where n—number of activities included in the production plan; m—number of constraints in production plan; j—sequential production activity; i—sequential constraint in the model; b_i—limitation of a single resource; a_{ij}—technological coefficients per production activity; c_j—objective function coefficients; x_j—known activities.

2.1 Typical cattle fattening farm with 60 animal heads and arable land

This study aims to demonstrate the methodological framework for feed ration optimization using the SiTFarm model, highlighting its analytical capabilities through a comparative assessment of different scenarios. The analysis quantitatively evaluates the effects of different feed formulation strategies on key economic indicators. The analysis quantitatively evaluates three feed optimization paradigms: (i) unconstrained nutritional optimization, formulating cheapest ration based solely on meeting cattle's dietary requirements without consideration of on-farm feed production constraints; (ii) production-constrained optimization, which adjusts ration formulation to reflect the actual feed production capacities of farm, and (iii) risk-adjusted optimization, which reformulates rations in response to agricultural damage events, such as yield loss, that reduce on-farm feed availability. Each scenario is analyzed for its effects on production costs and overall farm profitability. Prices of agricultural inputs are monitored and updated within the AIS framework and are recalculated for the 2022-2024 reference period [1].

The model-based analysis is conducted on a representative case study farm–a typical cattle fattening holding (TAH)–previously designed and described in a separate study [8]. The farm specializes in beef cattle production and crop cultivation for feed, including barley, silage maize, grass silage, and hay.

3 RESULTS AND DISCUSSION

A summary of the farm's production structure is presented in Table 1. It manages 6.1 ha of arable land and 9.9 ha of permanent grassland, with the additional capacity to rent land. The farm is semi-mechanized and operates with a labor input equivalent to 0.5 full-time equivalent (FTE). This case study farm is representative of semi-professional, medium-sized agricultural holdings in Slovenia. It operates at a scale below the threshold typically associated with fully commercial fattening farm enterprises. Labor requirements are modest and can be met through part-time work, often supplemented by family labor, making off-farm employment feasible. While the farm generates sufficient income to support incremental modernization, structural limitations and restricted capital flow characteristic of smaller-scale farms present ongoing challenges for substantial reinvestment and expansion.

Table 1: Production plan for analysed typical cattle fattening farm

Production activities	*Unit*	*Scope (unit)*	*Weight gain/Yield (kg/ha)*	*$GM_{2022\text{-}2024}$(€)*
Bulls for fattening	(heads)	60	1.3	1 043
Barley	(ha)	2.7	5 500	–1 171
Corn for silage	(ha)	4.1	50 000	–1 749
Grass silage	(ha)	9.5	19 429	–1 244
Hay	(ha)	2.3	7 907	–1 235
Utilized agric. area	(ha)	22.4		

In addition, we present selected economic indicators for all three scenarios analyzed using the SiTFarm model. This comparison highlights the significance of various constraints - whether stemming from the inherent biophysical limitations of a specific farm type (Scenario 2), or from external shocks such as natural disasters that necessitate adaptive management strategies (Scenario 3). The analysis focuses on two primary model outputs: (i) the structure of the annual feed ration, derived from the first stage of optimization, and (ii) selected economic performance indicators at the whole-farm level, generated in the second optimization stage. By isolating these two dimensions, we illustrate how feed formulation constraints–either structural or risk-induced–can substantially influence farm profitability and production planning.

Table 2: Feed ration quantities and economic indicators for typical cattle fattening farm

Feed ration quantities	*Baseline (BL)*	*Scenario 1*	*Scenario2*
Corn for silage	3 414	3 085	2 560
Grass silage	3 881	4 416	2 717
Hay	300	300	200
Corn	717	757	1 200
Specific feed for cattle fattening	478	445	702
Econ. indicators (€) Revenue	135 463	135 645	134 744
Variable costs	118 640	120 802	128 683
Gross margin	16 823	14 842	6 061
Gross margin/h (€/h)	16.4	14.2	6.5

The farm achieves relatively modest economic performance within the cattle fattening sector, despite generally favorable market conditions during the 2022-2024 analysis period, which was marked by elevated livestock purchase prices. Table 2 presents the annual feed ration quantities and key economic indicators for each scenario. The SiTFarm model conducts sequential annual optimizations, allowing the production plan to adjust while respecting agronomic constraints - particularly those related to crop rotation.

The Baseline scenario, which assumes theoretically unrestricted feed availability, yields the highest economic performance with a gross margin of 16.4 €/h, as expected. In Scenario 1, production-constrained optimization is applied, with feed rations tailored to the specific resource endowment and structural characteristics of the TAH. This scenario requires increased purchases of corn, leading to a slightly lower gross margin of 14.2 €/h. This part of the analysis clearly illustrates the annual-level impact per fattening animal. Given the natural conditions and resource endowments of the farm, the optimized ration includes a higher proportion of grass silage and a reduced share of corn silage. This shift reflects the model's capacity to align feed composition with on-farm forage production potential, while maintaining nutritional adequacy. The adjustment emphasizes the relevance of site-specific feed resource availability in determining economically viable and agronomically realistic feeding strategies. Scenario 2 models production risks through yield loss simulation, necessitating supplemental concentrate purchases to meet nutritional requirements. This reduces gross margin to 6.5 €/h, as increased variable costs (+8.5%) erode profitability despite stable revenues. The results demonstrate how adverse growing conditions and yield shocks propagate through production systems via forced feed substitutions, directly compromising farm economic viability. In this scenario, the reduction in forage yield remained within the farm's capacity to compensate by reallocating available production resources and purchasing supplemental feed. However, if forage losses were to exceed this compensatory threshold, the farm would be forced to reduce its livestock numbers in order to maintain nutritional sufficiency and economic viability. This highlights the vulnerability of forage-dependent production systems to climatic and agronomic shocks, and the critical importance of adaptive capacity in maintaining operational stability.

4 CONCLUSIONS

This study presents an analytical example demonstrating how feed ration composition significantly influences economic performance, while simultaneously showing how farm-specific conditions can alter the composition and, consequently, the cost structure of the ration. Using the LP, we first formulated a cost-minimizing, nutritionally adequate daily ration that also respects annual farm-level constraints, such as forage availability. In the second step, we applied gross margin maximization (LP) to derive the optimal production plan for a typical cattle fattening farm. The results reveal that, despite meeting daily nutritional requirements (BL), the resulting annual feed composition can diverge substantially from the actual on-farm forage production capacity. This discrepancy underlines the importance of integrating both nutritional standards and site-specific resource limitations in ration planning (Scenario 1) to ensure economically and operationally viable outcomes.

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THE IMPACT OF CLIMATIC FACTORS ON CROP YIELDS IN THE REPUBLIC OF SRPSKA

Nemanja Jalić
The University of Banja Luka, Faculty of Agriculture,
Bulevar Vojvode Petra Bojovića 1A, 78000 Banja Luka, Bosnia and Herzegovina
nemanja.jalic@agro.unibl.org

Maja Ralić
The University of Banja Luka, Faculty of Agriculture,
Bulevar Vojvode Petra Bojovića 1A, 78000 Banja Luka, Bosnia and Herzegovina
maja.ralic@student.agro.unibl.org

Karmen Pažek
The University of Maribor, Faculty of Agriculture and Life Science,
Pivola 10, 2311 Hoče, Slovenia
karmen.pazek@um.si

Črtomir Rozman
The University of Maribor, Faculty of Agriculture and Life Science,
Pivola 10, 2311 Hoče, Slovenia
crt.rozman@um.si

Aleksandar Ostojić
The University of Banja Luka, Faculty of Agriculture,
Bulevar Vojvode Petra Bojovića 1A, 78000 Banja Luka, Bosnia and Herzegovina
aleksandar.ostojic@agro.unibl.org

Željko Vaško
The University of Banja Luka, Faculty of Agriculture,
Bulevar Vojvode Petra Bojovića 1A, 78000 Banja Luka, Bosnia and Herzegovina
zeljko.vasko@agro.unibl.org

Abstract: This study aimed to analyze the impact of temperature and precipitation on agricultural crop yields in the Republic of Srpska. Data on crop yields and climatic variables from 2000 to 2021 were obtained from official statistical and meteorological sources. Regression and correlation analyses were applied to assess the relationships between yields of cereals, fruits, and vegetables and climate parameters during key growing months. Results indicate that cereals are significantly affected by summer temperatures and precipitation, while fruit crops show sensitivity mainly to spring temperatures. Vegetable yields showed limited dependence on climate variables, likely due to irrigation practices. These findings highlight the importance of targeted climate impact assessments to support sustainable agricultural planning in the region.

Keywords: temperature, precipitation effects, yield variability, crops

1 INTRODUCTION

Climate risks, driven by increasingly intense climate change, have become some of the most critical threats to agricultural production in the Republic of Srpska and globally. Bosnia and Herzegovina (BiH) is among the countries considered highly vulnerable to climate change (Brooks et al., 2005). Marković et al. (2012) analyzed temperature and precipitation data (2000–2005) for the Banja Luka region, finding frequent summer droughts despite adequate annual precipitation. Their results highlight that irregular rainfall distribution and high temperatures cause water deficits in the root zone, reducing crop productivity. Trbić et al. (2016) projected rising summer temperatures and declining precipitation for 2001–2030, particularly affecting spring–summer crops, increasing irrigation demand, and reducing yields

in northern BiH. Over the past two decades, BiH has experienced a rise in extreme weather events, with spatial and temporal anomalies in temperature and precipitation leading to yield losses and material damage (Stričević et al., 2017). A case study by Petković et al. (2015–2018) in Manjača linked dry periods to reduced green mass and hay yields, confirming the role of precipitation variability in forage production. Čereković et al. (2024) found that favorable water regimes significantly improved corn yields in two locations in BiH. Trbić et al. (2022) emphasized the need for fundamental shifts in fruit production planning, land use, and water resource management under changing climate conditions. Zurovec et al. (2015) advocated for the development of weather and climate information systems, crop improvements, and enhanced irrigation strategies. Ćustović et al. (2012) highlighted medium-term adaptation measures such as improving soil fertility, while long-term strategies include crop rotation changes, drought-resistant cultivars, shelterbelts, land-use changes, and irrigation infrastructure development. Majdančić et al. (2016) analyzed the impact of monthly temperature and precipitation on winter wheat yields in BiH and the Tuzla Canton. Similar challenges and the broader impacts of climate on agriculture in BiH and the Western Balkans have also been addressed by Trbić et al. (2018); Popov et al. (2019); Dejanović et al. (2019). The aim of this study is to assess the impact of climate variability and change on agricultural production in the Republic of Srpska.

2 MATERIALS AND METHOD

The dependence of agricultural crop yields on precipitation and temperature was examined using regression and correlation statistical analysis. Data were obtained from the Statistical Office and the Hydrometeorological Institute of the Republic of Srpska, covering the period from 2005 to 2020 for cereals, and from 2000 to 2021 for fruits and vegetables. The use of different time series is due to the availability of data in official databases. Only regression models that were found to be statistically significant at the $p < 0.05$ level are presented in the results, with preference given to those with higher coefficients of determination. The dependency assessment was conducted using a trial-and-error approach within regression analysis. For each trial, crop yield values in the territory of the Republic of Srpska were used as dependent variables, while precipitation and temperature values for key months were used as independent variables. In certain regression models, the data series were logarithmically transformed using the natural base ($e = 2.72$).

3 RESULTS AND DISCUSSION

Only models that showed statistical significance using the trial-and-error method are presented in the results. Notably, none of the analyzed crops exhibited dependence on annual averages or total sums of precipitation and temperature. Instead, statistically significant relationships were found for specific months and phases of the growing season.

Table 1. The Impact of Temperature and Precipitation on Cereal Yields (2005-2020)

Dependent variable Y (yield)	Independent variable X	Regression equation	R-squared (R^2)	Significance of the model
Wheat	Sum of temp. in June and July	Y = 12.49–0.22 lnX	0.27	0.037
Corn	Sum of temp. in May and June	Y = 0.73+0.59X	0.54	0.01
Corn	Total precipitation in June, July, August	Y = -1.25+1.72 lnX	0.31	0.024
Barley	Total precipitation in May and June	Y = -0.70+1.02 lnX	0.27	0.024

Wheat yield is most sensitive to temperature levels during June and July. A negative correlation was observed in years with higher cumulative temperatures in these months were associated with lower wheat yields, while cooler years recorded higher yields. Corn yield showed the greatest sensitivity to the cumulative temperature in May and June. In this model, 54% of yield variation is explained by temperature during late spring. The positive coefficient for the independent variable indicates that higher temperatures in this period positively affect corn yield. Additionally, increased precipitation in June, July, and August leads to higher corn yields. Based on the coefficient of determination, approximately one-third of the yield variability is explained by rainfall during the mid-vegetation period, typically the driest part of the season. Unlike temperature, which showed no statistically significant influence on barley yield, precipitation was found to have a significant positive impact. The model presented in Table 1 explains 27% of the variation in barley yield. The positive slope coefficient suggests that increased rainfall during these two months corresponds with higher yields, which is consistent with empirical observations.

Table 2. The Impact of Temperature and Precipitation on Fruit and Vegetable Crops (2000-2021)

Dependent variable Y (yield)	Independent variable X	Regression equation	R-squared (R^2)	Significance of the model
Potato	Sum of temp. in May, June, July, August	Y = 94.42–28.46 lnX	0.23	0.023
Potato	Total precipitation in July and August	Y = -1.62+2.68 lnX	0.17	0.048
Plum	Sum of temp. in April and May	Y = -85.08+29.86 lnX	0.22	0.029
Pear	Sum of temp. in April and May	Y = -58.65+21.98 lnX	0.18	0.047
Pear	Sum of temp. in February and March	Y = 17.24–0.50X	0.23	0.023
Pear	Precipitation in May	Y = 2.00+0.68X	0.43	0.000
Sweet cherry	Sum of temp. in April and May	Y = -64.27+23.71 lnX	0.19	0.039
Sour cherry	Sum of temp. in April and May	Y = -27.86+11.13 lnX	0.17	0.049
Apple	Sum of temp. in February and March	Y = 20.04–0.47X	0.22	0.027

According to the regression analysis, potato yield is negatively affected by increased temperatures from May to August, indicating that lower temperatures during this period are more favorable. As expected, higher precipitation in July and August positively influences potato yield. Elevated temperatures in April and May have a favorable effect on the yields of plum, pear, sweet cherry, and sour cherry, with an average coefficient of determination of around 20%. Increased precipitation in May significantly benefits pear yield, as indicated by a high R-squared value of 0.43. In contrast, higher temperatures in February and March negatively affect the yields of both pear and apple, likely due to early flowering and the occurrence of late frosts that damage fruit development.

4 CONCLUSIONS

Based on the research conducted in the territory of the Republic of Srpska during the last two decades, it can be concluded that precipitation and temperature affect different crops in different ways, depending on the phase of the growing season. Field crops, grown entirely outdoors, are most influenced by temperature and precipitation, particularly during the summer months. According to this study, fruit crops are generally less affected by precipitation, except for May precipitation, which has a positive effect on pear yield. Higher temperatures in April and May positively influence the yields of plum, pear, sweet cherry, and sour cherry, while elevated temperatures in February and March harm apple and pear yields due to early flowering and late frost damage. Except for potato, no statistically significant influence of precipitation and temperature was found on vegetable crops, likely because these

are commonly irrigated and often grown in protected environments. A recommendation for further research is to analyze climate and yield data at the regional level within the RS/BiH, where climate impacts on yields are likely more pronounced.

Acknowledgements: This research is partly based on the results of a completed master's thesis research "Project risk management with a focus on agriculture", student Maja Ralić, who is one of the co-authors of the paper.

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DECISION SUPPORT SYSTEM FOR GARDEN CENTERS: A RULE-BASED APPROACH TO IN-STORE CUSTOMER GUIDANCE

Martin Konestabo
Proventus d.o.o., Ajdovščina, Slovenia
martin.konestabo@tomappo.com

Jože Kropivšek
Biotechnical Faculty, University of Ljubljana, Ljubljana, Slovenia
joze.kropivsek@bf.uni-lj.si

Bojan Blažica
Department of Knowledge Technologies, Jožef Stefan Institute, Ljubljana, Slovenia
Proventus d.o.o., Ajdovščina, Slovenia
bojan.blazica@ijs.si

Abstract: We present the design of a lightweight decision support system (DSS) deployed in garden centers to assist non-expert customers in selecting appropriate plant nutrition products. Accessed via QR codes, the system simplifies decision-making using a structured decision tree. Insights from customer surveys and staff interviews informed the domain knowledge and interface logic. This work demonstrates the feasibility of in-store DSS tools that enhance customer experience and help the staff.

Keywords: decision support systems, urban gardening, product recommendation, retail technology, ICT in agriculture

1 INTRODUCTION

Urban and hobby gardening has grown in popularity, driven by its recreational, ecological, and health benefits. However, the diversity of available gardening products presents a challenge to less experienced customers. Many product types—such as fertilizers, or crop protection agents—require prior knowledge for correct use. Labels and packaging often provide insufficient guidance, while the availability of expert sales advice varies seasonally and is affected by staff turnover. This paper investigates how lightweight decision support tools, integrated into the physical retail environment of a garden center, can alleviate these issues. The goal is not to find the theoretically optimal solution for a customer's need, but to offer practical guidance within the constraints of available inventory.

DSS have long been applied in domains such as agriculture, healthcare, and retail to support structured or semi-structured decision-making. In agriculture, modern DSS support tasks ranging from fertilizer selection to pest control and land-use planning [1]. Notable examples include the Soil Navigator [3], which helps farmers optimize soil functions, and the IPM Adviser platform [2], which for integrated pest management decision support. In consumer-focused domains, lightweight and mobile DSS have proven valuable where decisions are frequent but users lack deep domain expertise. A prominent example is the VešKajJeš app [4], which simplifies food label interpretation through visual nutrition scoring.

To identify where digital support is most needed garden centers, we conducted a survey among users of a gardening app and run semi-structured interviews with garden center staff. In response, we designed and implemented a mobile-accessible DSS for one key product category: plant nutrition. The DSS is accessed via QR codes next to products on shelves and leads users through a structured selection process. This paper presents the development and early evaluation of this system.

2 METHODOLOGY AND SYSTEM DESIGN

2.1 Needs Assessment

To identify the most pressing decision-making challenges faced by hobby gardeners, we conducted a survey among 196 users of the Posadi.si (Tomappo) mobile gardening app [5]. The results 1 revealed that product categories such as fertilizers, plant protection agents, and growing media were the most problematic for users to select independently. The main difficulties included unclear labeling, insufficient knowledge, and a lack of advice from sales staff.

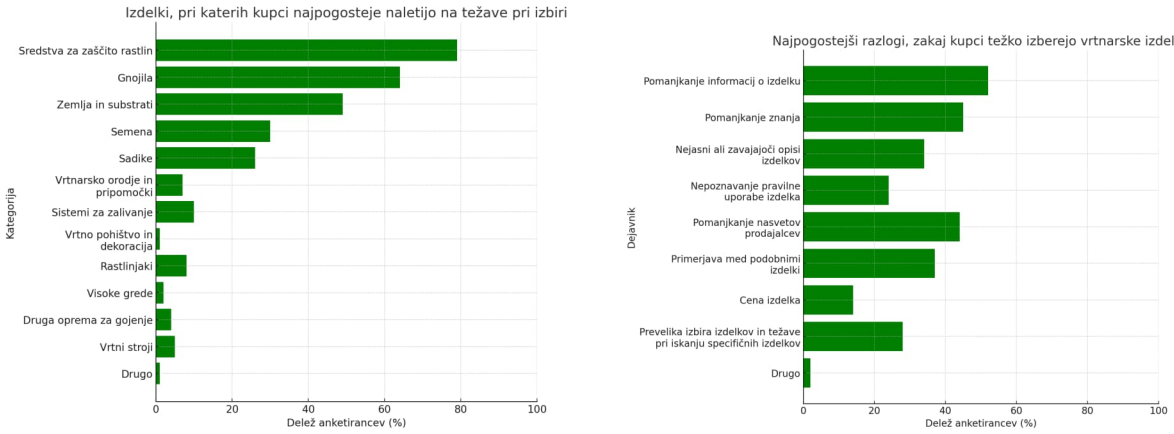


Figure 1: Survey responses (n=196) showing which product types hobby gardeners find most difficult to choose from (left) and why (right).

To understand the issue from the perspective of garden center staff, we conducted five semi-structured interviews with employees of a garden center chain in Slovenia. The findings highlighted several key challenges: customers most often require assistance with selecting fertilizers, plant protection products, and seeds, with the greatest difficulties arising in understanding plant protection product dosages and fertilizer types. Sellers reported relying heavily on practical experience, peer knowledge, catalogs, and manufacturer trainings, with formal education playing a lesser role. Customers base their decisions primarily on quality, price, personal experience, and brand recognition, often trusting familiar brands when unsure. From informal conversations with practitioners in the industry additional systemic constraints emerge: seasonal overloads, high staff turnover, onboarding complexity, and the lack of standardized knowledge about new or niche products. Even experienced staff reported difficulty keeping up with changes.

2.2 DSS Design and Implementation

Based on the identified needs, we developed a lightweight DSS focused on plant nutrition products. The system architecture consists of a React-based front end tailored for mobile use and a Node.js server backend linked to structured Excel-based product data. The DSS is accessible via QR codes displayed on product shelves, bridging the gap between physical inventory and digital guidance.

The decision logic is implemented as a rule-based tree with a shallow depth. Users are guided through 3-4 steps (Figure 2): selecting a plant category, specifying a target plant or plant group, selecting fertilizer type (e.g., organic vs. mineral), and optionally identifying desired product specific effects (e.g., improved rooting or disease resistance). The goal is to reduce the decision space quickly and provide one or more matching products from the current shop inventory. This structured filtering approach allows for transparent and explainable recommendations that are easily adaptable to changing product lines.



Figure 2: Screenshots of the mobile decision support system interface: four filtering steps leading from the initial list of 120 products to a final product recommendation.

2.3 User Testing

To evaluate the usability of the DSS, we conducted preliminary testing using the think-aloud protocol. Participants were asked to complete specific product selection tasks while verbalizing their reasoning, expectations, and any confusion encountered. Common themes identified during testing included the importance of clear question framing, the need for visual progress indicators, and the value of seeing the logic behind recommendations. The shallow structure of the decision tree was generally well received, and participants appreciated not needing to install a mobile application.

3 DISCUSSION AND FUTURE WORK

Our findings highlight the need and feasibility of a lightweight DSS for effective assistance of non-expert customers in purchase decisions in garden centers. By aligning decision logic with structured product catalogues and integrating the tool seamlessly into the in-store environment via QR codes, we lowered the barrier to digital in-store customer assistance. An additional benefit is the system's potential to support sales personnel, particularly new or seasonal staff, by acting as a reference for consistent, catalogue-aligned recommendations. This is especially relevant in a sector where staff turnover and product volatility are persistent challenges.

However, the current implementation has several limitations. The system is constrained to operate within the set of products available at a specific garden center and does not assess the intrinsic quality of the products. Recommendations are thus descriptive rather than evaluative. Additionally, the approach requires manual curation of product attributes, which limits scalability and responsiveness to inventory changes.

To address these constraints, future work will explore the following directions:

- **Recommendation quality enhancement:** Incorporating simple plausibility checks based on product composition (e.g., verifying consistency between declared effects and ingredient profiles) or integrating crowd-sourced feedback mechanisms.
- **Broader product coverage:** Expanding the DSS to other product categories such as pest control or pruning tools, where choice overload is similarly high.
- **Automated DSS generation:** Leveraging large language models (LLMs) to extract

structured attributes from online catalogues or manufacturer websites, enabling rapid deployment of similar DSS tools for different retailers or product lines.

- **Analytics and personalization:** Capturing usage patterns to refine system flow and adapt recommendations to user profiles or seasonal trends.

Overall, the results suggest that even simple ICT tools, when integrated into physical retail workflows, can enhance product decision-making and improve the quality of both customer and staff experience.

4 CONCLUSION

This paper presented the design, implementation, and preliminary evaluation of a mobile, rule-based DSS deployed in garden centers to assist customers with product selection. Developed in response to identified challenges from both users and sales staff, the system offers structured, catalogue-informed guidance for choosing plant nutrition products and is accessible via QR codes directly in-store. Although limited in scope, the tool demonstrates how even simple ICT interventions can support better decisions in retail environments. Future work will focus on automating system generation, validating product claims, and expanding to other product categories. These directions aim to further scale the benefits of decision support in contexts where customer expertise is limited and choice complexity is high.

5 ACKNOWLEDGEMENTS

The study was supported by the Slovenian Research Agency (grant P2-0103). We would like to thank the staff from the garden center chain KGZ Sloga Kranj and the users of the Posadi.si (Tomappo) application for their participation in interviews and testing.

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BENCHMARKING MODEL EVALUATION OF HOP INDUSTRY ECONOMICS

Martin Pavlovič
University of Maribor, Faculty of Agriculture and Life sciences
Pivola 10, 2311 Hoče, Slovenia
E-mail: martin.pavlovic@um.si

Darija Trpin Švikart
University of Maribor, Faculty of Agriculture and Life sciences
Pivola 10, 2311 Hoče, Slovenia
E-mail: darija.svikart@telemach.net

Abstract: On benchmarking workshops in 2025 input data about production technology and costs as well as revenues related to cultivation and marketing of hops were analyzed from 8% of farms in Slovenia for the 2024 growing season. We illustrate the three key economic parameters related to the competitive value of hop farms analyzed. The average revenue amounted to 10,463 EUR/ha, the average variable costs were assessed as 6,476 EUR/ha and the average gross margin was calculated as 4,169 EUR/ha. Results illustrate also average group and farms' individual data on yields and hop price as well as major cost groups related to farms analyzed.

Keywords: hop industry, farms, costs analysis, simulation model, benchmarking workshops, Slovenia.

1 INTRODUCTION

Hops are an international trading commodity and without hops, beer cannot be made, causing inelastic demand [2]. For export-oriented farms, the analytical monitoring and verification of business performance is essential for effective management and planning farm development. Existing reference sources in model assessment of agricultural production economics include a wide variety of operations research opportunities. The economic efficiency of agricultural production is determined by (a) the yield, (b) the price achieved and (c) payments from CAP measures/subsidies on the revenue side, and (d) production costs on the expenditure side. For hop growing, a capital and labor-intensive industry, the model calculation method of total production costs is more appropriate than the calculation method of gross margin [3].

Farmers can obtain an accurate estimate of the revenues and costs of a sector production by participating in the so-called workshops for benchmarking production economics at farm level. These workshops represent a form of group advisory work in agriculture [6]. Market-oriented farms, ready to cooperate and exchange their own experiences, can voluntarily join the industry circle.

In 2021, so called "sector/industry group/circle" (panožni krožek) in hop growing has been introduced. We economically evaluated the hop production technology using the simulation model - by analyzing farms' revenue, variable costs and gross margin.

2 MATERIALS AND METHODS

In the research, we used the simulation model PKH for economic analysis of hop production economics from the hop industry circle. We compared the technological and economic data of the 10 hop farms with the data of the calculated group average. For the participating Slovenian farms, we used data for the 2024 growing season from two workshops, which took place in February 2025.

2.1 Model PKH for assessment of economics at farm level

The simulation model PKH is based on the revenue inputs, analysis of variable production costs and the calculation of the gross margin (EUR/ha) of a hop production. The model offers pre-prepared standard input data for material, machine and energy costs - to be updated by farmers if needed, while the remaining data is completed by the hop growers themselves based on their notes and estimates [4]. The model has its six worksheets:

- To enter basic data about the hop production farm, year of data processing, agricultural land in use, hop field areas, variety structure and number of employees on the farm.
- To enter data of variable costs (costs of materials/services, hired labor and energy in hop production).
- To input revenues from hop sales, from potential insurance compensation for natural disasters and from the CAP (Common Agricultural Policy) subsidies related to a hop industry at farm level.
- To calculate the variable costs of using machines and attachments in hop production.
- The model PKH offers a calculation of the costs of hop production for the farm in the production year analyzed. The entire calculation with revenues, variable costs and gross margin is shown, which illustrates the economics of production. This calculation is then the basis for reviewing the data in the second workshop with the same farmers participating.
- To link data between different farms participating in workshops. The data is arranged in columns, where each farm obtains a code (CE1... CE10), which hop growers then use to identify themselves when interpreting the results of the model analysis of the participating farms.

2.2 Workshops and processing of data

A group of 10 farmers participated in research at the workshops held in 2025. At the first workshop, each hop grower had his own computer with a pre-prepared file with the model PKH for entering data from his own farm. Together, the farmers had entered their own technological and economic data on the revenues and expenses of hop production into the model in a guided manner [5].

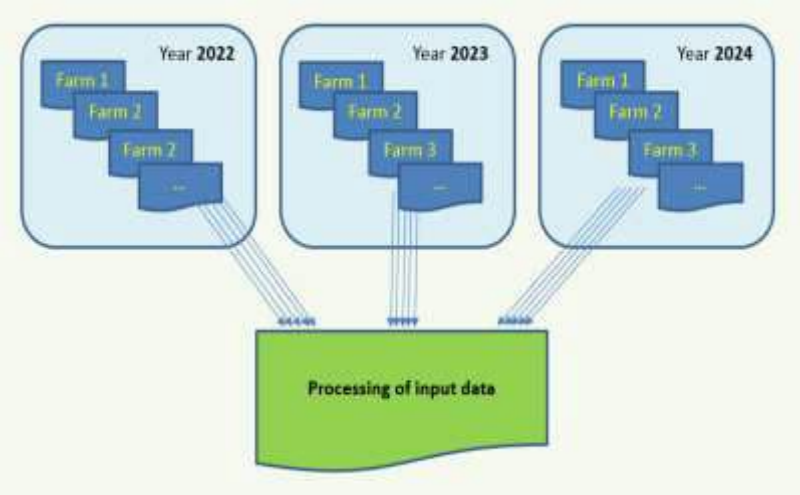


Figure 1: System links of Excel files

The workshop was followed by the analyzing of all data from the files of individual farms. The correctness of the data was checked, and the farms were coded (CE1...CE10). This was followed by a comparative analysis of the input data with an illustration of the cumulatively calculated average values according to the selected model items. In case of excessive deviation, the data were excluded from the calculation of the average.

The second workshop was intended primarily for the presentation of data from individual farms - in comparison with the average values of the group and a discussion of the results obtained. This was followed by an exchange of opinions and a group debate. Participating in hop growers looked for reasons for both poor and good results. They shared their experiences on how to improve practices on other farms.

Experts in a hop industry and economics play a key role in a debate with the intention to improve the economics of farms analyzed within the growing year as well as between the years (see Fig 1).

3 RESULTS

The 10 hop farmers in the Styrian-growing region are highly specialized with an average acreage of 15 ha of hops and with 1.83 full-time workers employed. We point out the following three key economic parameters related to competitive value of hop farms analyzed for the growing season 2024 (see Tab. 1):

- the average group revenue amounted to 10,463 EUR/ha,
- the average group variable costs were assessed as 6,476 EUR/ha and
- the average gross margin was calculated as 4,169 EUR/ha.

Table 1: Tab Benchmarking workshop results of the economic assessment of hop industry farms for 2024

RANK		5		1	2	3	4	5	6	7	8	9	10
ITEMS		Group average		Individual data (based on Gross margin 2024)									
				CE1	CE8	CE5	CE4	CE7	CE2	CE6	CE10	CE9	CE3
GROSS MARGIN	EUR/ha	4.169		169	2.111	2.233	3.653	4.970	5.055	5.110	5.265	6.382	7.081
Hop fields	ha	15		23	12	10	12	9	13	30	12	18	15
Hop Yield	kg/ha	1.553		1.624	1.490	1.678	1.161	1.487	1.346	1.858	1.455	1.643	1.780
Income from hops	EUR/ha	9.325		7.786	4.950	8.773	7.274	9.821	10.846	11.695	10.507	9.992	11.602
Price of hops	EUR/kg	6,0		4,8	3,3	5,2	6,3	6,6	8,1	6,3	7,2	6,1	6,5
INCOME total	EUR/ha	10.476		8.889	6.258	10.236	8.462	10.954	12.001	12.317	11.717	11.157	12.770
Stringing material	EUR/ha	249		260	133	374	169	322	200	348	216	193	274
Manure, Fertilizers	EUR/ha	631		687	417	737	682	631	765	477	830	600	480
Energy (drying hops)	EUR/ha	751		1.742	322	756	616	244	792	305	760	873	1.105
Plant protection material	EUR/ha	813		1.060	807	717	805	851	855	1.040	607	489	898
Labour - hired/seasonal	EUR/ha	1.718		2.142	1.165	2.239	1.103	1.879	1.617	2.281	1.925	1.069	1.736
Seasonal labour costs	EUR/h	5,9		5,2	5,5	4,1	6,0	7,1	5,4	6,4	5,7	6,1	6,7
Mechanization	EUR/ha	955		1.027	1.399	787	799	1.100	860	991	1.010	782	805
VARIABLE COSTS total	EUR/ha	6.476		9.058	5.618	8.004	4.810	5.985	6.947	7.428	6.457	4.775	5.689

Research results illustrate also average group and farms' individual data from farms coded from CE1 till CE10. Summarized, the model PKH illustrated information on average output data on acreage (15 ha), hop yield (1,553 kg/ha) and hop price (6.04 EUR/kg) as well as major hop production variable cost groups such as stringing material (249 EUR/ha), manure and fertilizers (631 EUR/ha), energy for drying hops (751 EUR/ha), plant protection material (813 EU/ha), hired seasonal labor (1,718 EUR/ha) and mechanization (955 EUR/ha) related to farms analyzed within the benchmarking workshop evaluation.

The tables with workshops' benchmarking results are also intended to determine whether the reason for the above-average successful farm in terms of gross margin (EUR/ha) is the highest yield and the highest selling price of hops. If we compare the data of all farms analyzed,

we find out that one farm may have the highest gross margin as the sum of average variable costs, above-average hop yield and slightly above-average selling price of hops. We may also conclude that the farm that is most successful in terms of gross margin may not necessarily have the highest yield per hectare and the highest selling price of hops. The level of variable costs counts and plays a key role as well.

The included farmers acknowledged benefits and have thus accepted for the fourth time (since 2021) the presented methodological approach of a group assessment of hop production economics at farms level.

4 CONCLUSIONS

Hops are an international trading commodity in global agricultural markets and beer production remains the only sector requiring hops in significant since breweries consume 98% of global hop production [1]. Hop growers in Slovenia, producing hop products in 2025 on 3% of the global acreage, face a strong business competition on global markets. Apart from a national hop industry research, development and advisory infrastructure as well as international promotion – also the economic competitiveness at farm level plays a crucial role.

Thus, the benefits of the camaraderie and freedom to participate in benchmarking workshops to analyze, to compare and to discuss the issues of production techniques and economics in agriculture, whatever they may be, far outweigh the costs and limitations of such analytical benchmarking activities. The workshop participants approved benefits and have thus accepted for the fourth time since 2021 the method of a the PKH simulation model assessment of hop production economics at farms level – in Slovenian called as “panožni krožek v hmeljarstvu”.

Acknowledgement
The authors appreciate the EIP FARM MANAGER and EKOHMELJ project teams as well as the European Agricultural Fund for Rural Development and the Republic of Slovenia for financing the research.

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THE IMPACT OF ESG STANDARDS ON THE COMPETITIVENESS OF AGRI-FOOD COMPANIES

Full. prof. dr. Karmen Pažek and **M.Sc. Teodora Marković**
Faculty of Agriculture and Life Sciences, University of Maribor
Pivola 10, 2311 Hoče, Slovenia
E-mail: karme.pazek@um.si, teodora.markovic@um.si

Abstract: This paper analyzes how ESG standards impact the competitiveness of Slovenian agri-food companies. With ESG reporting shifting from voluntary to mandatory under EU laws like CSRD, EU Taxonomy, and ESRS, companies must follow clear sustainability rules. The study compares ESG reports from 2021 to 2024, showing larger firms, especially multinationals, adopt ESG frameworks and measurable goals more than SMEs. SMEs face challenges due to limited resources and regulatory pressure. Although simplified rules like the upcoming "omnibus package" aim to help, there are concerns about reducing accountability.

Keywords: ESG standards, CSRD, ESRS, agri-food sector, competitiveness, sustainability reporting

1 INTRODUCTION

Reporting on the environmental, social, and governance (ESG) impacts of business operations,represents a significant factor for achieving competitive advantage, strengthening stakeholder relations, and improving corporate image [9]. ESG criteria encompass management of climate change and resources (E), relations with employees and the community (S), and corporate governance and transparency (G). Unlike traditional corporate social responsibility reports, ESG reports contain more quantitative and managerial information, thus better aligning with business goals [10, 13].The regulatory framework shaping the European Union's sustainable policy can be effectively explained through its core standards. The European Green Deal was introduced as a comprehensive policy initiative focused on reducing greenhouse gas emissions, promoting clean energy, improving energy efficiency, and protecting biodiversity. The Green Deal builds on the Paris Agreement and sets out concrete targets for transitioning the EU toward a green economy [3]. To ensure effective implementation of these objectives, the EU has developed several action plans that define operational steps toward achieving them [4]. This approach helps to transform political commitments such as the Paris Agreement and the European Green Deal into measurable and actionable outcomes.One of the most important legislative acts in this domain is the EU Taxonomy Regulation (2020/852), which establishes an official classification system defining which economic activities are considered environmentally sustainable. The taxonomy is designed to steer capital and investments toward businesses that preserve nature and reduce pollution, supporting the EU's ambition to become the first climate-neutral continent by 2050 [5]. For the agri-food sector, a particularly relevant legislative act is the Corporate Sustainability Reporting Directive (CSRD). This directive, which entered into force in 2023, significantly expands the scope and mandatory nature of sustainability reporting compared to the earlier Non-Financial Reporting Directive (NFRD). Under the CSRD, a large number of companies are required to report on how their business activities impact the environment, society, and corporate governance (ESG factors), including mandatory audits and compliance with standardized reporting frameworks [7].

To ensure consistency and comparability of such reporting, the European Sustainability Reporting Standards (ESRS) have been developed. These standards define in detail the information companies must disclose, including data on climate risks, emissions, human rights, resource consumption, and supply chains. The ESRS is a key tool for implementing the

CSRD and ensuring a coherent green transition across the EU [2]. In parallel with EU regulations, many companies continue to use internationally recognized frameworks such as the Global Reporting Initiative (GRI) standards, which offer comprehensive guidelines for sustainability reporting [10].

2 METHODOLOGY

This study uses a qualitative, comparative case study methodology to assess the implementation and maturity of ESG reporting in Slovenian agri-food companies between 2021 and 2025. The analysis relies on document research and content analysis based on publicly available ESG reports, environmental statements and corporate sustainability disclosures [1, 8, 11, 13–15, 17]. The aim is to assess how companies with different organizational profiles respond to regulatory developments, in particular the CSRD, ESRS and the EU Taxonomy Regulation.

A purposive sampling strategy was used to select a representative group of seven companies, including large companies (e.g. Comp. 1, Comp. 2), medium-sized companies (e.g. Comp. 3, Comp. 5) and smaller companies (e.g. Comp. 7). This stratification enables a meaningful comparison between companies that differ in size, ownership structure and international involvement. The methodological framework includes:

- Development of a coding scheme to assess six key dimensions: (1) existence and completeness of ESG reports, (2) standards used (GRI, ESRS, internal), (3) existence of measurable KPIs, (4) time-based progress tracking, (5) regulatory compliance status, and (6) integration into group-level ESG systems.
- Comparison of the information with the legal thresholds in the EU (number of employees, turnover, total assets) to determine whether de jure or de facto compliance obligations exist.
- Contextualizing the results in relation to the broader EU regulatory environment, in particular the timeline for implementation of the CSRD. An overview of the sampled companies, ESG reporting characteristics, applied standards and progress is presented in Table 1.

The results were interpreted using a thematic analysis based on a framework that draws on established ESG assessment criteria from the literature. In particular, the analytical perspective is guided by the methodology proposed by Gillan, Koch and Starks (2021) [9], who emphasize consistency of ESG content, transparency and responsiveness to stakeholder expectations as primary indicators of ESG quality. Their framework underpins the comparative structure of this study and enables informed conclusions on ESG performance and strategic positioning.

3 RESULTS AND DISCUSSION

The analysis of ESG reports of Slovenian agri-food companies for the period 2021 to 2024 shows a growing gap between large and small companies in their approach to sustainability reporting. At the same time, there is a clear trend towards greater formalization and alignment with modern European regulations. Larger companies, such as Comp. 1 and Comp. 2, are leading the way in this area. They show consistent reporting practices, a transition from the voluntary GRI standards to the more demanding ESRS framework, and clearly defined targets and key performance indicators (KPIs). This change will be particularly visible after 2023, when the CSRD comes into force, transforming ESG reporting from a voluntary choice to a legal obligation. Some companies within larger corporate groups, such as Comp. 4, also apply ESG standards (mainly GRI), but to a more limited extent and with partially defined targets. This may reflect the challenges of harmonizing reporting within complex group structures

Table 1: Overview of ESG Reporting by Companies (2021–2024)

Company	Year	ESG Report	Applied Standards	Measurable Goals (KPI)	Progress Over Time	Type of Report
Comp. 1	2022 – 2024	Present	GRI (2022), ESRS (2024)	Present	Present	ESG reporting
Comp. 2	2021 – 2024	Present	ESRS	Present	Present	ESG reporting
Comp. 3	2022	Partial	Not applied	Partial	No data	Overview of sustainability achievements
Comp. 4	2022 – 2023	Present	GRI	Partial	No data	Separate reports within Atlantic Group
Comp. 5	2022	Partial	Not applied	Not present	Not present	Environmental statement
Comp. 6	2024	Present	GRI	Present	No data	ESG reporting, led by the German entity
Comp. 7	2023	Present	Not applied	Not present	No data	ESG report from a small company
Comp. 8	-	Not present	Not applied	Not present	Not present	Sustainability bag

It is important to note that none of the companies surveyed are currently formally required to report under the CSRD, as most do not meet the legal thresholds in terms of company size (more than 250 employees), turnover (more than €40 million) or total assets (more than €20 million) on a consolidated basis, nor are they listed on a stock exchange. Companies that operate as part of larger international groups, such as Comp. 4, are not directly subject to the CSRD, but their data may be included in the consolidated ESG reports of their parent companies. This is the case with the parent group, which is listed on the Zagreb Stock Exchange and is subject to CSRD obligations. In contrast, smaller companies such as Comp. 5 and Comp. 7provide only basic environmental statements without a structured methodology or measurable targets. This probably reflects limited internal resources, but also less pressure from regulators and stakeholders. A special case is Comp. 6, whose reporting is managed from Germany. While the GRI Standards and certain KPIs are applied, there is a lack of time-based progress indicators, suggesting that the company is still in the early stages of developing a structured reporting system. At the other end of the spectrum is Comp. 8, which does not publish ESG reports at all, but instead occasionally reports on individual sustainability-related activities. This illustrates the difference between promotional communication and a comprehensive ESG strategy. In summary, the analysis confirms that company size, group affiliation, and regulatory engagement are important factors influencing the scope and quality of ESG reporting. The trend towards the adoption of ESRS and greater transparency is evident, but remains uneven across the industry. Nevertheless, challenges remain, particularly due to regulatory complexity and compliance costs, which are particularly burdensome for small and medium-sized enterprises (SMEs). We are waiting for the “omnibus package”, especially for SMEs. Nevertheless, this simplification package has met with criticism. Some fear that relaxing the requirements could weaken corporate accountability and slow down the EU’s progress towards its climate and sustainability goals.

4 CONCLUSION

The study shows that ESG standards are becoming increasingly important for the competitiveness of Slovenian agri-food companies. Larger companies, especially those in international groups, are leading the way in structured ESG reporting and compliance with EU regulations such as CSRD and ESRS. In contrast, SMEs face obstacles due to limited resources and regulatory complexity. While the trend towards standardized reporting is evident, implementation remains inconsistent. The forthcoming "omnibus package" could make compliance easier for SMEs, but the risk of reduced accountability remains. ESG integration is not just a regulatory requirement but a strategic advantage, and companies that proactively implement it will be better positioned for sustainable success.

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HARNESSING LAND AND SEA: STRATEGIC MODELLING OF HYBRID BIOMASS SUPPLY CHAINS

Maryam Roudneshin and **Amanda Sosa**
UCD, Ireland, 8 Glenard Hall, Frialand, Dublin 14, D14W9E5, Ireland
rudneshinmaryam@yahoo.com

Abstract: The optimisation of bioenergy supply chains is critical for achieving sustainability in both agricultural and marine resource management. This study develops an integrated spatial and mathematical optimisation framework for designing a sustainable supply chain network using agricultural residues and marine biomass, specifically seaweed. The proposed model incorporates economic, environmental, and social objectives and is solved using a hybrid method combining the augmented ε-constraint algorithm and Data Envelopment Analysis (DEA) for Pareto-efficient outcomes. A GIS-based case study in Ireland identifies optimal facility locations and transport routes while considering seasonal feedstock availability, cost dynamics, and policy interventions such as the Straw Incorporation Measure. The results demonstrate how strategic use of seaweed, combined with agri-waste, can enhance supply chain resilience, reduce emissions, and stimulate rural employment. The framework provides decision-makers with actionable insights for aligning local bioresource utilisation with national decarbonisation targets.

FRAMEWORK OF A MULTI-CRITERIA DEXI MODEL FOR HOLISTIC SUSTAINABILITY ASSESSMENT OF THE WINE SUPPLY CHAIN: THE CASE OF RADGONSKE GORICE

Martina Senekovič
University of Maribor, Faculty of Agriculture and Life Sciences
Pivola 10, 2311 Hoče, Slovenia
E-mail: martina.senekovic@student.um.si

Črtomir Rozman
University of Maribor, Faculty of Agriculture and Life Sciences
Pivola 10, 2311 Hoče, Slovenia
E-mail: crt.rozman@um.si

Maja Borlinič Gačnik
University of Maribor, Faculty of Tourism
Cesta prvih borcev 36, 8250 Brežice, Slovenia
E-mail: maja.borlinic@um.si

Abstract: The paper presents a framework for sustainability assessment of the Radgonske Gorice wine supply chain using the multi-criteria DEXi decision model. The model covers ten stages of the supply chain, integrating environmental, social, and economic criteria according to the triple bottom line principle. Indicators are assessed using discrete qualitative scales, aggregated by expert-defined rules. The paper outlines the model design, criteria selection, and a decision matrix example for the grape production stage. This approach facilitates transparent, systematic, and comparative sustainability evaluation, suitable for strategic decision-making and monitoring progress in agri-food chains.

Keywords: sustainable supply chain, DEXi, multi-criteria decision-making, wine industry, agri-food sector, sustainability indicators

1 INTRODUCTION

The concept of sustainable development is based on balancing environmental, economic, and social aspects, aiming to meet the needs of current generations without compromising future generations' abilities to meet their own needs (Brundtland, 1987). Sustainable supply chain management in the agri-food sector, particularly in the wine industry, requires integrating these three dimensions through a triple bottom line approach that combines reducing environmental impacts, ensuring economic efficiency, and promoting community welfare (Bentahar & Benzidia, 2018; Rozman et al., 2020). Additionally, the wine industry faces challenges such as climate change, limited natural resources, and growing consumer expectations regarding sustainable practices (Flores, 2018; Dressler & Paunović, 2021). This paper presents the application of the multi-criteria DEXi method to assess the sustainability of the Radgonske Gorice wine supply chain.

2 LITERTAURE REVIEW

Sustainability is typically addressed through three key pillars: environmental, social, and economic. These pillars are essential for a balanced evaluation, ensuring that no single dimension is neglected. The environmental pillar encompasses natural resource use, emission reduction, waste management, biodiversity conservation, and active mitigation of negative environmental impacts. The social pillar covers fairness, working conditions, employee safety, training and human resource development, community engagement, and satisfaction of consumers and stakeholders. The economic pillar focuses on economic efficiency, profitability,

long-term competitiveness, market stability, and the financial sustainability of the company (Santiago-Brown et al., 2015; Flores, 2018). In the DEXi model, sustainability pillars are represented through clearly defined attributes or indicators, enabling precise evaluation of each sustainability dimension and consequently ensuring transparency, traceability, and applicability of results in decision-making (Bohanec, 2008; Rozman et al., 2020).

Multi-criteria decision analysis (MCDA), such as the DEXi method, enables transparent integration of these dimensions into a comprehensive assessment (Bohanec & Rajkovič, 1999). DEXi employs hierarchically structured criteria, qualitative evaluation scales, and expert-defined rules. This approach ensures balanced sustainability assessment, as it prevents poor results in one dimension from being offset by good results in another. The model provides clear interpretation of results and facilitates their practical application in decision-making aimed at improving the sustainability of agri-food supply chains (Bohanec & Rajkovič, 1999; Rozman et al., 2020; Prišenk et al., 2024).

3 METHODOLOGY

To evaluate the sustainability of the Radgonske Gorice wine supply chain, we applied the qualitative multi-criteria **DEXi method**, enabling integration of diverse sustainability indicators through a clearly structured, hierarchical approach. The model includes ten key phases of the wine supply chain:

- Grape production
- Procurement
- Winemaking
- Packaging
- Logistics
- Sales
- Marketing and tourism
- End customers
- Local community
- Support services

Each phase was further divided into three primary sustainability dimensions**: environmental, social, and economic.** Within each sustainability dimension, specific attributes or indicators essential for assessing the performance of individual phases were identified. Environmental indicators encompass water usage, pesticide application, greenhouse gas emissions, and sustainable land management. Social indicators include assessments of working conditions, employee health and safety, worker satisfaction, and local community impact. Economic indicators focus on profitability of production, cost efficiency, income stability, and economic resilience of the company.

Criteria are evaluated using a qualitative assessment scale (poor, medium, good), where each phase is assessed according to its performance on specific indicators. Expert-defined decision rules enable a clear judgment of results, determining how individual indicator assessments combine into an overall evaluation for each phase. A negative assessment of any individual indicator can significantly lower the overall phase rating, underscoring the importance of balancing all sustainability dimensions (Bohanec, 2008; Rozman et al., 2020).
Figure 1 illustrates the hierarchical structure of sustainability assessment criteria for two key segments of the Radgonske Gorice supply chain: **grape production and procurement.** Each segment encompasses environmental, social, and economic dimensions with clearly defined indicators, including resource usage, working conditions, supplier engagement, economic costs, and profitability.

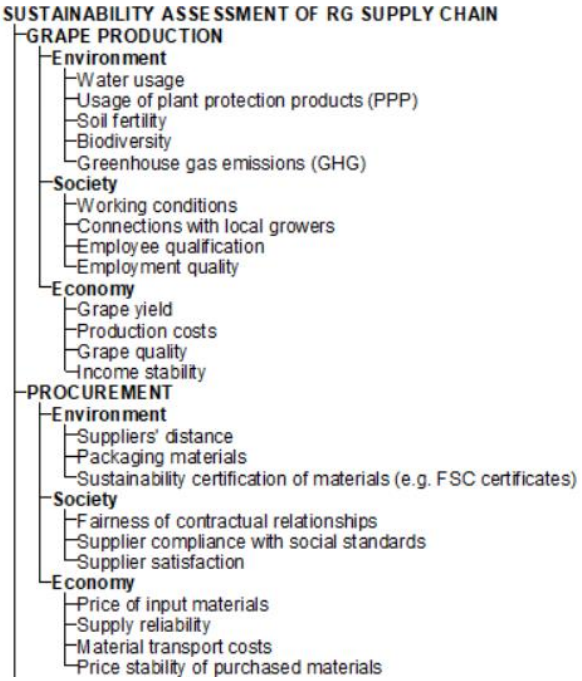


Figure 1: Sustainability assessment framework for grape production and procurement at Radgonske Gorice

4 RESULTS AND DISCUSSION

The evaluation example in Table 1 summarizes the sustainability assessment for grape production by aggregating detailed environmental, social, and economic indicators into a unified rating (Borsato et al., 2020). The applied decision rules adopt a cautious and balanced approach, ensuring that any criterion rated as "poor" significantly lowers the overall assessment. Achieving a "good" rating requires that at least two sustainability dimensions be evaluated as "good," with none rated as "poor." This structured method prevents the compensation of negative outcomes by positive evaluations in other areas, thus ensuring a holistic, transparent, and balanced sustainability assessment across all dimensions (Rozman et al., 2020).

Table 1: Decision rule matrix for assessing sustainability of grape production based on environmental, social, and economic criteria in the DEXi model.

Environment	Social	Economy	Overall sustainability assessment “Grape production”
Poor	Poor	Poor	**Poor**
Poor	Medium	Medium	**Poor**
Medium	Medium	Medium	**Medium**
Good	Good	Medium	**Good**
Good	Good	Good	**Good**

The results indicate that all dimensions must achieve at least a medium evaluation to receive a good overall sustainability rating, highlighting the importance of balanced sustainability (Rozman et al., 2020).

5 CONCLUSION

DEXi has proven to be an effective model for sustainability assessment in agri-food supply chains. The Radgonske Gorice case demonstrates the model's utility in identifying critical areas and supporting strategic decision-making by clearly defining areas with the greatest potential for improvement. The presented model serves as a comprehensive and flexible framework for practical sustainability assessment, enabling companies to transparently track progress and compare efficiency across different supply chain phases. Future work will include collecting empirical data from the company, quantifying these data, and integrating them into the DEXi model for deeper analysis and additional validation of results. Additionally, the development of an interactive decision-support tool is planned to facilitate easier tracking of results and enhance practical value, thereby enabling broader adoption within the agri-food sector.

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BIO-ECONOMIC FARM MODELLING TO ASSESS THE EFFECTS OF CAP MEASURES: THE CASE OF SUCKLER COW FARMS

Jaka Žgajnar and **Lidija Zadnik Stirn**
University of Ljubljana, Biotechnical faculty
Jamnikarjeva 101, 1000 Ljubljana, Slovenia
E-mail: jaka.zgajnar@bf.uni-lj.si, lidija.zadnik@bf.uni-lj.si

Abstract: This paper presents an analysis conducted using the SiTFarm model on a selection of representative farms specialized in beef cow production. The SiTFarm model is based on mathematical programming and allows for production plan analysis by optimizing a defined objective function. The analysis focused on evaluating the performance of selected representative farms during the period 2022–2024 and on assessing the significance of Agricultural Policy measures, with a particular emphasis on coupled support for suckler cows. The results indicate that these measures are economically highly significant and, despite reducing farm-level autonomy, contribute substantially to the economic sustainability of the farms.

Keywords: linear programming, farm model, suckler cows, common agricultural policy measures

1 INTRODUCTION

In Slovenia, the suckler cow sector plays a significant role in agriculture. This is a specific branch of livestock production, characterized by an unfavourable farm size structure. A large proportion of the sector is composed of small-scale farms, with fewer large-scale operations, and only a limited number of farms achieving solid economic performance comparable to other livestock sectors in the country. Suckler cow farms are often located in Less Favoured Areas (LFA), where permanent grassland dominates, significantly influencing production conditions, which are generally poor. In many cases, particularly among smaller farms, the sector has developed through a transformation process—former small-scale dairy farms that ceased milk production and switched to suckler cow husbandry.

The economic contribution of the suckler cow sector is generally weak. According to estimates, about 26% of all farms in Slovenia are engaged in this type of production, yet they account for only about 5% of total agricultural income [5]. Despite its modest overall economic relevance, the sector holds considerable social importance. Even more pronounced is its environmental role: farms in this sector manage approximately 42% of Slovenia's permanent grasslands. These are predominantly located in LFAs and often include mountain farms, where abandonment could result in significant negative consequences for biodiversity. Notably, not all of these farms are small in scale; for example, some large karst farms manage extensive land areas in a highly extensive production system [3].

These are the key reasons why the sector is closely monitored by agricultural policymakers. In the most recent programming period, it was granted specific coupled support measures aimed at assisting vulnerable sectors. This support is available to farmers raising at least two suckler cows, with higher payments awarded if production takes place within an LFA region (97.25 EUR per cow).

Given the socio-economic structure of these farms, it is not surprising that farm management practices often lack formal monitoring. Most farmers only maintain the mandatory records required for receiving budgetary payments. Business performance is not usually tracked through agricultural (e.g., FADN) or fiscal accounting systems. As such, assessments of farm economic performance rely on modelling approaches. Simpler and more general models—such as calculation-based tools (e.g., AIS Model Calculations)—are commonly used, although their major limitation lies in their lack of adaptability to specific farm conditions, which may lead

to significant discrepancies in results. In recent years, bio-economic farm models (BEFM) have increasingly been used for such assessments [2]. One such tool is the SiTFarm (Slovenian Typical Farm Model Tool), which is tailored to Slovenian conditions [4]. This tool employs mathematical programming to identify optimal production plans by maximizing an objective function under various farm-specific constraints and production conditions.

In this paper, we aim to investigate the role of budgetary payments for selected small, medium, and large representative farms in the suckler cow sector. We further examine the importance of coupled income support and its actual impact on the economic sustainability of these agricultural holdings.

2 MATERIAL AND METHODOLOGY

2.1 Use of the SiTFarm Model

For the purpose of analysing typical suckler cow farms in Slovenia, the SiTFarm model was employed [4]. This model, based on mathematical programming, facilitates a wide range of analyses at the level of a farm's production plan. SiTFarm is modular in design and incorporates three distinct modelling components.

The first component comprises static models of Typical Agricultural Holdings (TAHs), which represent various farming systems (i.e., production models) commonly observed in practice. Each TAH reflects the expected structure and output of a particular type of suckler cow farm, thereby serving as a representative proxy for a broader group of similar farms.

The second component involves budget models, or Model Calculations (MCs), which provide the primary source of economic and technological data at the level of individual production activities. These static models, developed independently by the Agricultural Institute of Slovenia [1], cover all major agricultural sectors—including fodder production, cash crops, vegetable production, and livestock farming. MCs allow for real-time adjustments in production technologies, yield intensity, and price-cost ratios, ensuring alignment with the specific conditions of the analysed typical farm.

The third component is the comprehensive Farm Model (FM), which integrates the TAH and MC modules. It enables the autonomous calibration of production plans based on technological parameters and each farm's specific production constraints and resource endowments. In this study, FM is applied using linear programming (LP), which serves as the analytical foundation for solving the farm's production plan. The model's production matrix captures the range of feasible activities, with the primary objective being the maximization of Gross Margin (GM).

Economic indicators presented in the analysis are based on average input prices, labour costs, and farm-gate prices from the years 2022, 2023, and 2024. Budgetary support is modelled according to the provisions of Slovenia's Common Agricultural Policy (CAP) Strategic Plan for the 2023–2027 programming period (BL Scenario). This includes both direct payments and compensatory payments for farming in areas with natural constraints (LFA). The direct payments analysed consist of non-coupled support, specifically: the Basic Income Support for Sustainability, the Complementary Redistributive Income Support for Sustainability, selected eco-schemes, and coupled support for suckler cows (this is the only one not taken into account in the CP-0 scenario). Where applicable, additional payments for animal welfare are included. Environmental payments are excluded from this particular analysis. All variable costs are considered, including costs related to herd renewal, which is classified as a variable cost. The magnitude of this cost is determined by herd replacement rates and farm management efficiency and is parameterised specifically for each TAH.

2.2 Typical Suckler Cow Farms modeled with SiTFarm

This section briefly summarizes the key characteristics of the typical farm types included in the analysis. All farms are primarily oriented toward suckler cow production. Calves are sold after the grazing period and are not fattened to finishing weight. The analysis includes four typical farms. The first, TAH03, represents small-scale farms raising only three suckler cows. These are typically tie-stall systems with large-frame cows, usually of the Simmental breed. The farm manages just under 5 hectares of land, with the feeding ration based primarily on hay (60%) and grass silage (40%). Grazing is not practiced. Permanent grassland is typically mown three times annually. The farm also includes a small area of traditional meadow orchards. The required labour input is 0.3 full-time equivalents (FTE), usually provided by a retired household member or someone employed off-farm who farms part-time.

The second farm, TAH10K, is a medium-sized operation with 10 suckler cows and a sufficient number of replacement heifers. The farm manages 32 hectares of permanent grassland in the Karst region. Of this area, nearly 60% is allocated to grazing, just over 10% to hay production, and 30% to grass silage. The total labour requirement amounts to just under 1 000 hours annually. The farm is operated by an individual who is employed at least part-time off-farm, with additional support provided by family members. The stocking density is extremely low, at 0.35 LU/ha.

The next two farms represent larger-scale suckler cow farms, with the primary distinction between them being location and land availability. The third farm, TAH40K, is also located in the Karst region. Due to poor soil fertility and extremely low pasture productivity, it requires significantly more land to support the same herd size (121 ha). As a result, the stocking density is similar to that of TAH10K. Grazing is practiced much more extensively here, with 61% of the land allocated to pasture, while the remainder is used mainly for hay production to meet winter feeding needs. Labour demand is high, exceeding 1 FTE. The farm is typically managed by a full-time insured agricultural worker, with family members helping during peak workload periods. Finally, TAH40 refers to a large farm situated in an area with permanent grassland but with more favourable production conditions. These farms can generally produce higher-quality forage and manage to mow their grasslands twice per year on average. This type of farm requires approximately 1 FTE to manage the suckler cow operation.

3 ECONOMIC RESULTS AND DISCUSSION

The analysis reveals substantial differences among the typical farms in terms of economic indicators, efficiency, and economic sustainability. The smallest farm (Tab. 1) achieves a modest GM per hour of only 4.58 € (BL), which deteriorates by 12.6% in the absence of coupled payments (CP-0). Although the absolute value of these payments is relatively small, their impact on the farm's viability is significant. Budgetary support accounts for as much as 92% of the farm’s total GM. This illustrates the fragile economic structure of such small-scale farms, which still represent the majority in practice. The low hourly GM underlines the sector’s economic vulnerability and supports its classification as a sector in difficulty.

The performance of the other three typical farms is considerably better—especially when viewed in the context of the livestock sector as a whole during the analysed period. TAH10 benefits primarily from its large land base and, with all budgetary support included, achieves a result that aligns with the average profitability of livestock farms in Slovenia. The share of subsidies in total revenue remains high (57%), but the removal of coupled payments (CP-0) has only a marginal impact, reducing the hourly GM by 1 €. Such farms are relatively few in number, and despite achieving a solid GM per hour, questions remain regarding their long-term structural direction in the face of ongoing sectoral transformation.

The final two farms show very strong results, though their foundations differ (Tab. 1). The large Karst farm (TAH40K) is significantly more reliant on budgetary payments, due to the extensive land requirements—managing 61% more land than its counterpart. Consequently, the share of subsidies in its total GM is higher (74%), although the importance of coupled payments specifically is lower. While the difference in hourly GM between the two large farms is minimal, this is largely offset by the higher labour requirements associated with more extensive operations and land structure challenges.

Table 1: Selected economic indicators for typical suckler cow farms (TAH) under baseline (BL) and no coupled payments (CP-0) scenarios.

Economic Indicators	TAH03		TAH10K		TAH40		TAH40K	
Scenario	BL	CP-0	BL	CP-0	BL	CP-0	BL	CP-0
Revenue (EUR)	5 273	4 981	22 395	21 422	70 682	66 792	85 149	81 259
Market revenue (EUR)	3 149	3 149	9 536	9 536	39 710	39 710	38 503	38 503
Budgetary payments (EUR)	2 124	1 832	12 859	11 886	30 972	27 082	46 646	42 755
Variable costs (EUR)	2 956	2 956	5 625	5 625	19 885	19 885	22 506	22 506
Gross margin (EUR)	2 317	2 025	16 770	15 798	50 797	46 907	62 643	58 753
Gross margin per hour (EUR/h)	4.58	4.00	16.80	15.83	27.20	25.12	26.12	24.49
Gross margin per hectar (EUR/ha)	490.56	428.79	524.07	493.68	677.29	625.42	517.71	485.56

4 CONCLUSIONS

Based on the conducted analysis, we conclude that the SiTFarm model, which is based on LP and maximization of GM, enables detailed and representative analysis of TAH that reflect real-world farming conditions. The results indicate substantial variability in economic performance across different farm types.

Although the suckler cow sector is generally characterized by weaker economic outcomes—as confirmed by one of the farms in our analysis—the other three farms exhibit above-average performance and fall within the top quartile of the most economically successful farms in the suckler cow sector. This highlights the structural and contextual diversity within the sector itself.

The analysis also shows that coupled payments play a relatively smaller direct role in determining profitability for some farms. Nevertheless, they provide an important policy signal to farmers. These supports are especially relevant in areas at risk of land abandonment and subsequent natural overgrowth, where maintaining agricultural activity is essential for environmental and landscape management.

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TYPOLOGY OF B2B AND B2C DECISION SUPPORT TOOLS IN THE AGRI-FOOD SECTOR

dr. Tanja Dergan, dr. Marko Debeljak
Institute Jožef Stefan, Department of Knowledge Technologies
Jamova Cesta 39, 1000 Ljubljana, Slovenia
E-mail: tanja.dergan@ijs.si, marko.debeljak@ijs.si

Abstract: The increasing complexity of agri-food systems requires decision support tools (DSTs) tailored to different business contexts. This paper presents a typology for classifying B2B and B2C DSTs, developed through an iterative mixed-method approach and applied to a set of 134 tools. The typology is structured around five key dimensions and enables the identification of critical gaps in terms of usability, accessibility and relevance to real-world conditions, in addition to functional diversity. These results provide a structured basis for analysing existing DSTs and for developing more targeted, context-specific DSTs**.**

Keywords: typology, decision support tools (DSTs), agri-food sector, B2B, B2C, digital transformation

1 INTRODUCTION

Alongside traditional management criteria, enterprises must also incorporate the results of various digital agricultural tools and a wide range of sustainability criteria into their decision-making processes [5]. Market globalisation and climate risks have additionally intensified the need for the development of more adaptive and precise farm management strategies [6]. The COVID-19 pandemic further accelerated digital transformation, but also revealed the unequal pace of adoption and persistent barriers to the inclusion of digital tools [1]. In this context, Decision Support Tools (DSTs) are emerging not only as technological aids but also as essential mechanisms for managing the increasing complexity of modern agricultural and food systems [1,5,7]. DSTs encompass a wide range of formats, from spreadsheets and diagnostic checklists to integrated, web-based platforms and mobile apps [2]. These tools support decision-making across various steps of business planning, such as production planning, logistics, traceability, risk management, and customer engagement [4]. However, their adoption is often hindered by poor usability, insufficient training, and weak alignment with practical needs [7]. Furthermore, small-scale and financially under-resourced actors face particular challenges in adopting DSTs effectively [3]. To understand the different adoption and effectiveness of DSTs, it is also important to consider the distinction between business-to-business (B2B) and business-to-consumer (B2C) settings. B2B DSTs often facilitate enterprise internal operations, supply chain coordination, or certification processes, while B2C DSTs focus on market interaction, customer communication, and user-facing services. Each of these pathways presents unique challenges in aligning DSTs with specific organisational needs and user expectations [4]. Despite the growing availability of DSTs, both user feedback and sector-specific reviews highlight a lack of systematic categorisation tailored to the agri-food sector [5]. Existing classification schemes are often too general or technology-focused, failing to reflect their purpose, user environment, and interaction dynamics. This fragmentation is especially problematic in the agri-food sector, where tools are often developed in isolation and show limited potential for reuse or transfer across different contexts. This paper addresses these issues by developing and testing a typology of DSTs tailored to the agri-food sector.

2 METHODOLOGY

2.1 Typology development process

The development of the typology followed a two-phase iterative process: (1) conceptual grounding through literature review, and (2) verification of the typology on a set of DSTs. The first phase involved a comprehensive review of both academic and professional sources, covering studies on DST use and adoption in the agri-food sector, the application of multi-criteria decision analysis (MCDA) methods for tool classification, and emerging digital business models based on platform technologies (i.e. apps, web-tools, data sharing systems) in agri-food systems. In addition, results from EU-funded projects were reviewed, which provided insight into user-oriented challenges in tools adoption. The second phase involved the practical verification of the developed typology through its application to a broad and diverse list of DSTs (as described in Section 2.2). Key functionalities of each tool were analysed and mapped against the proposed typology dimensions to verify whether the typology was able to categorise a broad spectrum of DSTs. When relevant DSTs could not be classified, the typology was recursively refined. This iterative process between conceptual design and empirical verification ensured that the typology effectively captured real-world variations of DSTs properties, such as decision context (e.g. B2B vs B2C), decision scope (e.g. operational vs strategic), and user interface types (e.g. web or mobile platforms). The proposed methodology aims to produce a typology for classifying DSTs in agri-food systems that is both theoretically sound and practically useful.

2.2 Collection of DSTs

The DSTs were collected from EU project repositories (e.g. SmartAgriHubs, EU FarmBook), published scientific literature and national innovation platforms. The main inclusion criteria were accessibility, functionality, diversity and availability in the English language. Tools were included in a list if they supported decision-making processes within agi-food systems and were suitable for both B2B and B2C contexts. Tools with highly technical solutions, limited transferability or insufficient available description information were not included. The final selection included 134 DSTs, representing a broad spectrum of decision-making purposes, user groups, and technological formats.

2.3 Classification of collected DSTs

Following the collection, each DST was analysed and classified by the typology developed in Section 2.1. A standardised Excel-based template was used to document the key dimensions of each tool, including its decision domain, decision context, decision scope, decision type, and user interface. The typology supports the identification of patterns, clusters and gaps in the collected DSTs by helping to recognise common design features or use cases (patterns), grouping tools with similar functions or user needs (clusters) and identifying under-represented areas where certain types of tools, users or functionalities are missing (gaps).

3 TYPOLOGY AND CLASSIFICATION OF DSTs

3.1 Developed typology

The developed typology is structured around five interconnected dimensions that enable a comprehensive description and classification of DSTs in terms of their context, functionality and usability (Figure 1). The first dimension, ***decision domain***, defines the overarching sectoral scope in which the DSTs are applied. In case all tools are positioned within the agri-food sector,

serving actors across production, processing, distribution, market and consumption links of the food supply chain. ***Decision context*** refers to whether a tool is intended for business-to-business (B2B) or business-to-consumer (B2C) use, or it is a hybrid of both. This is an important distinction, as B2B tools typically support coordination between businesses, such as managing inventory or planning supply chains, while B2C tools focus more on communication, branding and direct interaction with consumers. The third dimension, ***decision scope***, considers whether the tool addresses operational or strategic levels of decision-making. Operational tools support day-to-day tasks such as scheduling or input management, while strategic tools are oriented toward long-term planning or investment decisions. ***Decision type*** captures the functionality of the tool, distinguishing between analytical, advisory, and participatory modes. For instance, while some DSTs rely on algorithmic scoring models or decision trees, others are designed to facilitate group discussion or simulate complex outcomes. Finally, the ***user interface*** dimension categorises tools based on how users interact with them. This includes offline tools, such as paper-based methods, spreadsheets, or desktop applications; online tools, including downloadable software and web-based platforms; and mobile apps. These technological formats play a major role in determining the accessibility, usability, and adoption potential of a tool. These five dimensions underpin a structured framework designed to map and compare DSTs in the agri-food sector.

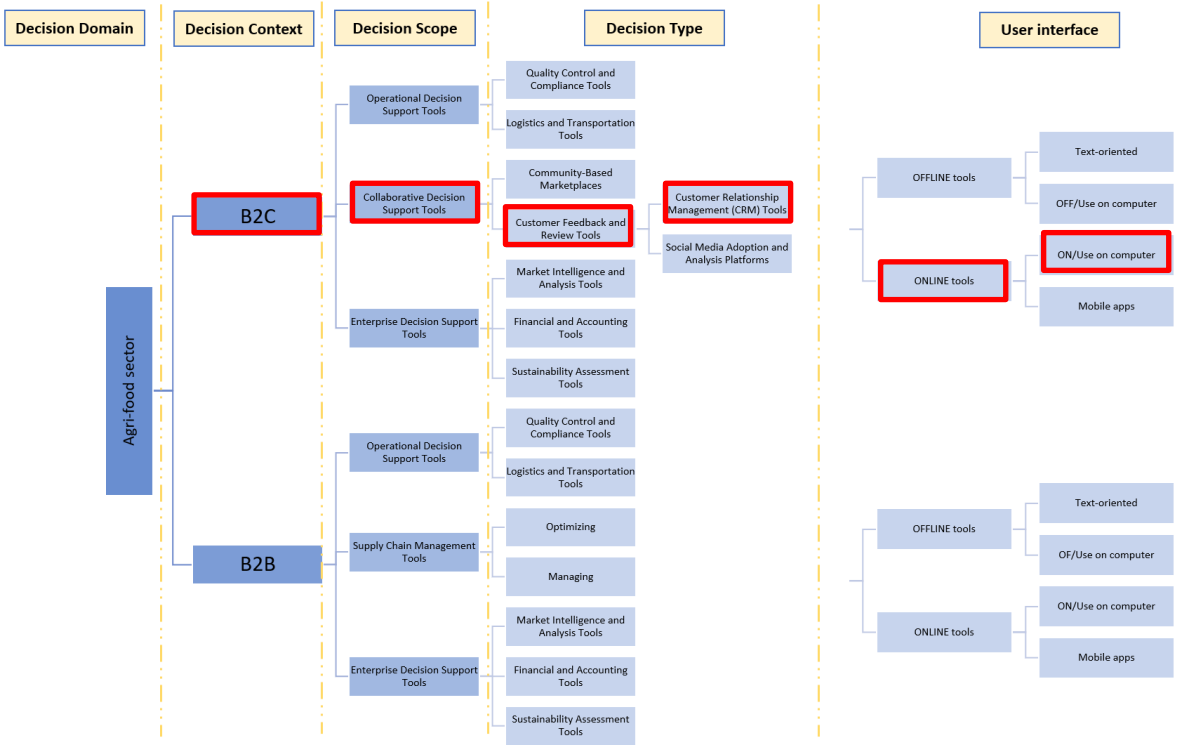


Figure 1: Typology of the DSTs. Dimensions of the typology are given in yellow boxes. Classification of the DST "HubSpot" is given as an example. Its classification path is shown in red.

3.2 Classification of DSTs and identification of gaps

The developed typology was used to categorise 134 DSTs and to identify clusters based on common characteristics. This classification revealed a strong functional orientation towards operational support, particularly in the areas of production, logistics, and compliance, while showing limited focus on traceability and consumer engagement. Most DSTs were developed for B2B to facilitate internal processes such as planning or certification. In contrast, B2C tools

were more diverse in terms of user interface design. They were limited in number and mainly available as smartphone apps or web-based platforms. Among the collected DSTs, analytical tools based on structured models or scoring systems are the most prevalent. In contrast, advisory tools that could support co-creation and collective learning have been less common. Furthermore, many tools are implemented as stand-alone offline solutions, such as spreadsheet-based tools or desktop applications, which limits their integration into web-based systems. Overall, the classification of DSTs using the developed typology reveals an underrepresentation of B2C tools, those supporting strategic-level decisions, and enterprise-level applications. In addition, results show a lack of interoperability of tools and limited user guidance. Addressing these issues is critical to support a transition from paper-based methods to web-based DSTs which will improve the ability of agri-food sector to navigate through the complex decision-making process better.

4 CONCLUSION

This paper presents a typology for the classification of B2B and B2C DSTs in the agri-food sector, which was developed in an iterative process that combines top-down theoretical insights with a bottom-up empirical analysis of 134 selected DSTs. The typology is structured around five key dimensions in order to comprehensively capture the diversity of DSTs in terms of their purpose, usability and technological form. The application of this typology enabled the systematic classification of the collected tools, revealing that operational, B2B oriented and analytical solutions predominate, while relatively few tools support strategic planning, B2C interaction or participative approaches. These findings emphasise the practical value of the typology not only in mapping current tools, but also in developing future DSTs that are better aligned with the user context and the need for digital transformation of the agri-food sector.

Acknowledgement
The research presented in this paper was funded by the EU- Horizon 2020 funded project, 'Connecting COnsumers and producers to REbalance farmers' position through AmbassaDOrs trainings' (COCOREADO), under Grant Agreement 101000573; and the Slovenian Research Agency (grant number P2-0103).

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FACTORS INFLUENCING THE USE OF SPORTS EQUIPMENT SHARING PLATFORMS: A USER-CENTRIC STUDY

Nikola Drinjak
University of Belgrade, Faculty of Organizational Sciences, Department of Operational Research and Statistics
Jove Ilica 154, 11000 Belgrade, Serbia
E-mail: nikola.drinjak@fon.bg.ac.rs

Milica Maričić
University of Belgrade, Faculty of Organizational Sciences, Department of Operational Research and Statistics
Jove Ilica 154, 11000 Belgrade, Serbia
E-mail: milica.maricic@fon.bg.ac.rs

Abstract: The sharing economy represents a contemporary paradigm aimed at optimizing the utilization of underused resources through collaborative access models. This study examines the key determinants influencing individuals' willingness to engage with sports equipment sharing platforms, emphasizing a user-centric perspective. Empirical data were obtained through a structured online survey conducted in early 2025, targeting residents of Belgrade, Serbia. The analysis employed a multinomial logistic regression model to identify statistically significant behavioral and contextual factors. Findings reveal that, although overall awareness of specialized platforms remains limited, user interest and openness to participation are considerably high. The study contributes to the emerging body of knowledge on platform-based consumption in the context of sport and provides a foundation for future research, policy development, and business model innovation within the circular and sharing economy domains.

Keywords: Sharing Economy, Sports Equipment Platforms, User Behavior, Multinomial Logistic Regression, Sustainable Business Models

1 INTRODUCTION

The Sharing Economy (SE), driven by digital transformation, has redefined how individuals access goods and services by enabling the temporary exchange of underutilized resources through peer-to-peer platforms [1]. This economic model reduces transaction costs, improves resource allocation, and supports sustainable consumption by promoting access over ownership [2]. Over the past decade, the SE has evolved into a disruptive force across various sectors, including transportation, accommodation, and increasingly, sports. Within the sports domain, sharing-based models have emerged to address challenges such as limited financial resources, infrastructure underuse, and environmental concerns. The shared use of sports facilities, equipment, and accommodation during major events contributes to cost reduction, operational efficiency, and long-term sustainability [3]. Additionally, initiatives like the Olympic Agenda 2020 emphasize decentralized planning and the reuse of existing infrastructure to enhance efficiency and minimize unnecessary investments [4].

Having in mind the growing interest in SE and its application to various spheres of life this study examines the key determinants influencing individuals' willingness to engage with sports equipment sharing platforms, emphasizing a user-centric perspective. Sport equipment rental platforms are in their initial stages, so information on user behaviour and perceptions on the business model are welcome. The empirical data which was used for modelling and determining the factors which impact the decision to use sport equipment rental platforms were obtained through an online survey conducted in early 2025, targeting residents of Belgrade, Serbia. This paper is organized into six sections. Following the introduction, Section 2 explores the theoretical background. Section 3 outlines user engagement factors. Section 4 details the methodology and modeling approach. Section 5 presents the findings, and Section 6 concludes with key insights and future research directions.

2 SHARING ECONOMY IN SPORT

The sharing economy is increasingly transforming the sports industry by redefining access to equipment, infrastructure, and services through digital platforms. Instead of emphasizing ownership, users benefit from short-term access, promoting affordability, flexibility, and sustainability [5]. Platforms such as PELOTON, MIRROR, and Equip enable personalized, home-based training and simplified rental experiences [6]. Community-driven models like Fritidsbanken function as libraries of sports equipment, supporting reuse, equal access, and waste reduction. Shared-use sports venues further increase efficiency by accommodating multiple activities, enhancing infrastructure value. Bike Sharing Programs (BSPs) exemplify how such models contribute to public health, sustainable urban mobility, and reduced emissions [7]. On a larger scale, decentralized sporting events like UEFA Euro 2020 reduce host burdens and foster regional cooperation, aligning with the Olympic Agenda 2020's goals [4]. Overall, the sharing economy promotes inclusion, active lifestyles, and social impact by lowering financial and accessibility barriers in sport. As it can be seen from the presented examples, the SE platforms in sports and sport equipment rentals are promising business models. Although these platforms are not as popular and used as Airbnb or Uber, they are attracting the attention of sport organizations, local governments, wider population, and investors.

3 FACTORS IMPACTING PARTICIPATION IN SHARING ECONOMY

Participation in the sharing economy is shaped by a multifaceted set of demographic, economic, and psychological factors, each contributing to the decision-making process of potential users. Age remains a prominent determinant, as younger individuals are generally more inclined to participate in sharing economy activities, in contrast to older populations who tend to engage less frequently [8]. Gender differences also play a significant role, with women often being more active in consumer roles, while men more frequently assume provider roles [9]. While income level is not always a decisive factor, lower-income individuals are typically more driven by economic necessity [10]. Trust in digital platforms and peer providers is a fundamental enabler of participation, reinforced by mechanisms such as user ratings and reviews. Moreover, perceived benefits—economic, social, and environmental—strongly influence user engagement [11]. Entrepreneurial aspirations, particularly among gig economy workers, further encourage active participation [12]. Employment-related considerations, including job flexibility, income security, and opportunities for skill development, are especially relevant among students and economically vulnerable groups. Collectively, these factors highlight the need for tailored strategies that reflect the heterogeneous motivations of sharing economy participants. The presented literature review points out that factors impacting participation in sharing economy are multiple, that their level of impact depends on the form of SE, as well as from socio-economic and cultural characteristics and beliefs.

4 RESEARCH METHODOLOGY

This study employed a quantitative research approach using a structured online questionnaire distributed via Google Forms. The data collection took place in Belgrade, Serbia between December 2024 and March 2025. To identify the key factors influencing user intention to adopt sharing economy platforms for sports equipment, logistic regression analysis was applied, enabling the modeling of multiple outcomes based on several predictor variables.

To model consumer behaviour logistic regression was used. Logistic regression is a statistical method used to examine the relationship between one or more independent variables

and a categorical outcome. It estimates the probability of an event by modelling the log-odds as a linear function of the predictors. Suitable for categorical outcomes, such as a respondent's intention to use sharing economy platforms, this method yields odds ratios that indicate how changes in predictors affect the odds of the outcome. When multiple predictors are included, adjusted odds ratios offer more precise insights by accounting for confounding factors. In this study, logistic regression identifies key variables influencing acceptance of sports equipment sharing platforms.

The conducted survey included both closed- and open-ended questions aimed at assessing the perceived benefits and drawbacks of sharing economy platforms, prior usage experiences, purchase and rental behaviour, and willingness to adopt such services in the future. The target population comprised individuals from diverse demographic backgrounds, and participation was entirely voluntary and anonymous. Respondents evaluated statements using a five-point Likert scale and provided numerical and categorical responses where applicable. The survey was motivated by the increasing relevance of platforms such as Equip Sport, which promote peer-to-peer access to sports equipment in a sustainable and socially responsible manner. The collected data provide a foundation for understanding user readiness and the potential market acceptance of these innovative service models.

5 RESULTS

The sample included 210 respondents—103 male and 107 female—ranging in birth years from 1962 to 2007, with most born between 2000 and 2003. A majority (65.7%) reported familiarity with the concept of the sharing economy, while 34.3% had not encountered the term before. Regarding platform usage, 37.6% had experience with two sharing economy platforms, while 22.4% had never used any. Most respondents discovered these platforms through friends or social media, while traditional media and foreign sources were less influential. Half of the respondents reported using such platforms both in Serbia and abroad; 22.4% had no prior usage. On average, respondents spent 80 EUR annually on purchasing sports equipment (median: 60 EUR) and 25 EUR on rentals (median: 17 EUR). Awareness of platforms dedicated specifically to sharing sports equipment was low (31.9%), though user interest was relatively high, with 47.6% expressing potential willingness to use such services and 38.6% indicating they would certainly do so.

The logistic regression model shows a significant improvement over the intercept-only model, with a decrease in -2 Log Likelihood from 393.509 to 293.254 (χ^2=100.255, $p<0.001$), indicating strong overall fit. Goodness-of-fit statistics support model adequacy, as both the Pearson Chi-Square ($p=0.107$) and the Deviance statistic ($p=0.993$) are non-significant, suggesting no major differences between observed and predicted outcomes. Pseudo R-squared values Cox & Snell (0.380), Nagelkerke (0.440), and McFadden (0.240) indicate moderate to substantial explanatory power. Key predictors include annual spending on sports equipment (χ^2=25.328, $p<0.001$), awareness of sports gear sharing platforms (χ^2=22.892, $p<0.001$), and Uncertainty about the quality and cleanliness of shared goods (χ^2=21.864, $p=0.005$). In contrast, prior platform use ($p=0.059$) and another drawback factor ($p=0.111$) were not statistically significant (Table 1).

Multinomial logistic regression reveals that respondents without prior experience using platforms like CarGo or Airbnb were nearly nine times more likely to reject using sports sharing platforms (Exp(B)=8.918, $p=0.010$). Additionally, lack of awareness (B=-1.845, $p<0.001$) and disagreement with limitations (B=-3.064, $p=0.008$) decreased adoption likelihood. Conversely, higher annual spending on sports equipment ($p<0.001$) was associated with greater interest in shared alternatives, likely driven by cost considerations.

Table 1: Results of the multinomial logistic regression

Effect	AIC of Reduced Model	BIC of Reduced Model	-2 Log Likelihood of Reduced Model	Chi-Square	df	Sig.
Intercept	349.254	442.973	293.254	0.000	0	.
Annual spending on sports equipment	370.582	457.607	318.582	25.328	2	0.000
Prior use of sharing platforms	349.381	423.017	305.381	12.126	6	0.059
Awareness of sports gear sharing platforms	368.147	455.172	316.147	22.892	2	0.000
Uncertainty about the quality and cleanliness of shared goods	355.118	422.060	315.118	21.864	8	0.005
Uncertainty about the return of goods without damage	346.277	413.219	306.277	13.023	8	0.111

6 CONCLUSION

The study shows that sports-related sharing economy platforms are influenced by key factors such as spending habits, awareness, and perception of platform limitations. Awareness and openness toward shared use strongly impact willingness to engage, while concerns about quality and safety act as barriers. Despite limited awareness, interest in such platforms is notable, especially among cost-conscious users. A limitation of the study is its focus on a single location and early-stage platforms. However, the findings offer practical insights and serve as a valuable reference for future research on sharing models in sport.

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MAPPING RESEARCH TRENDS IN THE SHARING ECONOMY: A TOPIC MODELING APPROACH

Milica Maričić
University of Belgrade – Faculty of Organizational Sciences, Department of Operational Research and Statistics
Jove Ilića 154, Belgrade, 11000, Serbia
E-mail: milica.maricic@fon.bg.ac.rs

Veljko Uskoković
University of Belgrade – Faculty of Organizational Sciences, Department of Operational Research and Statistics
Jove Ilića 154, Belgrade, 11000, Serbia
E-mail: veljko.uskokovic@fon.bg.ac.rs

Veljko Jeremić
University of Belgrade – Faculty of Organizational Sciences, Department of Operational Research and Statistics
Jove Ilića 154, Belgrade, 11000, Serbia
E-mail: veljko.jeremic@fon.bg.ac.rs

Abstract: The sharing economy (SE) is a business model that allows individuals transact goods and/or services in the short term via an Internet platform among peers. Since it first emerged, different forms of "sharing" have been developed in various spheres of life: accommodation, mobility, food, fashion, finance, and sports. This paper aims to analyse the scientific research corpus on the topic of sharing economy, define the key research topics, and perform topics trend analysis. The research presents a topic modelling analysis of articles extracted from the Web of Science database, published between 2011 and 2025 and written on a topic related to the sharing economy. The applied LDA identified 15 topics and pointed out topics of Digital Work and Regulation, Shared mobility, SE consumer behaviour, Shared energy, and Shared accommodation as the promising ones in 2025.

Keywords: topic modelling, bibliometric analysis, sharing economy, platform economy, LDA

1 INTRODUCTION

Sharing among humans has been a phenomenon for thousands of years. However, in the last few decades, the approach to sharing has fundamentally changed due to the Internet, Web 2.0, Web 3.0 and social networks. Regular sharing sees an individual with a resource (provider) sharing it with another individual searching for resources (user). The new form of sharing, coined platform economy, besides the providers and users, envelops the platform and Internet as means of mass communication and connection among participants. Some forms of platform economy are shared mobility, shared accommodation, crowdfunding, and gig economy.

The reception of the platform economy has been versatile. For some individuals, it was an opportunity to extend the usage life cycle of personal belongings and earn additional income. For some communities, it was a big disturbance and disruption of everyday peaceful life. Public and governmental perceptions have changed significantly over time, up to the point that sharing economy practices have been banned in some cities and communities. From a scholarly research standpoint, interest in the SE has steadily grown over the past decade [9].

According to ResearchAndMarkets [10], the global market for Sharing Economy was estimated at US$287.9 billion in 2023 and is projected to reach US$1.4 trillion by 2030, growing at a CAGR of 25.1% from 2023 to 2030. The interest of users, providers, platforms, and policymakers in researching and understanding the sharing economy will unlikely diminish anytime soon. Therefore, this paper aims to analyse the scientific research corpus on the topic, define key research areas, and examine topic trends. The findings should help researchers and decision-makers identify gaps and promising directions for future study. Within this research,

we present and discuss a topic modelling analysis of articles related to the sharing economy, extracted from the Web of Science database and published between 2011 and 2025.

After the Introductory section, the second section presents a literature review regarding SE research directions and topic modelling. The third section presents the research methodology where the data-collection procedure was outlined as well as the topic modelling parametrisation. The fourth section is devoted to research results, descriptive statistics and detected topics. The last section concludes the paper and provides a discussion of the analysis presented in this paper, as well as the study's limitations.

2 LITERATURE REVIEW

Authors Mody et al. [11] applied a tri-method approach using bibliometric (co-citation) analysis, thematic content analysis, and a quantitative systematic literature review on the scientific corpus of sharing economy between 2012 and 2020. Their study is valuable for several reasons. First, the study analysed previous bibliometric research on SE and identified 14 studies. According to them, the limitations of these studies were the following: related to a short time span, focused on just one platform, lack of information on research procedure, and classification of papers to apriori-defined themes and topics. The analysis of the collected data showed that the studies have been mostly conducted in Western contexts and lack diversity in contexts, data collection, sample populations, and applied statistical analysis.

Merino-Saum and co-authors [12] did a bibliometric, epistemological and substantive analysis of 175 SE-related papers from the Scopus database. Besides doing a bibliometric study, the categorised and classified the constructs and items used in collected studies and provide a much-needed overview. As many as 3191 factors have been extracted. Their study is valuable as it went a step further from a bibliometric study and tried to systematise factors and impacts on usage and participation in SE.

Duan [9] conducted a bibliometric study on sharing economy quite recently. He observed two periods: pre-COVID (2014-2019) and post-COVID (2020-2023). Besides providing the descriptive statistics of the collected corpus, he presented the thematic evolution of keywords as well as the thematic map by author keywords. The author identified that a topic which evolved is the conversation about sustainability or sustainable development in the SE, as well as the topics of social effects, environmental impacts, and economic possession.

Although the presented studies are highly promising and emphasise the need for bibliometric studies of SE, the applied bibliometric models and methods are somewhat basic and envelop bibliometric and qualitative analysis without the application of advanced text mining and data mining techniques.

3 RESEARCH METHODOLOGY

The data for this study was collected from the WoS database. The defined time period was between 2011 and 2025. The year 2011 was chosen as the lower limit because one of the first papers on shared economy was published by Bostman and Rogers [2]. The authors opted for collecting the abstracts of documents type Article since articles undergo a more thorough review process, ensuring their quality. Only documents written in English were observed, as more than 90 % of the indexed scientific articles are published in English, presenting the dominant language of publishing [4]. Another highly important aspect of the search are the keywords [3]. The following keywords were selected based on researchers' experience and literature review: "sharing economy" "platform economy" "gig economy" "collaborative consumption" "Airbnb" "uber" "p2p" "peer-to-peer" "ride sharing" "shared mobility" "micro mobility" "access-based consumption" "carsharing" "car swapping" "on-demand economy"

After the data was collected, a text mining model was applied. There are various text mining models available, such as Latent semantic analysis (LSA), Probabilistic latent semantic analysis (PLSA), and Latent Dirichlet allocation (LDA) [1]. In this paper, we opted for LDA as it is an unsupervised generative probabilistic method for modelling a corpus and presents one of the most used models in the fields of NLP, used for the topic modelling method [8]. The specific method of LDA applied in the study is the Gibbs sampling [6].

4 RESEARCH RESULTS

The defined query resulted in 11566 documents. Most papers were published in 2022 (1571), followed by 2024 (1563). The number of papers published was stable, with around 350 papers in the period 2011-2016. After that, the number of papers published surged. At the moment of analysis, March 2025, there have already been 388 papers published. The journals that published the most papers on SE in the observed period are IEEE Access (411, 3.6%) and Peer-to-peer networking and applications (310, 2.7%). Nine journals have published more than 100 papers on the topic. Looking at the citation count in the WoS Core, 1702 papers (14.7%) have not been cited. On the other hand, the paper "You are what you can access: Sharing and collaborative consumption online" by Russell Belk is the highest-cited paper with as many as 1716 citations. The mean citation is 21.47, with a standard deviation of 52.970 and a median of 7.00, indicating a distribution skewed to the left. According to the research area, most papers are in the areas of Business & Economics (12.35), Computer Science (10.2%), and Computer Science; Engineering; Telecommunications (7.3%).

After getting familiar with the collected data, we applied structural topic modelling. When it comes to topic modelling, an important aspect which should be elaborated upon is data preprocessing. Namely, the raw data has to be converted and structured so that the LDA can be applied. The collected text was converted to lowercase, ensuring case insensitivity in subsequent analyses. Then, we removed common stopwords and punctuation while intra-word dashes were preserved. Also, numeric characters from the text were removed. Finally, we standardised the spacing between words. These transformations were intended to uniform the text corpus for further text analysis. The following phase involved stemming the words and reducing words to their root forms in the corpus using the English language. We removed a series of specific words or phrases deemed irrelevant for analysis. By eliminating these words, we aimed to reduce noise in the text, ensuring that only the most meaningful and relevant terms remain for subsequent analysis. We also removed terms appearing in very few documents, making the matrix more manageable and rows without words.

Finally, the LDA algorithm was applied to extract significant topics and to determine the optimal number of topics by applying the following evaluation metrics: Griffiths and Steyvers - Based on the log-likelihood of the model, Cao et al. - Based on coherence score, Arun et al. – Also a coherence-based metric, and Deveaud et al. - A topic coherence score [7]. The number of topics varied from 5 to 20, with increments of 5. We decided to retain 15 topics, as this number of topics had balanced four evaluation metrics.

Based on the extracted keywords by topic, the retained topics are named the following: Digital Work and Regulation, Shared mobility, SE consumer behaviour, Business models of SE, Service Quality and User Access, P2P network, SE evaluation using ML, Shared energy, Shared Internet, Crowdfunding, Blockchain-based SE, Shared accommodation, Optimisation of SE practices, Tech changes due to SE, and Social aspects of SE.

The topic trend analysis pointed the following topics as those on the rise and whose interest is increasing in the observed period: Digital Work and Regulation, Shared mobility, SE consumer behaviour, Shared energy, and Shared accommodation. The topics whose interest has been stable are Service Quality and User Access, SE evaluation using ML, Crowdfunding,

Blockchain-based SE, Optimization of SE practices, Tech changes due to SE, and Social aspects of SE. Finally, the remaining three topics are losing interest (Business models of SE, P2P network, and Shared Internet).

5 DISCUSSION AND CONCLUSION

The conducted bibliometric study on the corpus of data from the WoS database indicated that the topic of SE is a proliferate one with almost 12000 papers in the 15-year period. After the phenomenon of SE emerged in 2011, in the first years, the publication count was stable, but it significantly increased after 2016. The citation count of papers is quite low; however, that could have been expected as a significant number of papers have been published in the last few years, and those papers have not yet accumulated citations. As many as 15 topics have been detected, and promising topics in the field have been identified. Although this research is promising, some limitations to the study should be mentioned. The first is related to the keywords. The keywords used are generic, and in future research, the keyword list can be enlarged by accounting for the topics and themes identified in this paper. Also, the analysis time frame is quite long for a bibliometric study. Usually, the time frame is 5 to 10 years [5]. Therefore, the study could be replicated in a shorter time frame. Finally, in this study, we performed LDA and Gibbs sampling; however, other algorithms, such as Expectation-maximization (EM) could have been applied, and the results could be compared [8].

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FROM COEFFICIENTS TO CONNECTIONS: COMPARING REGRESSION AND NETWORK ANALYSIS FOR EXPLAINING EXPORT SHARE IN TURNOVER

Mirjana Pejić Bach
University of Zagreb, Faculty of Economics and Business, Department of Informatics
Trg J.F. Kennedya 6, 10000 Zagreb, Croatia
Faculty of Commercial and Business Sciences
Lava 5, 3000 Celje, Slovenia
IEDC-Bled School of Management
Prešernova cesta 33, 4260 Bled, Slovenija
E-mail: mpejic@net.efzg.hr

Helena Nikolić
University of Zagreb, Faculty of Economics and Business, Department of Trade
Trg J.F. Kennedya 6, 10000 Zagreb, Croatia
E-mail: hnikolic@net.efzg.hr

Ivan Miloloža
Faculty for Dental Medicine & Health, University of J.J. Strossmayer
Crkvena 21, Osijek, Croatia
E-mail: ivan.miloloza@fdmz.hr

Abstract: This paper compares regression and network analysis to explain export share in total turnover, using JASP as the analytical platform. The study employs both approaches on a dataset that includes potential determinants such as CAGE dimensions, competitiveness, innovation, and customer orientation. Results indicate that regression highlights key direct effects on export share, while network analysis provides additional insight into variable connectivity and centrality. Building on these complementary insights, we formulate a theory-driven structural equation model (SEM) that unifies the significant regression paths with the most informative network connections.

Keywords: export share, turnover, regression analysis, network analysis, JASP, CAGE framework

1 INTRODUCTION

Identifying the drivers of export share has long been a topic of interest in international business and strategic management [2][8][5]. Traditionally, researchers have relied on regression analysis to evaluate the impact of firm-level, industry-specific, and institutional variables on export outcomes. Regression models provide interpretable coefficients and support hypothesis-driven analysis, but they assume independence among predictors and may fail to capture the complexity of variable interrelationships. Recent advances in data analytics have introduced network analysis as a complementary approach that emphasises the structure and connectivity between variables rather than their isolated effects [7]. In business research, network models have been used to explore interdependencies in innovation systems [4], strategic alliances [10], and organisational learning [1].

This paper aims to compare regression and network analysis in the context of explaining export share in turnover, based on data collected from Croatian firms operating in foreign markets, using constructs grounded in the CAGE framework [3], innovation capacity, customer orientation, internal coordination, and perceived competitiveness. We treat the regression and network results as sequential inputs for a confirmatory structural equation model, formalising the direct effects identified by the regression while retaining empirically warranted covariances uncovered in the network graph. By illustrating this three-step workflow, which includes regression, exploratory network analysis, and confirmatory SEM, we respond to recent calls for multi-method analytical pipelines that balance parsimony with relational richness.

2 METHODOLOGY

The data for this study were collected through a structured online questionnaire, targeting Croatian companies that actively export their products or services to foreign countries. A total of 94 Croatian companies participated in the survey, among which 47.7% were significant, with more than 250 employees, 29.2% were small, with up to 50 employees, and 23.1% were medium-sized, with 51–250 employees. In terms of industry, companies were operating in manufacturing (33.1%), professional, scientific, and technical services (24.6%), followed by wholesale and retail trade (20%) and construction (11.5%). Most companies, at 85.4%, were independent firms, and 14.6% were subsidiaries of multinational corporations.

The questionnaire captured data measuring the company-level perceptions of distance dimensions of the Eastern European countries (based on the CAGE framework), internal organisational capabilities, and the export share in total turnover (Table 1). The variables were measured using 1-7 Likert-type scales, and all items were aggregated as mean values to form the constructs used in subsequent analysis. Since Croatian companies export the most to Eastern European countries, CAGE items were used to measure the perceived distance of these companies from their Eastern European counterparts.

All analyses were conducted in JASP 0.19.3, an open-source statistical software that supports regression and exploratory network modelling. The research utilises linear regression and network analysis on a dataset encompassing potential factors of export activities, including CAGE dimensions, competitiveness, innovation, and customer orientation. Leveraging these complementary ideas, we develop a theory-driven structural equation model (SEM) that integrates the salient regression lines with the most informative network connections.

Table 1: Variables used in the research

Variable code	***Variable description***	***# of items***	***Mean (St.Dev.)***
Dependent variable			
EXPORT SHARE	*% of export in company turnover*	***-***	***18.271 (25.434)***
Independent variables			
CAGE_CULT	Cultural distance from the EU market	***5***	***2.311 (0.939)***
CAGE_ADMIN	Administrative and legal barriers	***5***	***3.938 (1.113)***
CAGE_GEO	Geographical distance to the EU market	***5***	***2.752 (0.984)***
CAGE_ECON	Economic differences and barriers	***5***	***3.445 (1.146)***
CUST	Customer orientation of the company	***3***	***5.279 (1.002)***
COMPET	Perceived competitiveness on the EU market	***3***	***4.564 (1.325)***
COORD	Internal coordination and communication	***3***	***5.032 (1.081)***
INOV	Innovation capabilities and practices	***3***	***4.765 (1.099)***

Source: Authors' work

3 RESULTS

3.1 Regression analysis

Table 2 illustrates the impact of various factors on export performance. The perceived economic distance of Eastern European countries (CAGE_ECON) has a positive effect on exports, significant at 5% level ($p = 0.017$), suggesting that companies export more to economically similar countries. On the other hand, administrative distance (CAGE_ADMIN) has a substantial negative impact, significant at the 1% level ($p = 0.003$), indicating that differences in regulations, governance, or legal systems hinder export to these countries. In addition, competitive pressure (COMPET) also negatively affects export performance and is significant at the 5% level ($p = 0.018$), implying that firms facing intense competition may

struggle more in international markets. Other factors, such as cultural distance (CAGE_CULT), geographic distance (CAGE_GEO), customer orientation (CUST), internal coordination and communication (COORD), and innovation (INOV), do not show statistically significant effects. The coefficient of determination ($R^2 = 0.195$) indicates that the model explains approximately 19.5% of the variance in export performance, indicating a modest explanatory power.

Table 2: Variables used in the research

Variable	Unstandardised Coefficients	Standard Error	t	p
(Intercept)	18.516	21.694	0.854	0.396
CAGE_CULT	4.864	2.885	1.686	0.095
CAGE_ADMIN	-7.939	2.577	-3.080	0.003***
CAGE_GEO	-0.231	2.782	-0.083	0.934
CAGE_ECON	6.091	2.494	2.442	0.017**
CUST	4.106	3.596	1.142	0.257
COMPET	-6.113	2.544	-2.403	0.018**
COORD	0.008	3.097	0.003	0.998
INOV	1.179	2.838	0.415	0.679
Coefficient of determination				0.195

*Note: ** statistically significant at 5%; *** 1%; Source: Authors' work*

3.2 Network analysis

The network analysis, based on nine variables, revealed a moderately sparse structure with 15 non-zero edges out of a possible 36. This resulted in a sparsity index of 0.583, indicating that slightly more than half of the potential relationships among variables are absent.

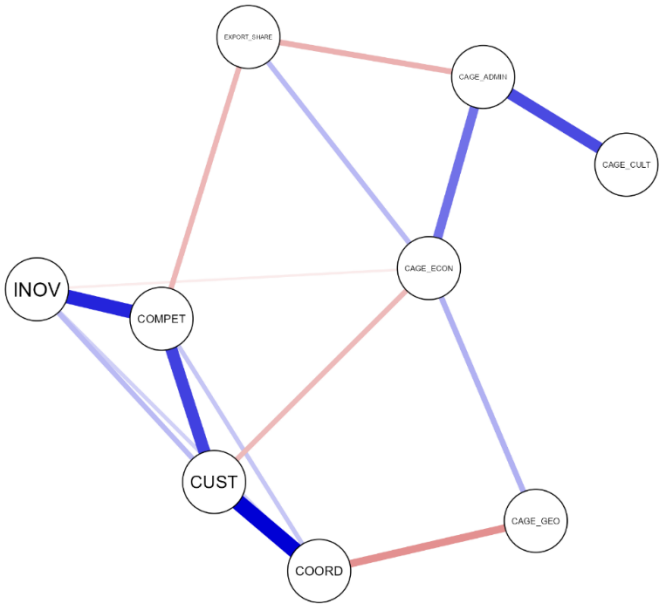


Figure 1: Network graph of the market share and its determinants

Figure 1 visualises the regularised partial-correlation network obtained with EBICglasso ($\gamma = 0.5$). Blue edges represent positive relations and red edges negative ones, while line width is proportional to effect size. Two compact blocks are immediately apparent: an internal capability cluster (INOV–COMPET–CUST–COORD) tied together by the three thickest positive links (COMPET ↔ CUST, CUST ↔ COORD, COMPET ↔ INOV), and an external-distance cluster centred on CAGE_ADMIN, whose **most potent** edge connects to

CAGE_CULT. CAGE_ECON acts as a bridge between the two blocks, linking to both CAGE_GEO and CUST, consistent with its intermediate betweenness score. Export share occupies a peripheral position; it is positively related to CAGE_ECON yet negatively linked to both CAGE_ADMIN and COMPET, mirroring the regression coefficients.

Table 3: Variables used in the research

Variable	Betweenness	Closeness	Strength	Expected influence
CAGE_ADMIN	0.643	-0.39	0.217	0.178
CAGE_CULT	-1.119	-1.704	-1.267	-0.18
CAGE_ECON	0.643	0.965	0.073	-0.071
CAGE_GEO	-1.119	-0.341	-1.211	-1.392
COMPET	1.147	0.609	1.151	1.034
COORD	-0.364	0.374	0.682	0.208
CUST	1.399	1.608	1.445	1.316
EXPORT_SHARE	-0.112	-0.266	-0.994	-1.676
INOV	-1.119	-0.855	-0.096	0.583

Source: Authors' work

Network centrality metrics for the variables in the research are presented in Table 3. Betweenness, closeness, strength, and expected influence indicate the position and influence of each variable within the overall network of determinants of export share. First, CUST and COMPET have the highest values in terms of betweenness, closeness, strength, and expected influence. Second, export share itself is positioned at the network's periphery and is related to CAGE_ADMIN, CAGE_ECON, and COMPET, which are also statistically significant in a regression model.

3.3 Comparison of variable importance and relationship

The findings from the regression analysis and network analyses converge on several key determinants of the export share, while offering complementary perspectives on their roles. In the regression model, four predictors (CAGE_ADMIN, CAGE_ECON, and COMPET) attain statistical significance and jointly explain approximately 19% of the variance in export share. Administrative distance and competitiveness exhibit negative coefficients, whereas economic distance shows a positive coefficient, indicating that regulatory barriers and intense competitive pressure inhibit exports, whereas economic similarity facilitates them. The network analysis confirms the importance of these variables but also clarifies their structural positions within the system of relationships. CAGE_ADMIN occupies a central location in the external-distance cluster and connects that cluster to the internal-capability cluster through CAGE_ECON. COMPET and CUST display the highest centrality values across betweenness, closeness, strength, and expected influence, suggesting that these two capabilities exert the broadest influence on other constructs, even though CUST does not achieve significance in the regression. Variables that are non-significant in the regression, such as CAGE_CULT, CAGE_GEO, CUST, COORD, and INOV, nevertheless contribute to the configuration of the network. Their peripheral or negative centrality scores suggest weaker integration with the core constructs. However, they may still moderate or mediate indirect pathways that affect export performance, providing a potential form of the path model. Export share itself is situated at the periphery of the network, consistent with the fact that only three variables have a statistically significant impact on it. The edges connected to export share replicate the signs of the significant regression coefficients: a positive association with CAGE_ECON and negative associations with CAGE_ADMIN and COMPET. Overall, the regression isolates the principal

direct effects, while the network perspective elucidates how those same variables interact with one another and with less prominent constructs.

3.4 Leveraging regression and network findings for initial SEM specification

Based on the analysis of multiple regression analysis and exploratory network modelling, we propose an evidence-driven strategy for specifying the structural equation model (SEM), which has been widely used in research to determine business performance [6][9]. Regression identifies the principal direct predictors of export share, while the EBICglasso network reveals how those predictors are embedded in a broader web of associations. We propose combining both outputs as the first step in determining the SEM model, retaining paths that are theoretically warranted and empirically supported. The resulting specification is therefore both parsimonious and comprehensive, since it captures the key causal directions suggested by the linear regression model and preserves statistically non-redundant covariances detected in the network. The full lavaan syntax, executable in JASP, is presented in Figure 2 and organized into four logically coherent blocks.

index.tsx

```
##########################################################################
#  SEM specification derived from the EBICglasso network in Figure 1   #
##########################################################################

# 1  Export performance (dependent variable)
EXPORT_SHARE ~  CAGE_ADMIN          # negative edge in the network
EXPORT_SHARE ~  CAGE_ECON           # positive edge
EXPORT_SHARE ~  COMPET              # negative edge

# 2  External-distance subsystem
CAGE_ADMIN   ~  CAGE_CULT           # strongest positive edge (admin ← cult)
CAGE_ECON    ~  CAGE_ADMIN          # positive edge (bridge role)
CAGE_ECON    ~  CAGE_GEO            # positive edge (econ ← geo)

# 3  Internal-capability subsystem
COMPET       ~  CAGE_ECON           # negative edge (external → internal)
CUST         ~  COMPET + COORD + INOV    # three positive edges
CUST         ~  CAGE_ECON                # negative edge (econ → cust)

# 4  Undirected (residual) associations retained as covariances
COMPET  ~~  CAGE_ECON
CUST    ~~  CAGE_ECON
CUST    ~~  INOV
COMPET  ~~  INOV
CUST    ~~  COMPET
CUST    ~~  COORD
CAGE_ECON ~~ CAGE_ADMIN
CAGE_ECON ~~ CAGE_CULT      # optional
COORD   ~~ CAGE_GEO
```

CodeImage

Figure 2: lavaan code of the market share and its determinants

Export share is treated as the outcome. The three predictors retained (CAGE_ADMIN, CAGE_ECON, and COMPET) were the only variables reaching statistical significance in the multiple regression, and they exhibit the same signed connections in the network graph. External-distance variables are entered as exogenous drivers. CAGE_CULT exerts its influence on export performance exclusively through CAGE_ADMIN; in turn, CAGE_ADMIN and CAGE_GEO shape economic distance, as reflected in their positive

partial correlations. Internal capabilities are specified as endogenous constructs. Economic distance is allowed to negatively affect competitive pressure, in line with the red edge in Figure 1. Customer orientation (CUST) is regressed on the three strongest capability edges (COMPET, COORD, and INOV) and additionally on CAGE_ECON to reproduce the negative external link. COORD and INOV remain predictors only, keeping the model parsimonious and recursive. All remaining network edges with no clear causal direction are preserved as covariances.

4 CONCLUSIONS

This study demonstrates that combining classical regression with exploratory network analysis brings a deeper understanding of the determinants of export share than either method alone. Regression isolates three statistically significant independent variables (administrative distance, economic distance, and perceived competitiveness). At the same time, the network model reveals how these variables occupy central structural positions and transmit influence through customer-oriented capabilities. Integrating both outputs into an evidence-based SEM delivers a parsimonious yet comprehensive specification that remains empirically identified and theoretically coherent. The approach, therefore, offers researchers a rigorous workflow for moving "from coefficients to connections." It provides practitioners with more precise guidance on which barriers to reduce and which capabilities to strengthen to enhance export performance.

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SEGMENTATION OF TOURISTIC DESTINATIONS BASED ON THE NUMBER OF DOMESTIC AND FOREIGN SHARED NIGHTS: INSIGHTS FROM SPAIN

Luna Radević
University of Novi Sad – Faculty of Philosophy, Department of Psychology
Dr Zorana Đinđića 2, Novi Sad, 21000, Serbia
E-mail: luna.radevic@ff.uns.ac.rs

Milica Maričić
University of Belgrade – Faculty of Organizational Sciences, Department of Operational Research and Statistics
Jove Ilića 154, Belgrade, 11000, Serbia
E-mail: milica.maricic@fon.bg.ac.rs

Veljko Jeremić
University of Belgrade – Faculty of Organizational Sciences, Department of Operational Research and Statistics
Jove Ilića 154, Belgrade, 11000, Serbia
E-mail: veljko.jeremic@fon.bg.ac.rs

Abstract: Sharing economy practices have transformed the tourism industry. While some cities and regions have adopted shared accommodation, others are lagging behind or even trying to ban it. European institutions are closely monitoring the adoption of shared accommodation, while the Eurostat publishes yearly data on the number of guest nights spent at short-stay accommodation offered via collaborative economy platforms by region. This paper, as a case study, segments cities of Spain based on the number of domestic and foreign shared nights in the period 2018-2024. The results indicate that the 59 cities could be segmented into three clusters named Tourist hubs, Urban tourist centres, and Local tourist centres. Tourist hubs are the leading tourist cities in Spain, while the Urban tourist centres are slightly smaller cities which attract a significant number of tourists. The cities which have drawn our attention are the Local tourist centres. Interestingly, these cities have seen a visible increase in domestic shared nights, indicating that the foreign tourists have not discovered them or that the local population is turning to domestic travelling.

Keywords: segmentation, clustering, Eurostat data, shared accommodation, Spain

1 INTRODUCTION

The term sharing economy refers to a digitally mediated peer-to-peer system through which individuals share underutilised goods or services without transferring ownership [6]. In recent years, the rise of sharing economy practices has significantly transformed the tourism industry, particularly the accommodation sector, by introducing new lodging models and reshaping conventional travel behaviours. Compared to traditional hotel stays, shared accommodation typically offers a more affordable and flexible form of lodging, while also encouraging tourists to engage in authentic local experiences and explore less popular travelling destinations [4,8]. Among these benefits, several other factors impact the choice of such accommodation, including sociodemographic characteristics of the consumer (education level, income, age), the nature of the destination (domestic or outbound) [10], as well as the general usability and previous experience in using sharing platforms [7].

Supporting the growth of the sharing economy in tourism, data from 2018 indicate that approximately 12% of European residents had already opted for shared accommodation when travelling [6], and this number is presumed to be even higher in 2025. Nevertheless, the extent of its adoption still varies significantly across European countries [5]. Within this broader European context, Spain emerges as a particularly relevant case given that it has recently become the world's second-largest international tourist destination [9]. However, the majority

of existing studies focus on tourism in large cities and coastal areas, where shared accommodation is more prevalent, while research on domestic travel and less prominent regions remains limited [1]. Therefore, the aim of this study is to classify cities in Spain according to domestic and foreign shared accommodation guest nights recorded from 2018 to 2024 using the k-means clustering algorithm.

2 RESEARCH METHODOLOGY

The data for this study have been collected from Eurostat [3]. The provided data is observed as experimental data since "*Unlike the traditional official accommodation statistics collected via business surveys or via local authorities, the data is collected using previously unexplored channels, namely data on listings and bookings obtained from four major booking platforms – Airbnb, Booking.com, Tripadvisor and Expedia Group – acting as intermediaries*." Currently, the data on shared nights is available for a timeseries from 2018 to 2024. Therefore, quite recent data have been published. The data is given for 1238 EU cities/regions/countries. Also, the data is aggregated for foreign tourists, domestic tourists, and the total number of tourists. For this study, we observed 59 cities in Spain over the period 2018-2024, including both foreign and domestic shared nights. To segment the chosen cities, we first used the *nbclust* package to determine the optimal number of clusters. Afterwards, the k-means algorithm was applied using the *cluster* package. The analysis was done in RStudio.

3 RESULTS

Spain is one of Europe's most popular tourist destinations. According to Eurostat's data on the number of shared nights, the year with the most foreign shared nights is 2024, with over 110 million shared nights (Table 1). Looking at domestic shared nights, the most was also measured in 2024, as many as over 60 million. During the COVID pandemic, foreign shared nights plummeted due to travel restrictions, while domestic shared nights were not as affected. Namely, in 2020 and 2021, there were more domestic shared nights than foreign. Following the COVID-19 pandemic, both domestic and foreign shared nights have increased. Namely, between 2021 and 2022, the number of foreign shared nights increased by 122.22%.

Table 1: Overall collected data for Spain on domestic and foreign shared nights, 2018-2024

Shared nights	**2018**	**2019**	**2020**	**2021**	**2022**	**2023**	**2024**
Domestic	29,078,012	34,348,850	24,493,054	36,398,589	44,902,360	52,534,357	60,387,142
% change		18.13	-28.69	48.61	23.36	17.00	14.95
Foreign	65,310,740	71,253,133	19,713,855	32,873,702	73,051,661	88,521,868	110,204,402
% change		9.10	-72.33	66.75	122.22	21.18	24.49
Difference	36,232,728	36,904,283	-4,779,199	-3,524,887	28,149,301	35,987,511	49,817,260

According to the results of the function NbClust(), which calculates as many as 24 clustering validation indices, the optimal number of clusters is three, as, by the majority rule, eight different metrics suggested retaining that many segments. In the next step, the k-means clustering algorithm, using Euclidean distance, was applied. Due to table size limitations, we present here only the cluster centres for the indicators 2022-2024 (Table 2). The first cluster is named Tourist hubs, with only five cities (Madrid, Barcelona, Alicante, Malaga, and Tenerife). These cities are the leading when it comes to shared nights, both domestic and international. Urban tourist centres, the second cluster, encompass 10 cities (Girona, Seville, Tarragona, Valencia, Mallorca, Cádiz, Granada, Lanzarote, Gran Canaria, and Fuerteventura). These cities

are also experiencing a significant number of shared nights. The largest cluster, with 44 cities, is the Local Tourist Centres cluster. Some of the cities within this segment are Salamanca, Ceuta, Almeria, Cordoba, Asturias, and Toledo. These cities are still considered "hidden gems" that are not as popular among foreign tourists.

Table 2: Retained cluster centres, reduced table showing indicators 2022-2024

Cluster	2022D	2023D	2024D	2022F	2023F	2024F
Tourist hubs	2,553,962.0	2,999,970.0	3,404,407.8	7,405,034.4	9,291,684.2	11,784,173.2
Urban tourist centres	1,398,208.8	1,594,771.0	1,785,373.3	2,688,124.7	3,137,181.6	3,826,693.9
Local tourist centres	412,510.5	490,609.0	579,803.9	207,846.4	242,991.6	295,831.8

The distribution of the cities per cluster is provided in Figure 1.

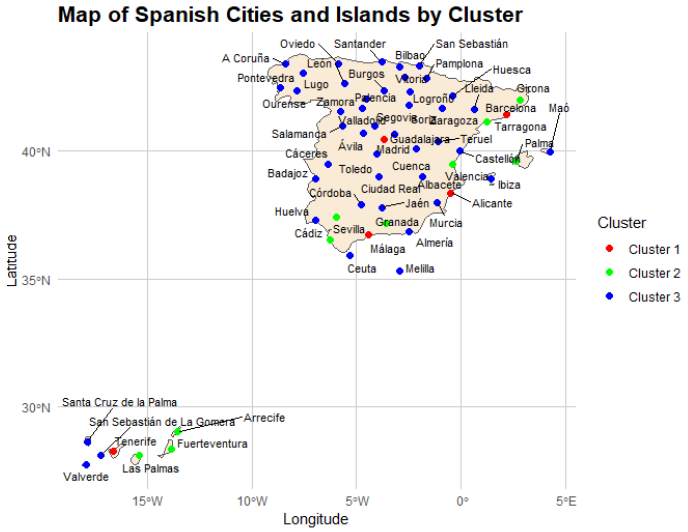


Figure 1: Distribution of the observed cities per cluster

Additional visualisation that has been produced is the trend analysis of cluster centres per year per (a) domestic and (b) foreign shared nights (Figure 2).

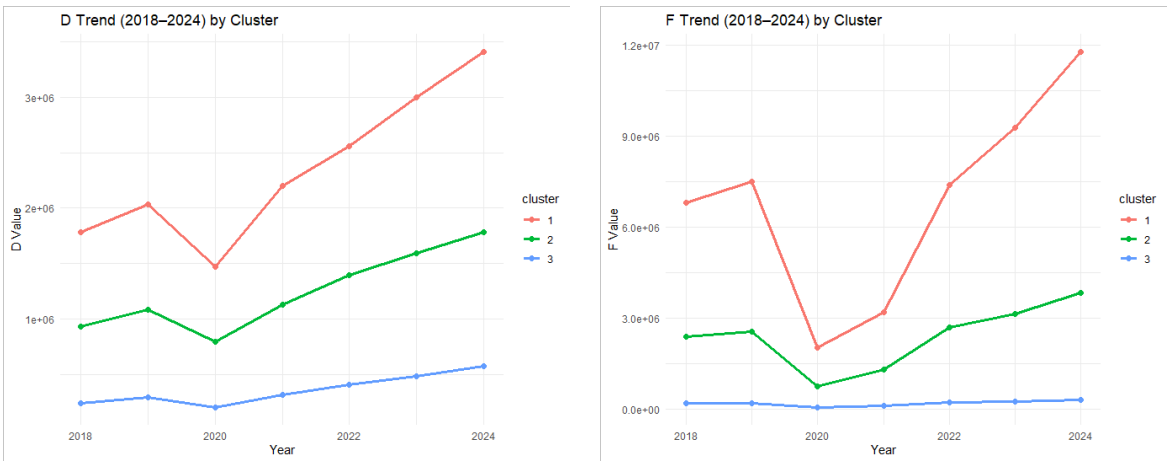


Figure 2: Trend analysis of cluster centres per year per (a) domestic and (b) foreign shared nights

It is visible that after COVID, the domestic shared nights, especially in the Tourist hubs, rose. This may indicate that residents in Spain have accepted shared accommodation practices.

Another interesting insight is that, following COVID, the interest and number of shared nights in smaller cities (cluster 3) are on the rise among domestic tourists. Looking at foreign shared nights, during the COVID pandemic, the decline was major in Tourist hubs, but in the remaining two clusters, the decline was not that drastic. The interest of foreign tourists in smaller cities has remained the same throughout the observed period.

4 DISCUSSION AND CONCLUSION

Our findings revealed three distinct clusters of Spanish cities based on shared accommodation usage, providing insight into spatial tourism dynamics. The results showed a post-pandemic recovery of both domestic and foreign shared nights, with domestic tourism playing a particularly resilient role during COVID-19. Smaller cities appear to be increasingly attracting domestic guests, indicating a hidden potential for internal tourism development. These insights provide a solid foundation for policy-making and strategic planning aimed at sustainable tourism development in Spain. Future research should explore strategies to increase foreign tourism beyond Tourist hubs, focusing on monitoring accommodation trends and promoting shared stays in underrepresented areas.

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USING SYSTEM DYNAMICS TO MODEL THE IMPACT OF PLATFORMISATION OF SERVICES ON TOURISM LABOUR MARKET IN THE EU

Blaž Rodič
University of Maribor, Faculty of Tourism, Slovenia
Rudolfovo - Science and Technology Centre Novo Mesto, Slovenia
E-mail: blaz.rodic1@um.si

Maja Turnšek
University of Maribor, Faculty of Tourism, Slovenia

Maja Breznik
The Peace Institute – Institute for Contemporary Social and Political Studies, Slovenia

Jerneja Šavrič
University of Maribor, Faculty of Tourism, Slovenia

Abstract: This paper presents the System Dynamics (SD) model developed within a research project focused on the phenomenon of platformisation of services in tourism and its impacts on the labour market, i.e. wages, working conditions and unionisation of tourism workers. Through consultations with domain experts we have developed a causal loop diagram representing the relationships between stakeholders and other influential factors, and used it as the starting point for the development of a System Dynamics (SD) model. The SD model aims to provide insight into the tourism labour market dynamics in Slovenia and EU27 through the interplay of a set of variables representing the tourists, employers and workers in tourism industry.

Keywords: tourism labour market, digital labour platforms, platformisation, simulation modelling, System Dynamics

1 INTRODUCTION

Tourism work is generally criticised as precarious and notorious for bad working conditions [1]. In recent years, tourism labour market has seen further deterioration of workers' rights and wages due to the proliferation of digital platforms. The resulting platform economy has shifted the employment practices from regular employment of workers to "work on demand" or gig economy, which has mainly benefited the employers. Next to the traditional issues of the industry, such as seasonality and concentration in urban and coastal destinations, the rise of the platform economy is feared to aggravate the working conditions, leading to joint efforts by both European hotel and trade union representatives for a level playing field and data collection [2], [3], being just one of the many pushes that led to the groundbreaking agreement of the European Commission with the four global accommodation platforms and the start of Eurostat collection of the data from the platforms.

This paper presents the development of a System Dynamics (SD) simulation model of tourism working conditions focused on the role of two particularly influential factors: state of collective bargaining, i.e. unionisation in the industry and country context and the level of tourism industry platformisation. The simulation model is aimed at providing insight into the tourism labour market dynamics through a set of variables representing the main stakeholders: employers, workers, job agencies and platforms, as well as influential factors such as level of unionisation and index of tourism seasonality. The model allows us to observe how the working conditions and wages in accommodation and food service change in response to shift from regular employment to agency and platform work.

Our research was performed within the project "The future of social dialogue in the platform economy: The case of Slovenia", funded by ARIS Slovenian Research and Innovation Agency. This project addresses two main problems identified with regards to reconfigurations of work in platform capitalism: the potential loss of bargaining power of workers over their pay and working conditions within the seemingly invisible process of algorithmic management of digital giants, and the issue of decent work due to potential precarisation of platform workers and the dissolution of workers' rights. Our research was focused on delivery workers in Slovenia working for two international digital platforms: Wolt and Glovo, which has since left Slovenia, leaving Wolt in a monopolist position.

2 METHODOLOGY

We have used System Dynamics (SD) [4] simulation modelling methodology for the development of our simulation model. The Anylogic simulation tool (www.anylogic.com) was used to develop the model. The simulation model is based on previous research on labour process theory, consultation with labour market experts, public data on collective bargaining coverage, and public statistical data on labour market and tourism industry from four different databases: Hotels, Restaurants & Cafés in Europe association (HOTREC), Eurostat, European Working Conditions Surveys (EWCS) and OECD.

2.1 SD simulation model development

Through discussion and consultation with the labour market experts within the project group we have identified the interactions and causal influences between stakeholders and other factors in the tourism labour market. While we were able to map some of the interactions directly, other interactions and influences in the labour market, e.g. labour bargaining influence required abstracting the relationships between variables (e.g. the relationship between labour's "control of output" and "control of income" and "wage levels" or "working conditions") and the introduction of additional variables (e.g. "degree of unionization", "degree of unemployment"). We have identified several variables and influences associated with platformisation and precarious employment, e.g. "share of platform jobs", "seasonality of tourists" and "share of migrant workers". Several reinforcing and controlling feedback loops were identified through discussion and the development of the causal influence diagram, affecting the development of the simulation model.

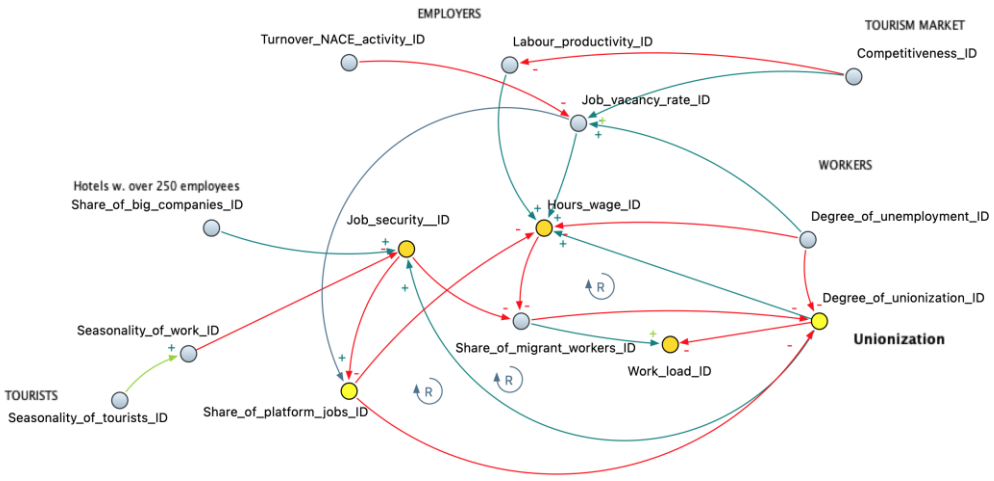


Figure 1: Causal Loop Diagram

The main interacting variables defined in the Causal Loop Diagram (Figure 1) are: Job security, Share of platform jobs, Degree of unionization, Hours wage, and Work load. Several reinforcing loops were identified, which may lead to erosion of working conditions and reduction of wages. The causal loop diagram was used as the outline of the SD model, and its elements were converted into stock and rate elements and variables. The model was then expanded with additional variables and parameters required to produce dynamic behaviour. The relationships between model elements were defined according with available data and domain knowledge. Initial values of stock elements and variables were defined according to the data from collected statistical sources, while parameter values were gradually calibrated in order to produce stable model behaviour replicating the status quo. The resulting SD model is more complex than the causal loop diagram, but still implements the same concepts and assumptions.

3 RESULTS

3.1 System Dynamics model

The central part of the model (Figure 2) are three parallel labour market processes of employment and employee attrition with stock elements representing employees with permanent employment contracts, temporarily employed, and platform workers. The stock elements are connected in a circular flow of employment and employee attrition with the stock element representing the unemployed.

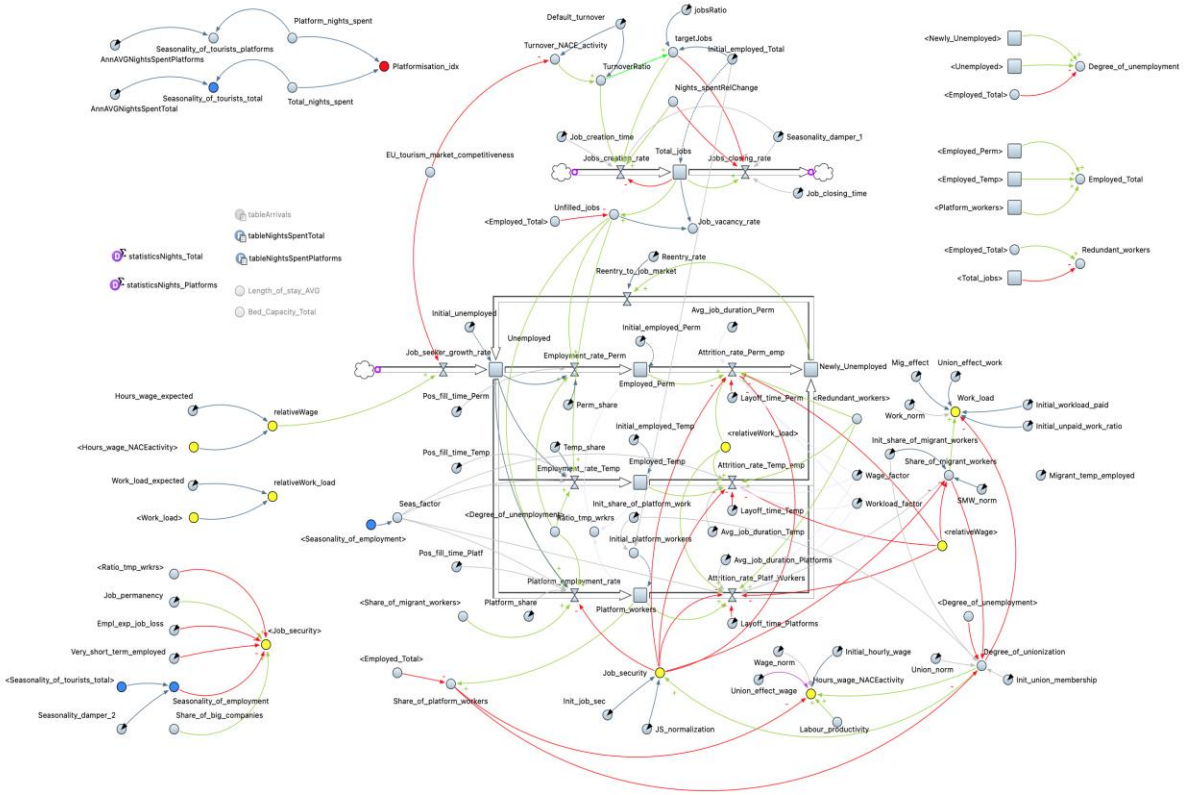


Figure 1: System Dynamics Model

3.2 Simulation scenarios

We have developed several simulation scenarios aimed to replicate the ongoing trends in the tourism industry, potential future interventions by the regulators and the impact of the major disruptions of tourism industry, such as suffered during the Covid-19 pandemic. Due to space limitations we present just one of the scenarios: decrease of tourism industry turnover due to a crisis such as the Covid-19 pandemic. In this scenario we simulate a drop of industry turnover by 75% in the second year of simulation (year 2024) to test the response of the model. In the lower chart (Figure 3) we can observe a marked drop of the Job vacancy rate from approx. 15% to zero, Degree of unionization (to 43.1%) and a negative trend of Job security (average dropped from 0.8 to 0.678), while in the upper chart the number of temporarily employed suffered the most perturbations (dropping by 36% before recovering to 85% of starting value) while the number of platform workers temporarily dropped by 33%, and permanently employed suffering a smaller (16%) but prolonged decrease.

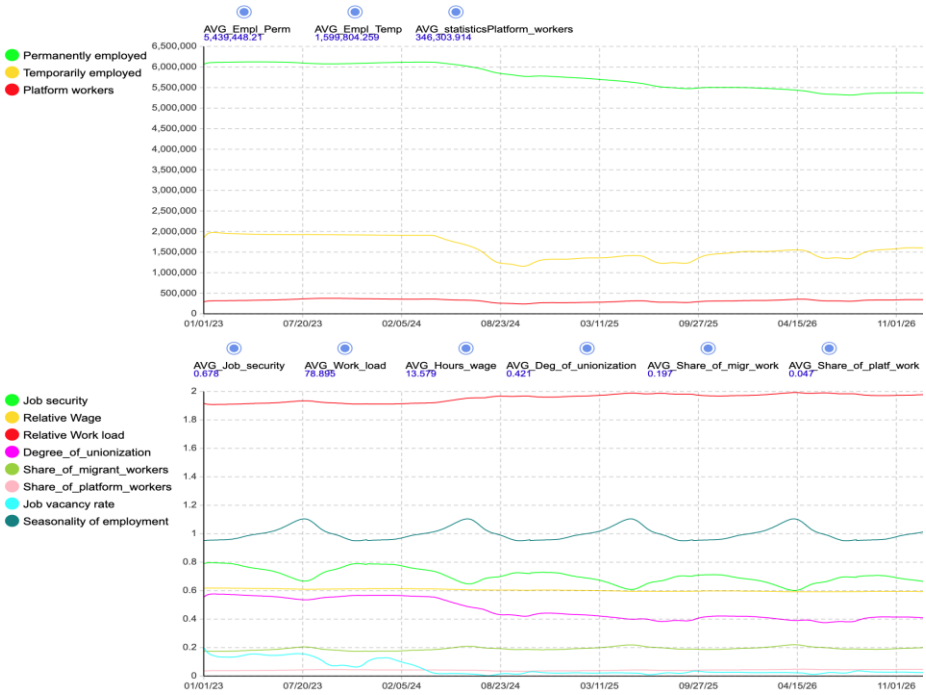


Figure 2: Scenario: Decrease Of Tourism Industry Turnover

4 CONCLUSIONS

The developed model allows us to observe the interaction of several tourism labour market variables providing insight into the relationships between working conditions and wages and platformisation of work. We should however note that due to the limitations of data on causal influences between some of the variables, the model accuracy is insufficient for quantitative analysis and prediction. It is however usable as a qualitative tool for exploration of labour market dynamics, and education of stakeholders. To this purpose we intend to develop a version of the model that will integrate several additional scenarios (e.g. “strike action”) with interactive simulation controls, which will allow the use of model for “serious gaming”.

Acknowledgement
This research was funded by ARIS Slovenian Research and Innovation Agency, grant number J5-4586 “The future of social dialogue in the platform economy: The case of Slovenia”.

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The 18th International Symposium on
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COMPARISON OF HIERARCHICAL CLUSTERING-BASED PORTFOLIO OPTIMIZATION FRAMEWORK IN MULTI-ASSET ETF

Qian Gao and **Aleš Kresta**
VSB - Technical University of Ostrava, Department of Finance
17. listopadu 2172/15, 708 00 Ostrava-Poruba, Czechia
E-mails: qian.gao@vsb.cz, ales.kresta@vsb.cz

Abstract: This study compares hierarchical clustering portfolio models, including Hierarchical Risk Parity (HRP), Hierarchical Equal Risk Contribution (HERC), and Nested Clustered Optimization (NCO), with the Critical Line Algorithm (CLA) across 42 Exchange-Traded Funds (ETFs) that span equities, bonds, commodities, real estate, and cryptocurrencies. The research utilizes daily data from 2020 to 2024, with models trained on an 80% in-sample period and their out-of-sample performance evaluated from May to December 2024. Results indicate that hierarchical methods, particularly HRP and NCO, consistently outperform CLA by enhancing diversification through correlation-driven clustering and reducing sensitivity to estimation error. These findings highlight the robustness and adaptability of hierarchical frameworks in managing complex, multi-asset portfolios.

Keywords: hierarchical clustering, multi-asset ETF, minimum variance, portfolio optimization, risk diversification.

1 INTRODUCTION

Modern portfolio optimization confronts inherent limitations in high-dimensional settings. Mean-variance frameworks reliant on precise covariance matrix estimation yield suboptimal solutions due to nonlinear asset interactions and regime shifts (García-Medina & Rodríguez-Camejo, 2024). Hierarchical clustering methods address this by grouping correlated assets into quasi-independent clusters, enhancing stability (Raffinot, 2018). While the Critical Line Algorithm (CLA) resolves quadratic optimization under constraints, its structural fragility emerges from extreme sensitivity to return estimation errors (López de Prado, 2016b).

Nested Clustering Optimization (NCO) enhances robustness by incorporating hierarchical noise filtering into risk parity frameworks (López de Prado, 2016a), with empirical evidence demonstrating superior Omega ratios through subclass-based clustering (León et al., 2017). Although validated in equity portfolios, these methods require systematic validation for alternative assets, such as cryptocurrency Exchange-Traded Funds (ETFs). Studies have shown that although CLA performs better than HRP on training data, HRP outperforms CLA in out-of-sample testing (Sen et al., 2021). However, computational scalability in high-dimensional heterogeneous asset spaces remains a critical implementation barrier.

This study focuses on the cumulative return trajectories across strategies, the distribution patterns of asset weights, and the dynamic clustering structures revealed through dendrogram analysis. It aims to evaluate the adaptability and effectiveness of hierarchical optimization frameworks in the context of diversified asset allocation.

2 METHODOLOGY

HRP allocates weights using a top-down, recursive risk-budgeting approach across the hierarchical dendrogram, where risk budgets are inversely proportional to cluster volatilities.

$$w_{V_{k1}} = \frac{\mathcal{R}(V_{k2})}{\mathcal{R}(V_{k1}) + \mathcal{R}(V_{k2})} \cdot w_{V_k}, \tag{1}$$

where V_k is the node level identifier, w_{V_k} is the weight of the parent node combination, $\mathcal{R}(V_{k1})$ is the overall risk of the child node V_{k1} combination, and the final weight is the product of all fork weights on the path.

HERC embeds local risk parity constraints within HRP's framework. Its optimization objective is formulated as

$$\min_{\mathbf{w}} \sum_{m=1}^{M} \left(\frac{\mathrm{RC}_m}{\sum_{n=1}^{M} \mathrm{RC}_n} - \frac{1}{M} \right)^2 + \sum_{c=1}^{C} \sum_{i \in \mathcal{G}_c} \left(\frac{\mathrm{RC}_i}{\sum_{j \in \mathcal{G}_c} \mathrm{RC}_j} - \frac{1}{\mathcal{G}_c} \right)^2, \tag{2}$$

where RC_m is risk contribution of asset m to the portfolio, M is the total number of assets, C is the number of clusters, $\mathcal{G}_c$ is the number of assets in the cth cluster. This enforces dual-layer risk equalization across macro and micro portfolio dimensions.

NCO addresses the numerical instabilities inherent in convex optimization by constructing a reduced covariance matrix, the formula is

$$w_k = argmin \frac{1}{2} w_k^{\top} \Sigma \mathrm{w} \quad \mathrm{s.t.} \quad w_k^{\top} a_k = a_{red,k}, \tag{3}$$

where $w_k^{\top}$ is the intra-cluster weight vector, Σ is the covariance matrix, a_k is the intra-cluster expected return vector, $a_{red,k}$ is the intra-cluster constraint scalar.

CLA efficiently computes the mean-variance efficient frontier by iteratively solving a sequence of quadratic programs under linear inequality constraints. At each iteration, the algorithm identifies a set of binding constraints (assets at their upper or lower bounds) and solves the Karush-Kuhn-Tucker (KKT) conditions of the optimization problem as

$$\begin{aligned} \min_{\mathrm{w}} \quad & \frac{1}{2} \mathrm{w}^{\top} \Sigma \mathrm{w} \\ \mathrm{s.t.} \quad & \mathrm{w}^{\top} 1 = 1, \quad \mathrm{w}^{\top} \mu = r_p, \\ & w_i \in \left[w_i^{\min}, w_i^{\max} \right], \quad \forall i = 1, \ldots, N. \end{aligned} \tag{4}$$

Where r_p is the target portfolio return, and $\left[w_i^{min}, w_i^{max} \right]$ are the lower and upper bounds for asset weights. CLA traces a piecewise-linear frontier by detecting breakpoints—values of r_p where the set of active constraints changes and recomputes optimal weights accordingly.

3 EMPIRICAL RESULTS

This study evaluates the HRP, HERC, NCO, and CLA methods across 42 ETFs spanning 13 asset classes, including equities, fixed income, commodities, and cryptocurrencies. The dataset comprises daily returns from 2020 to 2024. Ward linkage-based clustering and historical covariance optimization are employed, with 80% of the sample period used for training and an out-of-sample validation conducted from May to December 2024.

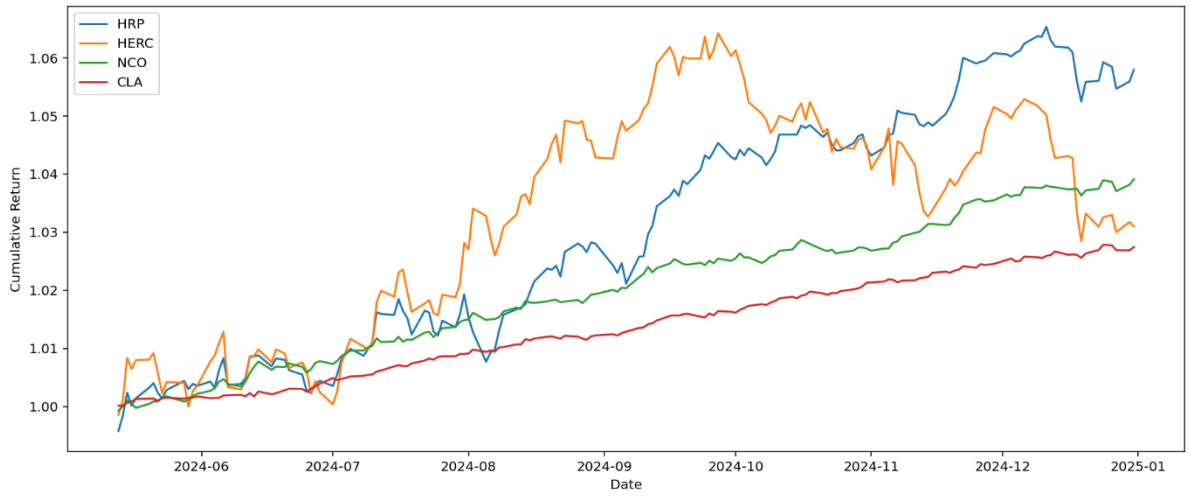


Figure 1: Cumulative returns of different model combinations on out-of-sample
Source: own research

The Figure 1 trajectories from May 2024 to December 2024 reveal critical performance among portfolio strategies. HRP achieves the highest terminal return (1.06) but exhibits pronounced volatility, reflecting its adaptive risk-budgeting capacity during market dislocations. HERC's intermediate outcome (1.04-1.05) demonstrates balanced risk allocation through clustered parity constraints, while NCO's steady ascent highlights its reduced error-robust nested architecture. CLA's minimal growth (1.02-1.03) highlights the limitations of mean-variance optimization in non-Gaussian multi-asset environments, where parameter sensitivity compromises efficiency. The hierarchy-dominated performance gradient provides quantitative validation of the structural advantages of clustering frameworks in asymmetric markets.

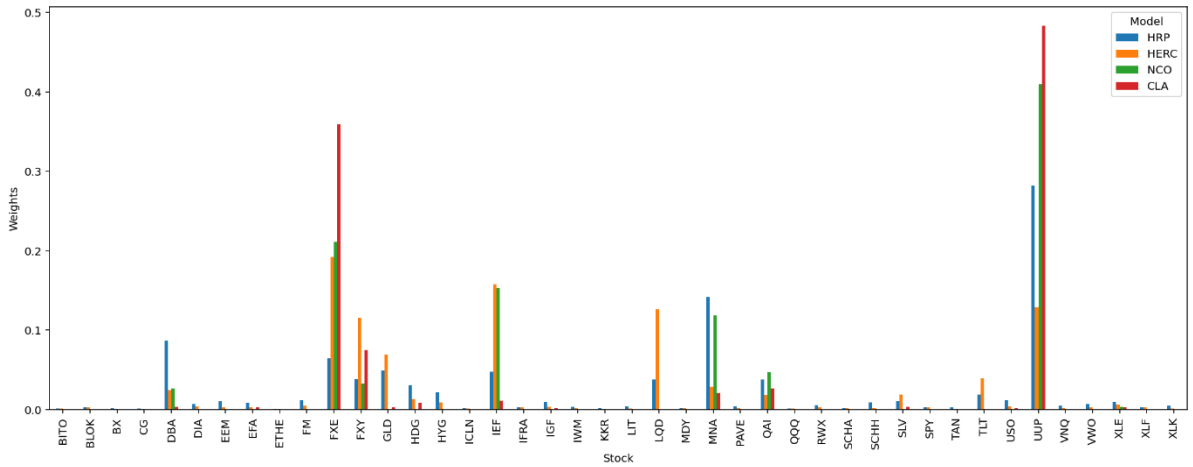


Figure 2: The weight distribution of the three portfolios on each stock
Source: own research

The Figure 2 derived from different models reveal structural differences in portfolio construction logic. The CLA tends to allocate a disproportionately high weight to just two stocks, which violates the principle of diversification and amplifies left-tail risk. In contrast, the NCO exhibits a hierarchical error-suppression structure that mitigates excessive exposure to individual risk factors, thereby achieving a more balanced allocation. The HRP allocates capital unevenly across clusters of assets with stable dependency structures based on their risk contributions, thus identifying potential returns that remain undetected under the traditional mean-variance framework. HERC further advances this approach by enforcing risk parity both across and within clusters, achieving a moderate level of risk exposure while avoiding excessive concentration in weights.

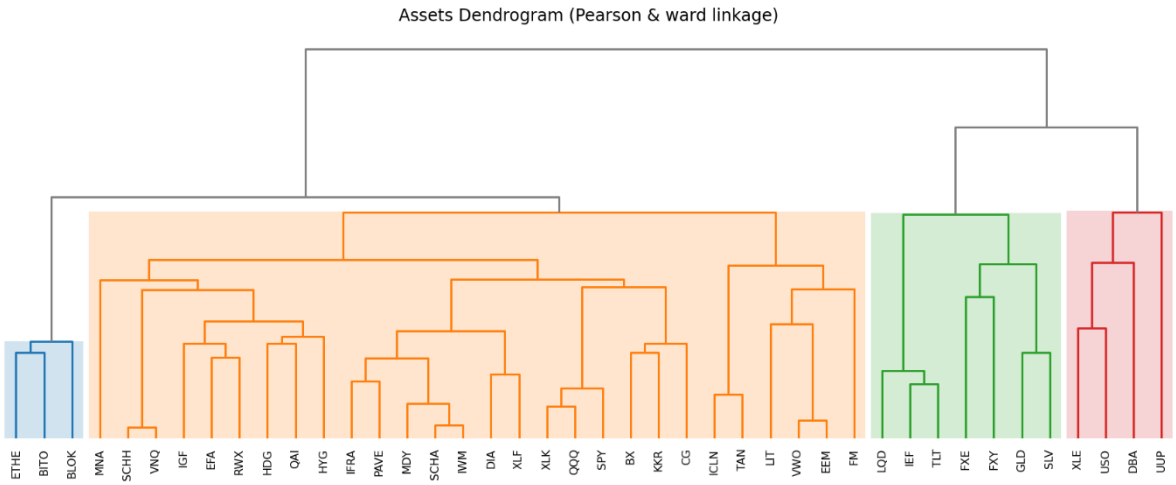


Figure 3: Hierarchical Clustering Dendrogram
Source: own research

The Figure 3 dendrogram reveals a four-level asset grouping structure based on Pearson correlation patterns, which differs from the traditional classification of 13 asset classes. In Cluster 1 (blue), cryptocurrency instruments primarily form an isolated cluster that reflects their unique risk characteristics, which is consistent with real-world observations. Cluster 2 (orange) comprises a mix of various style and sector groups that only become associated through deeper hierarchical levels, thereby avoiding information loss and indicating significant heterogeneity within the cluster. Clusters 3 (green) and 4 (red) consist of currencies, bonds, and gold ETFs, which generally exhibit more stable returns; however, they are separated and reorganized through hierarchical recombination. This demonstrates that the hierarchical method can capture mechanism-based co-movements that are systematically overlooked.

4 CONCLUSION

Empirical evidence demonstrates that hierarchical clustering methodologies (HRP, HERC, NCO) consistently outperform traditional CLA frameworks in multi-asset portfolios comprising equities, bonds, and alternative instruments. Hierarchical models achieve superior risk dispersion within portfolios and dynamically respond to non-stationary correlations that static CLA frameworks struggle to accommodate, particularly among highly volatile assets such as cryptocurrency ETFs.

By recursively partitioning assets into risk-balanced clusters while preserving inter-cluster diversification, hierarchical structures create implicit robustness against covariance misspecification. This architecture mitigates drawdowns during market dislocations by hedging correlated risk exposures more effectively than traditional optimization. Robustness is further enhanced by compounding noise-filtering weights in its independently optimized cluster hierarchy.

Therefore, hierarchical methods translate market microstructures into more stable allocation hierarchies. Future improvements include integrating regime-switching detection to dynamically adjust clustering parameters, further enhancing portfolio resilience to structural breaks.

Acknowledgement
This research was supported by VSB – Technical University of Ostrava SGS project under Grant No. SP2025/043.

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MATHEMATICAL FRAMEWORK FOR COST FUNCTION CONSTRUCTIONS IN SOME MULTI-PHASE QUEUEING SYSTEMS: REVIEW AND FURTHER IDEAS

Stefan Mirchevski[1,2], **Aleksandra Popovska-Mitrovikj**[3], **Verica Bakeva**[3]
[1] Ss. Cyril and Methodius University, Faculty of Civil Engineering, Skopje, North Macedonia
[2] European University, Faculty of Informatics, Skopje, North Macedonia
[3] Ss. Cyril and Methodius University, Faculty of Computer Science and Engineering, Skopje, North Macedonia
Emails: stefan_mircevski@outlook.com, stefan.mircevski@eurm.edu.mk,
{aleksandra.popovska.mitrovikj, verica.bakeva}@finki.ukim.mk

Abstract: In this paper, we provide a mathematical framework for new cost function constructions based on customer service speed for some multi-phase queueing systems with Poisson input stream. A review of already published results is made and new ideas for further research are provided. It can be concluded that all models are interconnected and represent one approach for analyzing multi-phase subsequent service.

Keywords: Poisson input stream, multi-phase subsequent service, cost function.

1 INTRODUCTION

Queueing theory is a branch of applied mathematics and probability theory that deals with the study of systems in which "customers" (people, data, objects, etc.) arrive at random and wait in a queue to be served by one or more servers. A queueing system is a collection of one or more input streams, one or more servers, and a queue (along with a service discipline). From this, it can be concluded that modeling systems is a rather complex mathematical process, especially when we want to make assumptions for the construction of the model that will be able to describe more real-life examples. As with other models based on probability theory, a very important part here is the construction of the cost function for such a system, through which the key elements in the system and the way the system operates will be reflected. The cost function should answer when and under what conditions the minimum cost will be achieved, while the system operates in a stationary mode. This indicates the need to include optimization techniques and methods in order to determine the points at which the cost function reaches a minimum value.

In this section we will briefly review some published papers related to the construction and analysis of various cost functions. The paper [11] examines an $M/G/1$ queue with working breakdowns under an N-policy, where the server initiates service only when the queue length reaches a threshold N. The cost function incorporates holding, server operation, breakdown, startup/shutdown, and service rate costs. A two-stage optimization method, combining direct search and quasi-Newton techniques, is employed to determine the optimal policy parameters. Similar proposed models and constructions exist for discrete systems, for example, in [5]. There are also papers that cover multi-phase customer service with Poisson input stream with corresponding cost function constructions. The paper [6] examines a tandem queueing system with Poisson arrivals, focusing on dynamic resource allocation to minimize long-run average costs. It employs Markov decision processes to derive optimal policies. There are many other interesting studies related to this issue, such as [1], [2], [3], [4] and [10].

In the rest of the paper we will provide a mathematical framework for new cost function constructions based on customer service speed in stationary mode with subsequent service.

2 MATHEMATICAL FRAMEWORK BASED ON CUSTOMER SERVICE SPEED

In this section, we provide a mathematical framework for cost function constructions based on customer service speed. We will use only phase-type distributions for subsequent customer service across the phases, such as Erlang, hypoexponential and Coxian distribution.

2.1 Erlang-k customer service time

In [8], we propose a construction of a cost function for a single-server k-phase system, where the variable is the traffic intensity $0 < \rho < 1$. Service time through the phases is exponentially distributed with common parameter $\mu > 0$. It has been shown experimentally that for such a construction of a cost function there will always be only one candidate for an optimal solution, i.e., the function reaches an absolute minimum at the point ρ^*. After finding ρ^*, the optimal value μ^* that generates the optimal speed of serving customers through the phases is found. This model is a variant where we have the same service speed throughout all phases, in a stationary mode.

2.2 Hypoexponential-2 customer service time

As a continuation of the results in the paper [8], we came up with the idea of forming a two-phase system where the service speed between phases will be different and linearly dependent. Namely, we have two-phase exponentially distributed customer service time with parameters $\mu_1 > 0$ and $\mu_2 > 0$, and we introduced the dependency $\mu_2 = \alpha\mu_1$, so that $0 < \alpha < 1$. This approach offers the possibility that in each subsequent phase the service speed will be lower than the previous phase. For the formed cost function, we proved that the function reaches an absolute minimum at the point ρ^*, from where the optimal speeds μ_1^* and μ_2^* in the stationary mode are found. The results of this research are published in the conference paper [7]. If we put $\alpha = 1$ and $\mu_1 = \mu_2 = \mu$, we obtain a special case of construction for Erlang-2 customer service time.

2.3 Hypoexponential-k customer service time

A generalized cost function for k-phase customer service is proposed in [9], as a continuation of the research [7] discussed in Subsection 2.2. In this case the dependence of the parameters is $\mu_i = \mu_1\alpha^{i-1}$, for $i = 1,\ldots,k$, so that $0 < \alpha < 1$. The service time through the phases is exponential with parameter $\mu_i >$, $i = 1,\ldots k$, such that $\mu_i \neq \mu_j$, for $i \neq j$. This cost function construction was written in terms of Lagrange basis polynomials. The existence and uniqueness of the minimum is also valid for this model, and the proof is derived using the intermediate-value theorem and the convexity of the function on the entire interval $(0,1)$. If we put $\mu_i = \mu$, for $i = 1,\ldots,k$, we obtain the cost function from the model in [8].

2.4 Coxian-k customer service time

In this subsection we present a new idea for the construction of a cost function that has not yet been published. Inspired by previous constructions and dependencies between parameters

for customer service speed, we continue our research in the direction of allowing customers to continue service to the next phase with a certain probability. In contrast to this idea, all previous models were based on the fact that a customer moves to each next phase with probability 1. Now, as a basis, we use the Coxian-k distribution for customer service time, with parameters $\mu_i > 0$, for $i = 1, \ldots, k$ and probabilities $0 \leq p_j \leq 1$, for $j = 1, \ldots, k-1$, where p_j is the probability that the customer will move from the j-th to the $(j+1)$-th phase. Although the probability density function of the random variable indicating the customer service time through all phases does not have a closed form, we used the analytical-matrix method and instead of matrix notation, we constructed it through symbolic notation. We constructed a cost function as a function of the traffic intensity ρ through variability in the speed of customer service across phases, by relation $\mu_i = \mu_1 \alpha^{i-1}$, for $i = 1, \ldots, k$ and $0 < \alpha < 1$. This construction allows us to combine the previous ideas. Namely, by introducing the dependence $\mu_i = \mu_1 \alpha^{i-1}$, for $i = 1, \ldots, k$, we obtain the following possibilities:

- For $\alpha = 1$, i.e., $\mu_i = \mu$, for $i = 1, \ldots, k$ and $p_j = 1$, for $j = 1, \ldots, k-1$, the model reduces to Erlang-k customer service time;
- For $0 < \alpha < 1$ and $p_j = 1$, for $j = 1, \ldots, k-1$, the model reduces to hypoexponential-k customer service time, thus providing a tendency for speed reduction in each subsequent phase. Otherwise, if $\alpha > 1$, the system provides a tendency to increase the speed in customer service in each subsequent phase.
- For $0 < \alpha < 1$ and $0 \leq p_j \leq 1$, for $j = 1, \ldots, k-1$, the model for Coxian-k customer service time and subsequent service speed reduction will be obtained, which represents a new idea for cost function construction. Otherwise, if $\alpha > 1$, we get the opposite situation with the subsequent service speed increasing.

3 BEHIND THE COST FUNCTION CONSTRUCTIONS

The basis for constructing the proposed cost functions from Section 2 refers to the expected waiting costs per customer per unit of time (C_w) and the expected system costs in terms of server depreciation (amortization) across phases per customer per unit of time (C_s). This can be represented as follows:

$$\Phi = C_w + C_s,$$

where Φ represents the expected total cost of the system. In more detail, the components of the cost function can be decomposed as follows:

- C_w is a product of the waiting cost per customer per unit of time $c_w > 0$ and the expected number of customers in the system per unit of time (for all proposed models from Section 2);
- C_s is a linear combination of the phases' depreciation costs $c_i > 0$, $i = 1, \ldots, k$ per customer per unit of time with the service parameters $\mu_i > 0$, $i = 1, \ldots, k$, respectively (for hypoexponential-2 customer service time, hypoexponential-k customer service time and Coxian-k customer service time when $0 < \alpha < 1$);
- C_s is a product of the phases' depreciation cost $c > 0$ per customer per unit of time (same cost for all phases), the service parameter $\mu > 0$ and the number of phases k (for Erlang-k customer service time and Coxian-k customer service time when $\alpha = 1$).

4 LIMITATIONS AND POSSIBILITIES FOR IMPROVEMENT

We list some limitations of the investigated and newly proposed cost function constructions, based on the basic assumptions and theoretical premises of the models.

- In all proposed models, the customers begin their service after the previous customers have completed service in all phases;
- Between service phases in the system, no additional delay due to technical or human factors is included.
- All customers must be served at all phases, without the possibility of skipping any of the phases. Additionally, the service of all customers begins from the first phase.

In order to overcome these limitations, and thereby ensure a more realistic application of multi-phase queueing systems and their corresponding cost functions, in the future we plan to deal with theoretical settings of new models structured as follows (but not limited to it):

To form systems in which each of the k phases will represent a separate server for serving that can serve one customer at a given moment, and in front of each server (phase), there is its own waiting queue (tandem queueing systems). This overcomes the limitation on the number of customers that are served simultaneously. On the other hand, in many real situations, some customers for justified reasons need to start serving from a phase other than the first, and continue subsequently to the end. To achieve this, it is planned to apply the method of coloring customers, so that the customers will be assigned an appropriate class in terms of color, as follows: through the input stream come customers with a "blue" color who must be served through all phases, starting from the first to the k-th, while the "red" color is used for customers who will start serving from the i-th phase with probability p_i, for $i = 1, \ldots, k$ and will continue subsequently until the final phase. At the very entrance to the input stream, customers with red color arrive with probability γ, while those with blue color, with probability $1 - \gamma$. Finally, it is planned to introduce the concept of random noise between the service phases, i.e., in addition to the basic time of serving customers through the phases, it is possible that a delay of service may occur due to technical reasons or human factors. Due to this uncertainty, an additional random variable with a known probability distribution will be introduced at each phase, which will determine the additional delay between two consecutive phases. For these new model settings, it is planned to form and analyze appropriate cost functions, which in special conditions, can be reduced to the previously obtained cost functions.

5 CONCLUSION

In this paper, we have made a brief review of published results and provided ideas for further research towards the construction of a cost function based on the customer service speed. It can be concluded that the introduced linear dependence between the parameters can describe many models involving multi-phase service. On the other hand, this dependence offers opportunities for rapid transformations of the form of the cost function and further generalizations. With the proposed plan for further research, the limitations of these systems can be largely overcome.

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DIGITAL DRIVERS OF HUMAN DEVELOPMENT IN THE EUROPEAN UNION: A DYNAMIC PANEL ANALYSIS

Renee Škulić Ažić
Faculty of Economics and Business, University of Rijeka, Department of National Economy and Development
Ivana Filipovića 4, 51000 Rijeka, Croatia
E-mail: renee.skulic.azic@efri.uniri.hr
Faculty of Economics and Business, University of Maribor, Razlagova ulica 14, 2000 Maribor, Slovenia
E-mail: renee.skulic@student.um.si

Abstract: This paper examines how digital technologies influence human development in EU countries (2008 – 2019) using dynamic panel data models. Results indicate that industry activity, internet access, and skilled human capital significantly improve HDI, while patents and R&D spending show no effect. The novelty lies in considering industry activity, science and tech workforce, patents, and R&D in the HDI context. The study improves understanding of EU human development and guides future actions. Findings support evidence-based policies for digital inclusion and human-centred innovation in Europe.

Keywords: ICT, HDI, economic development, technology, digital transformation

1 INTRODUCTION

In recent decades, information and communication technologies (ICT) have played a crucial role in driving both economic and human development. Digital transformation is among the key forces reshaping modern society and the economy [2], hence functioning without ICT is almost unimaginable [9]. Just like previous industrial revolutions that boosted productivity and wellbeing [11], the current digital era also carries similar potential. The COVID-19 pandemic accelerated digitalisation in all sectors, revealing the transformative power of digital tools and their irreversible impact on human capital [14]. Internet usage increased dramatically, with some countries reporting up to 60% growth shortly after the outbreak, emphasizing the importance of digital access for education, employment, and healthcare.

Technological advancement reduces production costs, enables task automation, and increases productivity, potentially improving quality of life and human wellbeing [4]. ICT contributes not only to economic growth but also to broader human development by enhancing education, health, and living standards [1]. Despite this, in the last two decades, most research has focused on the impact of ICT on economic growth, while its role in human development remains less explored. Moreover, different indicators are vastly chosen over measures such as the Human Development Index (HDI), which combines aspects of both human and economic progress [12, 9].

A growing part of economic and business activity is now driven by digital technologies [10]. Therefore, ensuring equal economic development through digitalisation has become a global priority [18]. ICT has the potential to boost productivity, growth, and employment, while also supporting human and social development [6]. Still, less developed countries fall behind, which highlights the importance of promoting ICT as a tool for both economic and human progress [1]. Since technological development improves quality of life [15, 7], its role will only grow [8]. That being the case, future research should keep focusing on the link between ICT and human development [3].

2 DATA AND METHODOLOGY

The analysis uses panel data from 27 EU countries covering 2008–2019. The dependent variable is HDI**,** sourced from UNPD, and used as a proxy variable for human development. Independent variables include: Industry Activity Index (IND – UNCTAD), Total Patents (PAT

– OECD), Human Resources in Science and Technology (HRST – Eurostat), Internet Access (INT – Eurostat), and R&D expenditure (RND, as% of GDP – World Bank). Dynamic panel estimator Blundell-Bond is used to control for endogeneity and unobserved heterogeneity. The analysis was conducted utilizing the statistical software package STATA 15.

Derived from the aforementioned variables, the econometric model equation is as follows:

$$HDI_{it} = \beta_0 + \gamma HDI_{i,t-1} + \beta_1 IND_{it} + \beta_2 PAT_{it} + \beta_3 HRST_{it} + \ +\beta_4 INT_{it} + \beta_5 RND_{it} + \alpha_i + \varepsilon_{it} \quad (1)$$

The objective of this paper was to investigate how digital technologies, specifically infrastructure, human capital, and innovation indicators, affect human development across EU countries. The study aims to identify which components of digital transformation contribute most significantly to improving human development. The main research question is - which specific dimensions of digital transformation (e.g., internet access, science and technology workforce, R&D investment, patent activity, and industry capacity) should be prioritized to generate measurable and positive impacts on human development in EU member states?

3 RESULTS AND DISCUSSON

The two-step system GMM estimator is employed due to its capacity to relax the assumption of error term independence, robustness against heteroscedasticity, and enhanced efficiency compared to the one-step estimator [16]. Table 1 summaries the key estimation results.

Table 1: Key estimation results (Blundell – Bond Two step robust)

Independent variables	*BB2robust Coefficients*	*BB2robust Standard Errors*
L.HDI	0.628***	(0.0835)
IND	0.0230**	(0.00972)
PAT	-0.000000144	(0.00000026)
HRST	0.00104***	(0.000238)
INT	0.000200**	(0.0000935)
RND	0.000511	(0.00170)
Cons	0.255***	(0.0635)
N	297	
Sargan Chi2	24.40482	p=0.1811
AB AR(2)test z	-0.88092	p=0.3784

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Within the context of the BB two-step robust estimator, statistically significant variables are lagged HDI, Industry Activity, Human Resources in Science and Technology and Internet Access. Moreover, as the lagged dependent variable is both less than 1 and positively and statistically significant, the stationarity condition is met, validating the application of a dynamic panel model in this paper. The Sargan test (p = 0.1811) confirms instrument validity and indicates no endogeneity in the two-step Blundell-Bond estimation. Finally, the AR(2) test (p = 0.3784) indicates no evidence of second-order autocorrelation in the model.

The results of the dynamic panel regression show that the lagged dependent variable, HDI, has a significant and positive impact on current HDI levels, confirming the persistence of

human development over time and aligning with previous studies. Industry activity also shows a strong positive effect, suggesting that the capacity to adopt and adapt advanced technologies within industry plays a critical role in improving human development. Although rarely included in similar models, this finding highlights the need for more focused research on the industry–HDI relationship. In line with this, human resources in science and technology (HRST) significantly contribute to HDI, reflecting the importance of tertiary education and a skilled workforce in fostering high-tech production and exports. This supports earlier findings linking education with both economic and social development [5, 8]. Previous research when including such variables as HRST opted for the number of ICT specialist, ICT professionals or technicians employed in a country. These variables would also prove to be significant but not necessarily in a positive relation. For instance, [17] came to the conclusion that the number of domestically employed ICT professionals is significant but negatively affects HDI. In this paper HRST has proven to be a good alternative variable and it deserves further attention. On the other hand, total number of patents did not show a statistically significant relationship with HDI, possibly due to measurement or conceptual mismatches, though this result remains relevant for future research. Households with internet access significantly and positively affect HDI, which is consistent with prior literature [13, 15, 17], highlighting the significance of digital inclusion. Lastly, gross domestic R&D expenditure did not show significance, though it is often expected to contribute to human development. Overall, the findings suggest that digital technology has a measurable impact on human development in EU-27 countries, although not all indicators yielded statistically significant results.

4 CONCLUSION

In this paper, the impact of different factors on human development within European Union countries was examined. The originality of this paper lies in including industry activity, human resources engaged in science and technology, total number of patents and research and development expenditure in the model. It is interesting and empirically important that industry activity and human resources engaged in science and technology have proven to be significantly and positively related to HDI. These insights contribute to a better understanding of the complex dynamics shaping human development and offer guidance for policy formulation and future research directions with implications for both organizational strategy and public policy in the EU. Finally, this study supports ongoing efforts toward inclusive digital transformation across the EU.

Acknowledgement:
This research was funded by the University of Rijeka, „Održivi razvoj i razvoj ljudskih potencijala“ – uniri-iskusni-drustv-23-275.

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PROJECT TASK DESCRIPTIONS QUALITY ASSESMENT USING ENTROPY AND TEXTUAL METRICS

Tatyana Unuchak
University of Maribor, Faculty of Organizational Sciences,
Kidričeva cesta 55a, 4000 Kranj, Slovenia
tatyana.unuchak@student.um.si

Yauhen Unuchak
University of Maribor, Faculty of Organizational Sciences,
Kidričeva cesta 55a, 4000 Kranj, Slovenia
yauhen.unuchak@student.um.si

Mirjana Kljajić Borštnar
University of Maribor, Faculty of Organizational Sciences,
Kidričeva cesta 55a, 4000 Kranj, Slovenia
mirjana.kljajic@um.si

Abstract: An approach to assessing the quality of textual descriptions of IT project tasks on the data set obtained from the Kaggle portal is presented. For this purpose, several metrics were used (semantic metrics, text structure metrics, stylistic metrics, linguistic metrics) and correlation analysis was conducted. Results of the study show that quality of project task descriptions can be objectively assessed, and a minimal informative set of metrics are suggested.

Keywords: task description, project management, text assessment, entropy, semantic metrics, structure metrics, stylistic metrics, linguistic metrics.

1 INTRODUCTION

The formulation of project task descriptions has a direct impact on the success of project team [3]. A clear, concise and understandable description of project tasks helps the team to accurately define the scope of work, avoid misunderstandings and shorten the project timeline. However, in real life, task descriptions are often redundant, vague or, on the contrary, too brief. Poor project task descriptions impede communication within the project team and can lead to deadline violations or improper implementation. In this regard, an important area of research is the formalized measurement of the quality of textual descriptions of project tasks using objective metrics [2].

We hypothesize that the quality of project task description can be assessed by using various text metrics such as length, vocabulary variety, readability, semantic coherence. To test the hypothesis, we conducted a quantitative analysis of project tasks description texts from the "Jira dataset-2025-Tamseel" published at https://www.kaggle.com/.

Testing metrics to assess the quality of the textual descriptions of project tasks in this study allows us to identify the most reliable and informative metrics. It also enables us to assess their applicability and determine the advantages, disadvantages, and redundancy of individual approaches. This allows us to select only the most useful metrics, confirm their effectiveness on real data, and ensure their further use in project management practice.

2 METHODOLOGY

We conducted the analysis on a sample of 156 task descriptions. Prior to analysis, we removed all special characters and converted them to lower case. We computed the following groups of metrics:

- Structural characteristics [1]: text length (in words), number of unique words, number of syllables and sentences.

- Linguistic metrics [5]: Shannon entropy (diversity score), Harkevich measure (informativeness), readability on Flesch, SMOG, Gunning Fog scales.
- Semantic metrics [4]: BERT Score – evaluation of semantic proximity to the reference description; ROUGE-L and SacreBLEU Score – coincidence with the reference at the level of word combinations.
- Style metrics [1]: conciseness (share of unique words), presence of clichéd expressions, number of linking words, share of stop words.

The BERT Score and SacreBLEU Score metrics were calculated in comparison to a reference description of the project task, which was manually prepared with the involvement of experts. We paid special attention to analyzing the correlation between the metrics, as it enabled us to uncover hidden dependencies between formal attributes and the content structure of the text.

3 RESULTS

Table 1 summarizes the main statistics on key metrics. It shows that most tasks have short, concise, and varied descriptions. Meaningful similarity to the benchmark as measured by the BERT Score and SacreBLEU Score remains low in most cases, indicating a lack of a unified style.

Table 1: Key values of the obtained metrics (sample).

Metric	***Min***	***Median***	***Max***	***Interpretation***
Description Length	2	8	41	Most descriptions are short and concise (5-13 words), single outliers are longer.
Shannon Entropy	1.0	3.0	4.8	Good variety of words, low entropy – repetition or boilerplate.
BERT Score	-0.59	-0.29	-0.06	The semantic similarity with reference is low, harmonization efforts are needed.
Concise Ratio	0.57	0.98	1.0	Almost every word is unique, with a high level of concision and precision.
Flesch Reading Ease	-125	47	120	Most descriptions are readable, individual descriptions are overly complex or overly simple.
Average Word Length	4.3	4.7	7.0	Words of medium length (4-5.5 characters), readability is high.
Gunning Fog Index	1.0	9.0	28.0	Optimal difficulty (6-14), rare tasks are difficult to grasp.
Flesch-Kincaid Grade Level	0.0	8.0	30.0	Reading level is appropriate for grades 6-12; isolated outliers are complex descriptions.
SMOG Index	3.0	9.0	16.0	Most tasks are understandable to specialists, some are overloaded.
Stopword Ratio	0.0	0.1	0.6	Most of the texts are concise, with a minimum of "water", isolated ones are colloquial.
Unique Words Count	2.0	8.0	32.0	Usually 6-11 unique words for brevity and accuracy.
Cliché Ratio	0.0	0.0	0.0	Clichés are practically not used, the style is strict and technical.
Cohesion (Connective) Count	0.0	0.0	0.0	There are almost no logical connections, which is typical for technical tasks.
Sentence Count	1.0	1.0	3.0	Most descriptions consist of a single sentence
Harkevich Measure	6.9	8.7	10.5	The informativeness is balanced, most descriptions are uniform and clear.

Correlation analysis (Figure 1) showed that some metrics of textual description quality have a very high degree of correlation among themselves (e.g., description length, number of unique words, and Shannon entropy), while other metrics reflect independent aspects of quality.

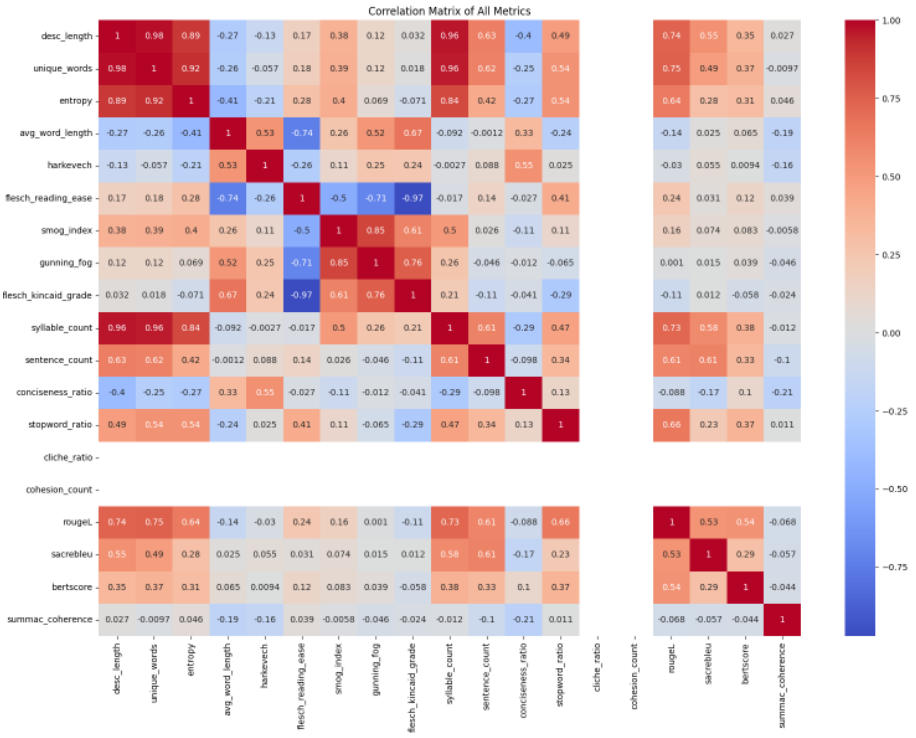


Figure 1: Correlation matrix between metrics.

Metrics like Description Length, Unique Words, and Shannon Entropy are strongly correlated ($r > 0.9$). For practical use, selecting just one of these (e.g., entropy or unique words) is sufficient to avoid redundancy and multicollinearity in quality analysis of textual descriptions of project tasks.

The strong correlation among readability metrics (Flesch Reading Ease, Gunning Fog, Flesch-Kincaid Grade, SMOG Index) allows us to use just one or two of the most informative ones. Flesch Reading Ease and Flesch-Kincaid Grade Level are widely used and provide easily interpretable values for text comprehensibility.

Semantic metrics (BERTScore, ROUGE-L, SacréBLEU) correlate weakly with linguistic metrics, indicating that they add unique information about text quality that is not overlapped with formal characteristics. Their combined use with linguistic metrics will help to obtain a more complete and objective assessment of the quality of the description of design tasks.

The combination of a high Concise Ratio metric value and a low Stopword Ratio metric value identifies project task descriptions that are characterized by conciseness and lack of redundant formal phrases. Based on the data obtained it is possible to form a minimal informative set of metrics for assessing the quality of textual descriptions of project tasks:

- *Unique Words or Shannon Entropy* (to assess diversity and informativeness).
- *Flesch Reading Ease and/or Gunning Fog* (to assess readability).
- *Conciseness Ratio* (to assess conciseness).
- *Stopword Ratio* (to detect excessive formalization).
- *Average Word Length* (to characterize text structure).
- *BERTScore and/or ROUGE-L* (to evaluate semantic similarity).
- *Summac Coherence* (to assess coherence).
- *Harkevich* (for information density).

Excluding redundant metrics can simplify analysis and improve the objectivity of results without losing key information.

4 DISCUSSION AND CONCLUSION

The presented analysis confirmed our hypothesis that the quality of task descriptions can be assessed using a set of objective metrics. These metrics have practical value: project managers can use them to automatically validate descriptions, while quality engineers can identify incomplete or ambiguous tasks. Metrics may also serve as an input for machine learning models to categorize or evaluate project tasks.

However, the findings may be limited by algorithm settings (e.g., tokenization, stop word lists) and the dataset used, as our analysis was based on a limited set of English-language design tasks without expert review. This restricts the generalizability of results to other text types and industries and may overlook subjective quality aspects.

In the future, expanding the dataset, analyzing the impact of descriptions on task completion time, and applying metrics to task prioritization are promising directions.

Acknowledgement
The authors acknowledge the financial support from the state budget by the Slovenian Research Agency (project No. P5-0018).

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ON QUBO RELAXATIONS TO THE SPARSEST k-SUBGRAPH PROBLEM

Omkar Bihani[†], Janez Povh[†,‡], Dunja Pucher[*1]

[†]Rudolfovo - Science and Technology Centre Novo mesto, Novo Mesto, Slovenia
[‡]University of Ljubljana, Faculty of mechanical engineering, Slovenia
[*]University of Klagenfurt, Department of Mathematics, Austria
email: omkar.bihani@rudolfovo.eu, janez.povh@rudolfovo.eu, dunja.pucher@aau.at

Abstract
We initiate a theoretical study of QUBO formulations for the sparsest k-subgraph problem, which seeks a subgraph on k vertices with as few edges as possible. While QUBO models are attractive for both classical solvers and quantum annealing, encoding cardinality constraint remains nontrivial. We explore several modeling approaches, including the exact penalty method, Lagrangian relaxation, and the augmented Lagrangian method, and analyze how these techniques influence the structure and theoretical properties of the resulting QUBO formulations. Our goal is to highlight key modeling trade-offs and support the future design of effective formulations for quantum and classical solvers.

Keywords: The sparsest k-subgraph problem, QUBO formulations
Math. Subj. Class. (2020): 90C27

1 Introduction

The sparsest k-subgraph (SkS) problem is a fundamental NP-hard problem in combinatorial optimization. Given a graph G on n vertices and a positive integer $k \leq n$, the task is to find a subset of k vertices that induces the smallest number of edges. This problem can be seen as a generalization of the maximum stable set problem and is NP-hard in general graphs; see, for instance, [10].

While various classical methods, such as exact algorithms, heuristics, and relaxations, have been explored for the SkS problem and related combinatorial problems, as summarized in the recent survey [8], the behavior of quadratic binary formulations in this setting remains largely unexplored. Reformulating the SkS problem as a Quadratic Unconstrained Binary Optimization (QUBO) problem enables the use of quantum annealing and other QUBO-oriented solvers. However, as discussed in [2], constructing effective QUBO models remains a significant challenge, particularly in the context of quantum annealing.

A standard approach to handling constraints in QUBO models is the exact penalty method, which introduces a quadratic penalty term into the objective function. More recently, [5, 6] proposed using an augmented Lagrangian formulation that combines this penalty term with Lagrange multipliers to enhance the performance of quantum annealers. An alternative strategy is to omit the quadratic penalty term altogether and instead apply a Lagrangian relaxation. A method for solving the Lagrangian dual of constrained binary quadratic problems using a quantum annealer was introduced in [9].

This motivates our study of different QUBO models for the SkS problem. We examine the aforementioned strategies, i.e., the exact penalty, Lagrangian relaxation, and augmented La-

[1] This research was funded in part or in whole by the Austrian Science Fund (FWF) [10.55776/DOC78].

grangian method, and provide parameter bounds that ensure exactness under appropriate conditions. For the Lagrangian relaxation, we investigate its behavior and identify instances where it yields solutions of the correct size. In this short communication, we outline our key findings; full proofs and extensions will be presented in a forthcoming paper.

Throughout, we adopt standard graph-theoretical terminology as in Diestel [4]. In particular, we recall that a stable set refers to a set of pairwise non-adjacent vertices, and the stability number $\alpha(G)$ denotes the size of a maximum stable set in G.

2 Problem Definition and QUBO Reformulations

The SkS problem asks for a subgraph on k vertices that induces the minimum number of edges. Given a graph $G = (V, E)$ with $|V| = n$, we represent vertex subsets by binary vectors $x \in \{0,1\}^n$, where $x_i = 1$ indicates inclusion of vertex i. The cardinality constraint is $\sum_{i \in V} x_i = k$, and the number of induced edges is $\sum_{\{i,j\} \in E} x_i x_j$.

Now, let A be the adjacency matrix of G, and let e denote the all-ones vector of length n. Then the SkS problem can be written as

$$m_k := \min \left\{ \frac{1}{2} x^T A x \mid e^T x = k,\ x \in \{0,1\}^n \right\}. \tag{SkS}$$

This problem generalizes the maximum stable set problem: if the optimal value of (SkS) is zero, the solution corresponds to a stable set. In other words, the stability number $\alpha(G)$ is the largest k for which the optimal value of (SkS) is zero.

2.1 Exact Quadratic Penalty Reformulation of SkS

The first QUBO reformulation of (SkS) we present is based on the standard quadratic penalty method. Hence, we penalize the cardinality constraint $e^T x = k$ by moving it as a quadratic term into the objective function, resulting in the following QUBO formulation

$$\min \left\{ \frac{1}{2} x^T A x + \frac{\mu}{2} (e^T x - k)^2 \mid x \in \{0,1\}^n \right\}, \tag{SkS-Q}$$

where $\mu \in \mathbb{R}^+$ is a penalization parameter.

Note that the term $\frac{\mu}{2}(e^T x - k)^2$ in the formulation (SkS-Q) penalizes all cases where $e^T x \neq k$, supporting that exactly k vertices are selected in an optimal solution of (SkS-Q). The following result provides a sufficient condition on the penalty parameter to ensure that the optimal solutions of (SkS-Q) also satisfy the original cardinality constraint.

Lemma 1. *Let G be a graph on n vertices and $k \leq n$. Let $\tilde{x} \in \{0,1\}^n$ such that $e^T \tilde{x} = k$, and let $\frac{\mu}{2} > \frac{1}{2} \tilde{x}^T A \tilde{x}$. Then,*

(i)

$$\frac{1}{2} \tilde{x}^T A \tilde{x} < \frac{1}{2} x^T A x + \frac{\mu}{2} (e^T x - k)^2$$

for all $x \in \{0,1\}^n$ with $e^T x \neq k$.

(ii) Any optimal solution of (SkS-Q) *is an optimal solution of* (SkS).

Hence, if we manage to find a feasible solution for SkS where the number of edges is rather small, then we know from Lemma 1 that we can choose a small penalty parameter. However, the number of edges in feasible solutions can still be large, which is why we continue to investigate whether it is possible to choose an even smaller penalty parameter.

By definition, we have $m_k \leq m_{k+1}$ for all $k \in \{1, \ldots, n-1\}$. Furthermore, note that a sparsest subgraph on $k+1$ vertices can have at most k edges more than a sparsest subgraph on k vertices, while a sparsest subgraph on $k+2$ vertices has at most $k+k+1 = 2k+1$ more edges than a sparsest subgraph on k vertices. Thus, if we deal with a graph on n vertices, then a sparsest subgraph on $k+i$ vertices for some $i \in \{1, \ldots, n-k\}$, can have at most $k+k+1+\ldots+k+i-1 = ki + \frac{i(i-1)}{2}$ more edges than a sparsest k-subgraph in G, i.e., we have

$$m_{k+i} \leq m_k + ki + \frac{i(i-1)}{2}.$$

Similarly, a sparsest subgraph on k vertices can have at most $k-1+k-2+\ldots+k-j = kj - \frac{j(j+1)}{2}$ more edges than a sparsest subgraph on $k-j$ vertices for any $j \in \{1, \ldots, k-1\}$, i.e., the following inequality holds

$$m_k \leq m_{k-j} + kj - \frac{j(j+1)}{2}.$$

This serves as the basis for the following lemma.

Lemma 2. *Let G be a graph on n vertices and $k \leq n$. Let $x^* \in \{0,1\}^n$ be an optimal solution for* (SkS) *and let $\frac{\mu}{2} > k-1$. Then,*

(i)

$$\frac{1}{2}(x^*)^T A x^* < \frac{1}{2}x^T A x + \frac{\mu}{2}(e^T x - k)^2$$

for all $x \in \{0,1\}^n$ with $e^T x \neq k$.

(ii) Any optimal solution of (SkS-Q) *is an optimal solution of* (SkS).

Finally, we demonstrate that in some cases, even lower penalty parameters than the ones proposed in Lemmas 1 and 2 can be chosen. For this purpose, we define sequence $\{\text{diff}_\ell \mid 2 \leq \ell \leq n\}$ as $\text{diff}_\ell = m_\ell - m_{\ell-1}$, and for $\ell = 1$, we set $\text{diff}_1 = m_1$. Furthermore, we say that the sequence $\{\text{diff}_\ell\}$ is monotonically increasing in G if $\text{diff}_\ell \leq \text{diff}_{\ell+1}$ for all $\ell \in \{1, \ldots, n-1\}$ and therewith

$$m_1 \leq m_2 - m_1 \leq m_3 - m_2 \leq \ldots \leq m_{n-1} - m_n \leq m_n - m_{n-1}.$$

Lemma 3. *Let G be a graph on n vertices and $k \leq n$. Suppose that the sequence $\{\text{diff}_\ell\}$ is monotonically increasing in G. Let $\frac{\mu}{2} > \text{diff}_k$ and $x^* \in \{0,1\}^n$ be an optimal solution for* (SkS). *Then,*

(i)

$$\frac{1}{2}(x^*)^T A x^* < \frac{1}{2}x^T A x + \frac{\mu}{2}(e^T x - k)^2$$

for all $x \in \{0,1\}^n$ with $e^T x \neq k$.

(ii) Any optimal solution of (SkS-Q) *is an optimal solution of* (SkS).

2.2 Lagrangian Relaxation of SkS

We now consider the Lagrangian relaxation of the sparsest k-subgraph problem (SkS), which means that we move the linear constraint $e^T x = k$, multiplied with a (Lagrangian dual) parameter λ, to the objective function; see, for instance, [1]. For the sake of simplicity, we move to the objective function $\lambda(k - e^T x)$ instead of $\lambda(e^T x - k)$. These are clearly equivalent because the constraint is an equality, and therefore λ is an unconstrained dual variable. Hence, the Lagrangian relaxation of (SkS) is therefore

$$\min \left\{ \frac{1}{2} x^T A x + \lambda(k - e^T x) \mid x \in \{0,1\}^n \right\}. \tag{SkS-L}$$

Note that the classical duality theory implies that for any $\lambda \in \mathbb{R}$, the optimal value of the relaxation (SkS-L) is a lower bound on the optimal value of (SkS); for details, see [1]. Since the term λk is a constant, we can leave it out of the objective function and solve only the following problem

$$\min \left\{ \frac{1}{2} x^T A x - \lambda e^T x \mid x \in \{0,1\}^n \right\}. \tag{SkS-L'}$$

We get the optimal value of (SkS-L) from the optimal value of (SkS-L') by adding λk.

Similarly to the exact quadratic penalty approach, we now theoretically investigate how to set the multiplier λ to obtain feasible solutions. We start with some simple observations. Recall that λ is not constrained and can take both positive and negative values. However, if $\lambda < 0$, then the problem (SkS-L') becomes

$$\min \left\{ \frac{1}{2} x^T A x + \tilde{\lambda} e^T x \mid x \in \{0,1\}^n \right\}$$

for $\tilde{\lambda} = -\lambda > 0$. Clearly, the minimum of this problem equals zero and is attained when x is the zero vector.

If $\lambda = 0$, then the problem (SkS-L') is reduced to minimizing the number of edges $\frac{1}{2} x^T A x$. Since the minimum value of this objective function is zero, any incidence vector of a stable set in the graph, as well as the zero vector, is a feasible solution.

If $\lambda = 1$, then solving the problem (SkS-L') becomes equivalent to solving the QUBO formulation for the stable set problem with the penalty parameter $\frac{1}{2}$. In particular, for a given G, the optimal value of (SkS-L') in this case corresponds to $-\alpha(G)$; for details, see [7, Lemma 2]. Moreover, for any $\lambda \in (0,1)$, any optimal solution of (SkS-L') is the incidence vector of a maximum stable set, as the next statement shows.

Lemma 4. *Let G be a graph on n vertices. If $\lambda \in (0,1)$, then any optimal solution of* (SkS-L') *is the incidence vector of a maximum stable set in G.*

Therefore, if we solve (SkS-L') with $\lambda \in (0,1)$ and $k < \alpha(G)$, then we can extract from computed optimal solution (which is a maximum stable set) an optimal solution of (SkS) with cardinality k by simply taking any subset of k vertices from the optimal solution.

As a next step, we establish an upper bound for the value of the Lagrangian multiplier λ.

Lemma 5. *Let G be a graph on n vertices, and let k be a positive integer such that $k \leq n-1$. If $\lambda > k$, then no optimal solution of* (SkS-L') *corresponds to an optimal solution of* (SkS).

Although we established lower and upper bounds for the penalty parameter λ, its value typically needs to be determined through an iterative process. As a result, solving the Lagrangian relaxation of the SkS problem may involve solving multiple QUBO problems. However, for certain instances, we show that tighter bounds on λ can be identified that directly yield solutions with the correct cardinality.

To this end, we revisit $\{\text{diff}_\ell\}$, introduced in Section 2.1, and demonstrate that if $\{\text{diff}_\ell\}$ is monotonically increasing in a given graph, then tighter bounds on λ can be determined. Furthermore, we show that choosing λ within these bounds yields an optimal solution for the SkS problem. Additionally, we can choose λ to be an integer, further emphasizing the practical effectiveness of Lagrangian relaxation for this problem.

Lemma 6. *Let G be a graph on n vertices, let $k \leq n-1$, and suppose that $\{\text{diff}_\ell\}$ is monotonically increasing in G. Then, choosing $\text{diff}_k \leq \lambda \leq \text{diff}_{k+1}$ in the formulation* (SkS-L') *yields an optimal solution x^* that is the incidence vector of a sparsest k-subgraph in G.*

2.3 Augmented Lagrangian Method for SkS

As a last step, we consider the augmented Lagrangian method, which combines the exact penalty approach with Lagrangian relaxation. This leads to the following QUBO relaxation for the SkS problem

$$\min\left\{\frac{1}{2}x^TAx + \lambda(k - e^Tx) + \frac{\mu}{2}(e^Tx - k)^2 \mid x \in \{0,1\}^n\right\}, \tag{SkS-AL}$$

where μ is referred to as the penalty parameter, and λ as the Lagrangian multiplier; see, for instance [3].

As with the Lagrangian relaxation, the values of μ and λ in the augmented Lagrangian method are generally not known in advance and are therefore typically determined through an iterative procedure. Furthermore, since we are dealing with a problem involving binary variables, there is no general theoretical recipe for selecting the penalty parameter and the Lagrangian multiplier. Nevertheless, we demonstrate that for the SkS problem, we can always choose in advance parameters that yield an optimal solution.

Lemma 7. *Let G be a graph on n vertices and $k \leq n$. Let $\lambda = \frac{1}{2}(k-1)$ and $\mu = k$. Then,*

(i) If $x^* \in \{0,1\}^n$ is an optimal solution of SkS, then

$$\frac{1}{2}(x^*)^TAx^* < \frac{1}{2}x^TAx + \lambda(k - e^Tx) + \frac{\mu}{2}(e^Tx - k)^2$$

for all $x \in \{0,1\}^n$ with $e^Tx \neq k$.

(ii) Any optimal solution of (SkS-AL) *is an optimal solution of* (SkS).

3 Conclusion and Future Work

This work presents an initial theoretical study of various QUBO formulations for the sparsest k-subgraph problem, focusing on how different approaches to enforcing the cardinality constraint affect the structure and properties of the resulting models. As shown in Section 2, multiple strategies for selecting penalty parameters can lead to exact solutions. However, a comprehensive computational study is necessary to assess the practical implications of these choices, particularly in terms of how the magnitude of the penalty term affects both the quality of the solution and the underlying computational effort. In addition, the design of efficient

iterative schemes for solving Lagrangian-based formulations presents a promising avenue for further investigation.

Our findings also highlight the impact of the underlying graph structure on selecting appropriate penalty values. A more in-depth investigation into this relationship would be valuable. Moreover, an interesting direction for future research is the identification of graph classes for which the sequence $\{\text{diff}_\ell\}$, introduced in Section 2.1, is monotonically increasing. In such instances, where only a linear penalty term is introduced, the QUBO matrix preserves the sparsity pattern of the original adjacency matrix; a property that may offer computational advantages, particularly for quantum annealers and solvers that exploit matrix sparsity.

Finally, it would be of interest to extend this line of investigation to other cardinality-constrained optimization problems and examine whether some of the insights gained here can be generalized to a broader class of problems.

Acknowledgements

We would like to acknowledge dr. Roman Kužel for fruitful communication.

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A REVIEW OF QUANTUM COMPUTING FOR VEHICLE ROUTING PROBLEM

Tomislav Erdelić
University of Zagreb, Faculty of Transport and Traffic Sciences, Vukelićeva 4, Zagreb, Croatia, terdelic@fpz.unizg.hr
Tonči Carić
University of Zagreb, Faculty of Transport and Traffic Sciences, Vukelićeva 4, Zagreb, Croatia, tcaric@fpz.unizg.hr
Nikola Mardešić
University of Zagreb, Faculty of Transport and Traffic Sciences, Vukelićeva 4, Zagreb, Croatia, nmardesic@fpz.unizg.hr

Abstract: The paper deals with reviewing recent work in the field of quantum computing applied to modeling and/or solving the well-known operations research problem - Vehicle Routing Problem (VRP). The goal is to investigate the latest advances regarding modeling techniques and algorithmic schemes.

Keywords: quantum computing, quantum annealer, quantum gates, vehicle routing problem

1 INTRODUCTION

As a part of the operations research community, the Vehicle Routing Problem (VRP) has been thoroughly studied over the last 60 years [1]. The VRP aims to determine a set of least-cost delivery routes from a depot to a set of geographically scattered customers, subject to side constraints such as vehicle load capacity or customer time windows. As an NP-hard problem, a vast number of exact, heuristic, and metaheuristic methods have been developed to solve the VRP. However, these approaches still require substantial computational resources and time, especially for large-scale instances [2]. In the last decade, Quantum Computing (QC) [3] has emerged as a promising approach for modeling and potentially solving VRP problems more efficiently. QC is performed on a specialized quantum computer that uses the laws of quantum mechanics to perform computations. Instead of the classical bit-based architecture, quantum computers rely on q-bits, which can exist in a superposition of multiple states simultaneously, each with a certain probability [4]. When a qubit is measured, it collapses to one of the possible pure states. Additionally, q-bits can be entangled with each other, creating correlations that are stronger than those possible in classical systems. This property of entanglement enables quantum computers to process complex information in fundamentally different ways compared to classical machines [5].

There exist two main QC algorithmic schemes for solving VRP: quantum annealing and quantum gate-based computing, [5]. Quantum Annealing (QA) follows a procedure similar to Simulated Annealing (SA) on classical computers, which gradually lowers the temperature of the system, transitioning it from an unordered to an ordered state; in operations research terminology, this means allowing larger jumps in the search space at the beginning and gradually reducing them as the search progresses - the so-called thermal jumps. Similarly, QA begins from an initial state (eigenstate) represented by a Hamiltonian, which encodes the energy objective function of a problem. According to quantum mechanics, the goal is to slowly transform the initial Hamiltonian at the lowest-energy state into a final Hamiltonian at the lowest-energy state that represents the problem, which corresponds to an optimal or near-optimal solution, [6]. Unlike SA, which relies on thermal jumps, QA uses quantum tunneling to escape local minima. The most commonly used model for the Hamiltonian in QA is the Ising model, which represents quantum spins of each q-bit and their coupling strength [7]. QA is well-suited for

solving Quadratic Unconstrained Binary Optimization (QUBO) problems, which involve binary variables and no explicit constraints. These problems can be naturally mapped to the Ising model.

The quantum gate-based scheme applies quantum circuits to transform the quantum states of the system. This approach is analogous to the use of logic circuits in classical computing-such as AND, NOT, and OR gates - but instead utilizes quantum gates (operators), which may include single-qubit operations, tensor products (for parallel application), and matrix products (for sequential application) [4, 8]. Examples of commonly used quantum gates include the SWAP gate (which exchanges the states of two qubits), CPHASE gate (which applies a phase shift to one component of the state), and the Hadamard gate (which creates an equal superposition when applied to a basis state) [8]. Quantum circuits composed of such gates can be used to implement Quantum Variational Algorithms (QVA), such as the Quantum Approximate Optimization Algorithm (QAOA), which encodes the Hamiltonian related to the problem into a quantum circuit and leverages adiabatic time evolution and layering to optimize the variational parameters of the circuit, such that the approximate solution to the problem can be constructed by measuring the QAOA circuit with the optimal set of parameters, [9]. The QAOA is also well-suited for solving QUBO problems, but it is not limited to this class of problems.

2 REVIEW

We investigated papers published in Google Scholar, Scopus, IEEE Xplore Digital Library, and Web of Science databases that include 'routing' and 'quantum' in their titles or keywords, and subsequently filtered a total of 24 papers that are related to both the VRP and QC.

Several researchers formulated the VRP as a QUBO and applied a QAOA or a similar variation of a QVA to solve it [10, 11, 12, 13, 14], while highlighting the importance of the qubit encoding scheme to reduce the number of binary variables, and noting that noise is considered the most problematic aspect of today's intermediate-scale devices. The authors in [15] compared the QAOA and QA for solving a VRP variant and concluded that QA shows better results for tackling medium-sized problems, although challenges with embedding and scaling to large instances still exist. In [12], the authors highlight that for a finite value of p (ansatz parameterized), there is no guarantee that the solution achieved by QAOA corresponds to the most optimal solution of the original combinatorial optimization problem.

From a modelling perspective, the authors in [16] investigated four mathematical formulations of the VRP: route-based, arc-based, sequence-based, and sequence-based with continuous time; transforming them to a QUBO formulation and comparing three solution methods: QAOA, VQA, and the Alternating Direction Method of Multipliers (ADMM, optimizing an augmented Lagrangian function over QUBO), with the route-based formulation and QAOA producing the best results. In [17], the authors took a different modelling approach to VRP and proposed the Quantum Path Integral Approach (QPIA), which combines Feynman path integral formulation with the first Born approximation and QC, overcoming some limitations of the Ising-Hamiltonian model.

QA on a QUBO formulation was applied in several research papers [18, 19, 20], with QA showing better time-scaling performance than some classical exact solvers [18]. An important aspect in these studies is the problem encoding in terms of a set of spins [19]. Some authors divided the VRP into a subset of TSPs and used a hybrid classical and quantum algorithm, where the classical part performs the partitioning and the quantum part solves the TSP problem modeled as QUBO or Constrained Quadratic Model (CQM) using QA [21, 22, 23]. In [24], the authors performed classical route separation, and on quantum-inspired hardware with a digital annealer, solved the QUBO model for TSP and the final set partitioning model.

Another stream of research is related to the application of classical evolutionary algorithms

which, in some parts, use quantum-inspired methods to solve the optimization problem [5]. In VRP solved with evolutionary algorithms, these methods are mostly related to solution encoding as Q-bit representation and the application of quantum-inspired crossover and mutation to enhance exploration [25, 26, 27, 28], maintaining the population of best solutions using specially designed Q-gates [29], updating the ant colony using a quantum revolving gate [30], or using quantum-inspired annealing [31].

With the explosion of machine learning in the last couple of years, the researchers have also investigated the application of quantum machine learning (leveraging the probing of a vast uncertain solution space) using VQA to solve VRPs [32].

3 CONCLUSION

From the review, it can be seen that most researchers apply quantum-inspired classical metaheuristic algorithms, such as genetic algorithms and ant colony optimization, to solve the problem, with truly quantum algorithm applications appearing more on the side of QA, which shows better scalability and performance than the use of QVA (QAOA).

4 Acknowledgements

The research has been supported by the project *Optimization of Routes and Charging Technologies for Electric Vehicle Fleets* and *Intelligent Transportation Systems for Dynamic Vehicle Routing in Urban Environments* founded by the University of Zagreb Faculty of Transport and Traffic Sciences.

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Introduction to Quantum Extensions of Classical Games

Anna Gorczyca-Goraj, Krzysztof Grzanka, and **Marek Szopa**
Department of Operations Research, University of Economics in Katowice,
ul. Bogucicka 3, 40-287 Katowice, Poland
Piotr Frąckiewicz
Institute of Exact and Technical Sciences, Pomeranian University in Słupsk
ul. Arciszewskiego 22A, 76-200 Słupsk, Poland

Abstract: This study presents our research on quantum extensions of classical 2×2 games within the Eisert–Wilkens–Lewenstein framework, supported by a Mathematica package developed by our team. The quantum extensions are introduced in this paper, while the package's functionalities and usage examples are detailed in the corresponding article.

Keywords: quantum game theory, Eisert-Wilkens-Lewenstein scheme, Nash equilibrium

1 Introduction

Since its inception, quantum game theory has attracted attention for its potential to reveal novel strategies and enhance our understanding of decision-making. A foundational framework by Eisert, Wilkens, and Lewenstein (EWL) [1] formalized the quantization of classical games, showing how quantum strategies can shift equilibria toward Pareto-optimal outcomes in dilemmas like the Prisoner's Dilemma [2, 7].

In classical game theory, the Nash equilibrium (NE) identifies stable strategy profiles, though it often fails to achieve socially optimal results [8]. Quantum extensions, particularly within the EWL framework, introduce unitary strategies that expand the strategic space and enable more favorable equilibria [1, 6].

This study investigates such quantum extensions by adding one or two unitary strategies to classical 2×2 games. We analyze classes of quantum games, each invariant under isomorphic transformations of the classical structure. A detailed discussion of the extensions' properties will be presented separately, with an introduction to the functionalities of the QEGS (*Quantum Extension Game Solver*) Mathematica package [5].

2 Rationale for developing a Mathematica package

The complexity of verifying properties of quantum extensions of classical games motivated the development of a Mathematica package for analyzing 2×2 games within the EWL framework. It generates all relevant quantum extensions with one or two unitary strategies, preserving isomorphism invariance. Key functionalities and examples are presented in the accompanying article entitled "Quantum Extensions of Classical Games with QEGS".

3 Preliminaries

Definition 1. *A bimatrix game is a strategic interaction between two players, each possessing a finite set of strategies, where the outcomes are defined by a pair of payoff matrices - one corresponding to each player. This type of a game can be represented by:*

$$\Delta = \begin{pmatrix} (\Delta^1_{11}, \Delta^2_{11}) & (\Delta^1_{12}, \Delta^2_{12}) & \cdots & (\Delta^1_{1m}, \Delta^2_{1m}) \\ (\Delta^1_{21}, \Delta^2_{21}) & (\Delta^1_{22}, \Delta^2_{22}) & \cdots & (\Delta^1_{2m}, \Delta^2_{2m}) \\ \vdots & \vdots & \ddots & \vdots \\ (\Delta^1_{n1}, \Delta^2_{n1}) & (\Delta^1_{n2}, \Delta^2_{n2}) & \cdots & (\Delta^1_{nm}, \Delta^2_{nm}) \end{pmatrix} = (\Delta^1, \Delta^2). \tag{1}$$

A specific combination of strategies, where player 1 chooses i and player 2 chooses j, is denoted by the ordered pair (i, j) and referred to as a strategy profile. The outcome of this interaction yields a payoff of Δ^1_{ij} for player 1 and Δ^2_{ij} for player 2. One of the fundamental solution concepts in game theory is the NE, which can be expressed in terms of bimatrix games as follows:

Definition 2. *A strategy profile* (i^*, j^*) *is a (pure) NE in* (Δ^1, Δ^2) *if* $\Delta^1_{i^*j^*} \geq \Delta^1_{ij^*}$ *for every* $i \in \{1, \dots n\}$ *and* $\Delta^2_{i^*j^*} \geq \Delta^2_{i^*j}$ *for every* $j \in \{1, \dots m\}$.

A NE represents a stability property in which no player has an incentive to unilaterally deviate from their chosen strategy.

The EWL scheme is a foundational approach to modeling quantum games, integrating quantum mechanics into classical game theory. In the EWL model, players' strategies are represented by unitary operators acting on a shared entangled state within a Hilbert space, i.e.

$$U_i(\theta_i, \alpha_i, \beta_i) = \begin{pmatrix} e^{i\alpha_i}\cos\frac{\theta_i}{2} & ie^{i\beta_i}\sin\frac{\theta_i}{2} \\ ie^{-i\beta_i}\sin\frac{\theta_i}{2} & e^{-i\alpha_i}\cos\frac{\theta_i}{2} \end{pmatrix}, \quad \theta_i \in [0, \pi] \text{ and } \alpha_i, \beta_i \in [0, 2\pi), \tag{2}$$

acting on qubits. A full description of the EWL scheme is presented in [4].

4 Quantum extensions by one or two unitary strategies

This chapter outlines the necessary conditions for quantum extensions of classical games. As shown in [3, 4], such extensions can be constructed using one or two unitary operations alongside classical strategies, allowing players to apply unitary transformations to their qubits.

The QEGS package initiates its analysis with an arbitrary 2×2 classical game.

$$\Gamma = \begin{pmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{pmatrix}, \text{ where } \Delta_{ij} = (\Delta^1_{ij}, \Delta^2_{ij}). \tag{3}$$

The quantum extended matrix of game 3 can be expressed by:

$$\begin{array}{c} \\ I \\ iX \\ U \end{array} \begin{array}{c} \begin{array}{ccc} I & iX & U \end{array} \\ \begin{pmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} \\ \Delta_{21} & \Delta_{22} & \Delta_{23} \\ \Delta_{31} & \Delta_{32} & \Delta_{33} \end{pmatrix} \end{array}, \tag{4}$$

where $U(\theta, \alpha, \beta) \equiv U$ is an arbitrary but fixed unitary strategy, $I = U(0, 0, 0)$, and $iX = U(\pi, 0, 0)$. It was proven in [4] that there are only three permissible types of 3×3 extensions:

$$A_0 = \begin{pmatrix} \Delta_{11} & \Delta_{12} & \frac{\Delta_{11}+\Delta_{12}}{2} \\ \Delta_{21} & \Delta_{22} & \frac{\Delta_{21}+\Delta_{22}}{2} \\ \frac{\Delta_{11}+\Delta_{21}}{2} & \frac{\Delta_{12}+\Delta_{22}}{2} & \frac{\Delta_{11}+\Delta_{12}+\Delta_{21}+\Delta_{22}}{4} \end{pmatrix}, \text{ where } U = U\left(\frac{\pi}{2}, \alpha, \beta\right), \ \alpha, \beta \in \{0, \pi\}. \tag{5}$$

The second class is

$$B_0 = \begin{pmatrix} \Delta_{11} & \Delta_{12} & \frac{\Delta_{21}+\Delta_{22}}{2} \\ \Delta_{21} & \Delta_{22} & \frac{\Delta_{11}+\Delta_{12}}{2} \\ \frac{\Delta_{12}+\Delta_{22}}{2} & \frac{\Delta_{11}+\Delta_{21}}{2} & \frac{\Delta_{11}+\Delta_{12}+\Delta_{21}+\Delta_{22}}{4} \end{pmatrix}, \text{ where } U = U\left(\frac{\pi}{2}, \alpha, \beta\right), \ \alpha, \beta \in \{\frac{\pi}{2}, \frac{3\pi}{2}\}. \tag{6}$$

And the third class is

$$C_0 = \begin{pmatrix} \Delta_{11} & \Delta_{12} & \frac{\Delta_{11}+\Delta_{12}+\Delta_{21}+\Delta_{22}}{4} \\ \Delta_{21} & \Delta_{22} & \frac{\Delta_{11}+\Delta_{12}+\Delta_{21}+\Delta_{22}}{4} \\ \frac{\Delta_{11}+\Delta_{12}+\Delta_{21}+\Delta_{22}}{4} & \frac{\Delta_{11}+\Delta_{12}+\Delta_{21}+\Delta_{22}}{4} & \frac{\Delta_{11}+\Delta_{12}+\Delta_{21}+\Delta_{22}}{4} \end{pmatrix}, \tag{7}$$

where $U = U\left(\frac{\pi}{2}, \alpha, \beta\right)$, and $\alpha, \beta \in \{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\}$.

When the classical game (3) is extended with two quantum strategies, the resulting extension matrix takes the form of a 4×4 extended matrix. In such case there exist eight distinct classes detailed in [3].

5 Summary

Quantum game theory is inherently complex due to its interdisciplinary nature and the analytical challenges of quantum extensions. With growing interest in quantum strategies like those in the EWL framework, there is a need for tools that simplify analysis. The QEGS Mathematica package addresses this by automating calculations and enabling users to explore both classical and quantum games without manual derivations. The package is available under the following link: https://github.com/k-grzanka/QEGS

6 Acknowledgements

Computations were carried out using the computers of Centre of Informatics Tricity Academic Supercomputer & Network.

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Quantum Extensions of Classical Games with QEGS

Krzysztof Grzanka, Anna Gorczyca-Goraj, and **Marek Szopa**
Department of Operations Research, University of Economics in Katowice,
ul. Bogucicka 3, 40-287 Katowice, Poland
Piotr Frąckiewicz
Institute of Exact and Technical Sciences, Pomeranian University in Słupsk
ul. Arciszewskiego 22A, 76-200 Słupsk, Poland

Abstract: With the growing popularity of quantum games, there has been an increasing demand for analytical tools to simplify their studies. Our Mathematica package QEGS (*Quantum Extension Game Solver*) falls in this place. The package functionalities include quantum extensions of classical 2×2 games within the Eisert-Wilkens-Lewenstein scheme, as well as tools for the study of any bimatrix game. While the permissible quantum extensions are introduced in the accompanying article, this paper focuses on the usage of the package.

Keywords: quantum game theory, Eisert-Wilkens-Lewenstein scheme, Nash equilibrium, Mathematica package

1 Introduction

Over the years, quantum games have become increasingly popular in decision-making dilemmas. Quantum extensions introduce additional unitary strategies to the game that can move equilibria closer to Pareto-optimal payoffs [1]. However, generation and detailed analysis of the permissible quantum extensions within the Eisert-Wilkens-Lewenstein (EWL) scheme [2] may turn out to be extremely difficult without the analytic tools.

This paper presents QEGS (*Quantum Extension Game Solver*), a Mathematica package to explore quantum extensions of classical 2×2 bimatrix games in the EWL scheme and make their analysis easier. It allows for straightforward generation of all possible quantum extensions of the initial game with one or two additional, unitary strategies. These extensions are demanded to remain invariant under the isomorphic transformations, which results in three and eight classes of extensions with one and two unitary strategies, respectively. Besides generating the extensions, the package also allows a user to analyse the features of any bimatrix games. These features include the determination of Nash Equilibria (NE) in pure strategies, dominated strategies elimination, finding the maximin strategies and reporting to a PDF file.

2 Preliminaries

A ***bimatrix game*** is a two-player game in which each player has a finite set of strategies. The outcomes for each player are defined by the payoff matrix and it can be written as

$$\Delta = \begin{pmatrix} (\Delta^1_{11}, \Delta^2_{11}) & (\Delta^1_{12}, \Delta^2_{12}) & \cdots & (\Delta^1_{1m}, \Delta^2_{1m}) \\ (\Delta^1_{21}, \Delta^2_{21}) & (\Delta^1_{22}, \Delta^2_{22}) & \cdots & (\Delta^1_{2m}, \Delta^2_{2m}) \\ \vdots & \vdots & \ddots & \vdots \\ (\Delta^1_{n1}, \Delta^2_{n1}) & (\Delta^1_{n2}, \Delta^2_{n2}) & \cdots & (\Delta^1_{nm}, \Delta^2_{nm}) \end{pmatrix} = (\Delta^1, \Delta^2). \tag{1}$$

The ***strategy profile*** is the ordered pair (i, j) denoting that the first player (row player) chooses strategy i, while second, column player uses strategy j. Δ^1_{ij} and Δ^2_{ij} denote the payoff for the first and second player respectively for the (i, j) strategy profile.

The pure strategy i of the row player is ***strictly dominated*** if there exists another strategy i' such that $\Delta^1_{i'j} > \Delta^1_{ij}$ for every strategy j of the column player, similarly for the column players strategies.

A strategy profile (i^*, j^*) is a (pure) ***Nash Equilibrium*** in (Δ^1, Δ^2) if $\Delta^1_{i^*j^*} \geq \Delta^1_{ij^*}$ for every $i \in \{1, \dots n\}$ and $\Delta^2_{i^*j^*} \geq \Delta^2_{i^*j}$ for every $j \in \{1, \dots m\}$.

A NE signifies a stability point at which neither of players can improve their payoff by changing the strategy unilaterally [3].

A strategy i is a ***maximin strategy*** of player 1 in a bimatrix game if

$$\min_{j\in\{1,\dots m\}} \Delta^1_{ij} \geq \min_{j\in\{1,\dots m\}} \Delta^1_{i'j} \quad \text{for all} \quad i' \in \{1, \dots, n\}. \tag{2}$$

Similarly for the column players strategies.

The EWL scheme serves as a fundamental framework for modeling quantum games by merging quantum mechanics with classical game theory. In this approach, players operate within a Hilbert space, where their strategies are represented by unitary operators $U_i(\theta_i, \alpha_i, \beta_i)$ acting on qubits.

$$U_i(\theta_i, \alpha_i, \beta_i) = \begin{pmatrix} e^{i\alpha_i}\cos\frac{\theta_i}{2} & ie^{i\beta_i}\sin\frac{\theta_i}{2} \\ ie^{-i\beta_i}\sin\frac{\theta_i}{2} & e^{-i\alpha_i}\cos\frac{\theta_i}{2} \end{pmatrix}, \quad \theta_i \in [0, \pi] \text{ and } \alpha_i, \beta_i \in [0, 2\pi), \tag{3}$$

More details about the EWL scheme and permissible quantum extensions of 2×2 classical games by one and two unitary strategies can be found in [4, 5].

3 Quantum extensions

Quantum extensions of the classical 2×2 game can be constructed by adding one or two unitary operations alongside classical strategies, enabling players to perform unitary transformations on their qubits. Consider an arbitrary 2×2 classical game

$$\Gamma = \begin{pmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{pmatrix}, \text{ where } \Delta_{ij} = (\Delta^1_{ij}, \Delta^2_{ij}). \tag{4}$$

The quantum extension of the game with one additional unitary strategies can be expressed by:

$$\begin{array}{c} \\ I \\ iX \\ U \end{array} \begin{array}{c} \begin{array}{ccc} I & iX & U \end{array} \\ \begin{pmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} \\ \Delta_{21} & \Delta_{22} & \Delta_{23} \\ \Delta_{31} & \Delta_{32} & \Delta_{33} \end{pmatrix} \end{array}, \tag{5}$$

where $U(\theta, \alpha, \beta) \equiv U$ is an arbitrary unitary strategy, $I = U(0, 0, 0)$, and $iX = U(\pi, 0, 0)$. It was proven in [4] that there are only three permissible types of 3×3 extensions, one of which is

$$A_0 = \begin{pmatrix} \Delta_{11} & \Delta_{12} & \frac{\Delta_{11}+\Delta_{12}}{2} \\ \Delta_{21} & \Delta_{22} & \frac{\Delta_{21}+\Delta_{22}}{2} \\ \frac{\Delta_{11}+\Delta_{21}}{2} & \frac{\Delta_{12}+\Delta_{22}}{2} & \frac{\Delta_{11}+\Delta_{12}+\Delta_{21}+\Delta_{22}}{4} \end{pmatrix}, \text{ where } U = U\left(\frac{\pi}{2}, \alpha, \beta\right), \ \alpha, \beta \in \{0, \pi\}. \tag{6}$$

The classical game (4) can also be extended by two unitary strategies U_1 and U_2 resulting in the 4×4 extended matrix. In this case, eight classes have been distinguished, and more details can be found in [5]. The necessary conditions for the validity of quantum extensions are discussed in [4, 5]

4 Package features

The package facilitates the exploration of quantum extensions of classical games defined by 2×2 payoff matrices and simplifies the investigation of properties of bimatrix games using

numerical or parametric representations. Users can study the existence of NE in pure strategies for both classical and quantum games. Furthermore, the package allows for the identification of dominated and maximin strategies applicable to both quantum extensions and classical games.

To present the basic functionalities of the package let us consider a following game in traditional bimatrix form:

$$\mathrm{PD} = \begin{pmatrix} (3,3) & (0,5) \\ (5,0) & (1,1) \end{pmatrix}. \tag{7}$$

The corresponding Mathematica syntax for the payoff bimatrix is:

$$\mathrm{PD} = \{\{\{3,3\},\{0,5\}\},\{\{5,0\},\{1,1\}\}\}. \tag{8}$$

First, we can generate extensions of this game with one additional unitary strategy

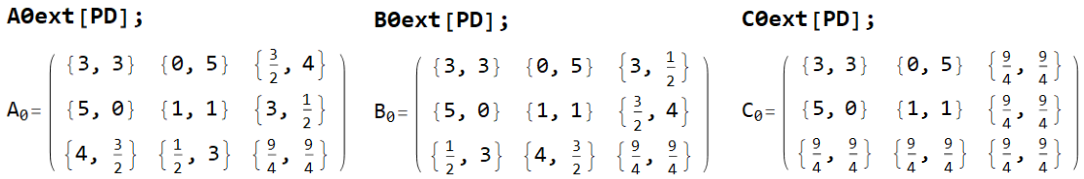


Figure 1: All three permissible extension classes of the game given in eq. (7) by one unitary strategy generated with QEGS.

Similarly extensions with two quantum strategies are generated. For these extensions as well as for any other bimatrix game, the package allows one to find and highlight NE, maximin and dominated strategies as presented in Fig. 2.

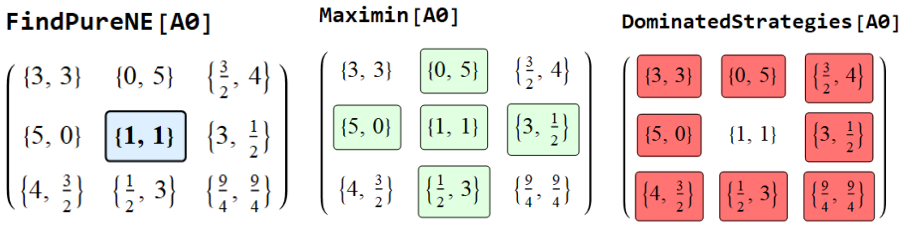


Figure 2: Example of search for NE (left), maximin (center) and dominated strategies (right) for the A_0 extension class from Fig. 1.

These properties, shown for a numerical matrix, can also be analyzed for the matrix with one continuous parameter involved, such as, for example, EWL extension parameter. This parameter can be controlled with a dynamic slider. Below an example for the A_1 class, one of eight permissible extensions, as defined in [6] is presented. The corresponding payoff matrix depends on the parameter a_1, its value can be changed dynamically with a slider.

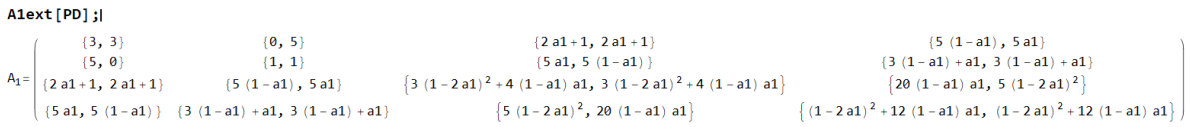


Figure 3: A_1 extension of the game given in (7) by two unitary strategies generated with QEGS.

All package functionalities are summarised in our recent work [7]. The package with two exemplary notebooks is publicly available on GitHub under the following link: https://github.com/k-grzanka/QEGS

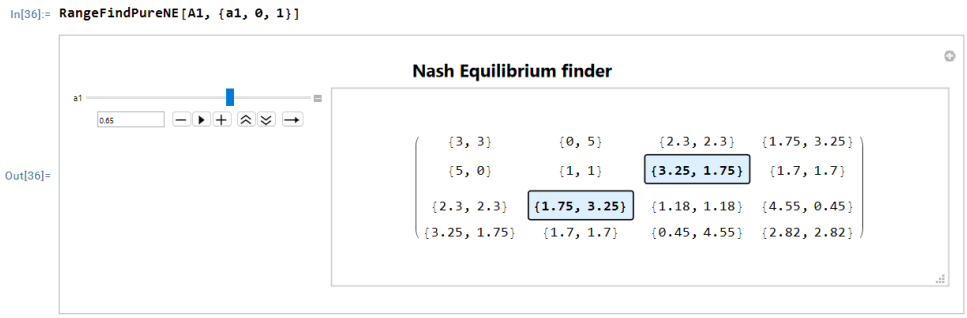


Figure 4: Pure NE finder for the A_1 matrix as in Fig. 3 allowing for dynamic change of the value of a_1 parameter.

5 Summary

Because of the intricate nature of quantum game extensions, a modern tool is required to facilitate analytical computations. This is the reason behind creating a specialized Mathematica package to simplify the study. The QEGS package serves for two main purposes. First, it allows user to generate all permissible quantum extensions of 2×2 classical game with one or two unitary strategies added. Secondly, it can be used to analyse these extensions, as well as any classical game, finding pure NE, maximin and dominated strategies. In the future we plan to extend the package with additional functionalities, such as investigating NE in mixed strategies or finding dominated strategies iteratively.

6 Acknowledgements

Computations were carried out using the computers of Centre of Informatics Tricity Academic Supercomputer & Network.

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HPC-ENABLED BILSTM ARCHITECTURE FOR MULTI-STEP ENERGY CONSUMPTION FORECASTING

Jelena Joksimović[†,‡], Martin Zagorc[†]

[†]Rudolfovo Science and Technology Center
Podbreznik 15, 8000 Novo mesto, Slovenia
{jelena.joksimovic, martin.zagorc}@rudolfovo.eu
[‡]corresponding author

Abstract

This paper presents the use of BiLSTM neural networks for energy consumption modeling in buildings. Buildings account for 30–45% of global energy use, and traditional models remain limited in accuracy and scalability. Robotina, a Slovenian SME, currently applies data-driven methods but is constrained by CPU-only infrastructure. By leveraging high-performance computing (HPC), more complex architectures, such as BiLSTM NNs, can be trained efficiently, enabling multi-day forecasts and real-time optimization. The approach supports thousands of end-users, reduces computation from weeks to 10 hours, and provides scalable predictions for buildings.

Keywords: high performance computing, energy management, recurrent neural networks, BiLSTMs, deep learning

1 Introduction

Buildings are among the largest energy consumers, accounting for 30–45% of global energy use, much of which originates from subsystems such as HVAC, lighting and safety systems. Their growing complexity, driven by distributed generation, dynamic pricing models and electrification, requires a transition from static control to adaptive, data-driven optimization. Accurate multi-day energy consumption forecasting is essential for aligning demand with intermittent renewable sources, enabling load shifting and improving cost-effectiveness. Traditional rule-based or tree-based models [2] are insufficient to capture the nonlinear dynamics of building energy demand. Deep learning models, and in particular Bidirectional Long Short-Term Memory (BiLSTM) neural networks, address this challenge by modeling long-range temporal dependencies. However, deploying BiLSTMs at scale requires optimized architectures and significant computational capacity [8, 9]. To address these challenges, we combine BiLSTMs with high-performance computing (HPC) and neural architecture search (NAS). This integration enables systematic exploration of model configurations and efficient large-scale training. The resulting framework is designed to deliver robust, low-latency Software-as-a-Service (SaaS) forecasting solutions for thousands of buildings, supporting intelligent energy management across residential, educational, healthcare and industrial domains.

2 BiLSTM neural networks

BiLSTM networks are an extension of traditional LSTMs, which belong to the class of Recurrent Neural Networks (RNNs), designed to capture past and future temporal dependencies for improved sequence modeling and forecasting performance. They extend standard LSTMs by capturing both past and future dependencies in sequential data [10]. Unlike traditional LSTMs,

where the hidden state at time t depends solely on inputs up to time t, BiLSTMs employs two independent LSTM layers: a forward LSTM that processes the sequence from $t = 1$ to $t = T$ and a backward LSTM that processes it from $t = T$ to $t = 1$. At each time step, the outputs from both directions are merged, typically via concatenation, giving a context enriched representation. This is particularly effective in tasks such as text analysis or energy consumption forecasting, where future inputs can be as informative as past ones. Formally, the forward and backward hidden states are computed as $\overrightarrow{h_t} = \text{LSTM}(x_t, \overrightarrow{h_{t-1}})$ and $\overleftarrow{h_t} = \text{LSTM}(x_t, \overleftarrow{h_{t+1}})$, respectively [5]. The final hidden state is given by $h_t = [\overrightarrow{h_t}; \overleftarrow{h_t}]$. It captures temporal spatial dependencies in sequential data through its internal memory cells and gating mechanisms, which are specifically designed to retain both short and long-term patterns [1]. Each LSTM unit uses three gates [11]: **forget gate:** that eternines which information from the previous cell state should be discarded, **input gate** that regulates how much new information is added to the cell state and **output gate** that controls how much of the updated cell state is revealed to the hidden state, which serves as the LSTM output at the current time step. The gating mechanism enables the BiLSTM to selectively retain relevant information and discard noise across time steps, allowing it to model complex temporal dynamics. In load prediction, for instance, this ability helps the model capture both short-term fluctuations (e.g., sudden spikes in consumption) and long-term trends (e.g., recurring peak-hour patterns). The bidirectional structure further enhances this by providing a full temporal context, since past and future data influence the representation of each time step.

2.1 Importance of HPC

Despite their modeling capabilities, BiLSTMs are computationally intensive and face several bottlenecks that make standard hardware unsuitable for large-scale deployment [7]. Each LSTM cell performs four matrix multiplications per time step. In BiLSTMs, this doubles to eight, creating massive computational overhead when applied to large sequences. Memory limitations of standard workstations restrict feasible sequence lengths and the number of concurrent sequences that can be processed. The "memory wall" problem and slow CPU-GPU data transfers hinder scalability for high-frequency, long-horizon time series [3]. For example, in our project, processing over 1,000 building datasets with 15-minute intervals generates tens of millions of timesteps. Running neural architecture search (NAS) with BiLSTM models on a CPU-only machine becomes infeasible. A single trial with a 298-timestep input and one BiLSTM layer may take 45 minutes per epoch, leading to over 11 hours per trial in the final bracket of Hyperband. With hundreds of combinations to explore, total training time on CPUs can easily exceed 900 hours, highlighting the necessity of GPU or HPC infrastructure.

HPC deployment strategy

We use the VEGA HPC cluster (1,000 node-hours and 128,000 GPU core-hours annually) to scale training and deployment of BiLSTM models. Each training job typically utilizes three GPUs within Singularity containers and TensorFlow 2.15.0 for portability and reproducibility. To support this workflow, we implemented an end-to-end pipeline:

- Automated Data Ingestion: Real-time smart meter data via MQTT, weather via OpenWeatherMap and Open-Meteo APIs, electricity market prices and tariff rules in Slovenia.
- Structured Storage: PostgreSQL with TimescaleDB extensions for efficient time series operations and aggregation.

- Remote HPC Training: SLURM-based batch execution through SSH, with secure API-based dataset access.
- Efficient Synchronization: Monthly retraining cycles with consistency checks and incremental updates.
- Deployment Automation: Trained models and scalers are retrieved via PostgreSQL LISTEN/NOTIFY triggers and deployed into Robotina's production systems for real-time energy optimization.

3 Data infrastructure

Our data platform integrates diverse data sources to support large-scale time series modeling. Smart meter data is collected via the MQTT protocol, transmitting 17 distinct metrics per minute in JSON format. These metrics include grid exchange (in both directions), total electricity load, photovoltaic (PV) generation and battery state of charge.
Weather data is acquired from multiple APIs to support both historical analysis and real-time forecasting. Two years of hourly historical data for five Slovenian cities were obtained through the Open-Meteo API, while real-time weather conditions are synchronized with the smart meter updates using OpenWeatherMap. Additionally, five-day weather forecasts are retrieved daily, with a resolution of three-hour intervals.
Market and fuel price information is integrated into the platform to account for economic signals in the modeling process. Day-ahead electricity prices are accessed via secure HTTPS and bi-weekly fuel prices are retrieved from the public `goriva.si` API. To model the economic impact of usage patterns more precisely, we also implemented Slovenia's complete five-tier tariff system. This system includes logic for seasonal changes, day-type classification and hourly block pricing.
All the data is stored and managed within a PostgreSQL–TimescaleDB hybrid infrastructure. This setup enables efficient partitioning and long-term storage retention, while maintaining high-performance capabilities for frequent inserts and complex time series queries. To ensure data quality, automated validation pipelines handle missing value interpolation, detect outliers and verify consistency across different data sources.

4 Results

We applied Neural Architecture Search (NAS) using Keras Tuners Hyperband algorithm[1] to optimize BiLSTM architectures for 96-step energy forecasting. The dataset was enriched with time-based features (season, day of week, weekend indicators etc.) and lagged target values to capture temporal dependencies. All features were scaled using the `MinMaxScaler` and the last 3,840 samples were reserved as the test set. Each NAS trial dynamically generated a BiLSTM model architecture by exploring combinations of the following hyperparameters: Input sequence length, number of units per BiLSTM layer, number of BiLSTM layers, dropout rate and learning rate. Each model architecture included **BiLSTM layers** with ReLU activation to capture bidirectional temporal patterns, **dropout layers** applied optionally for regularization and **output layer** - a dense layer predicting the next 96 timesteps. Model training was carried out using the Adam optimizer with gradient clipping (`clipnorm = 1.0`) and the Huber loss

[1] `https://keras.io/keras_tuner/`

function [4], ensuring stable and robust convergence. For each trial, the training and validation datasets were reconstructed dynamically according to the selected sequence length, aligning the input data with the model's temporal requirements. The hyperparameter search space, summarized in Table 1, includes key architectural and training parameters such as sequence length, layer depth, dropout rate and learning rate. The Hyperband algorithm employs a successive halving strategy[2], where all candidate configurations were trained for two initial epochs, with only the most promising models receiving further training (up to 15 epochs). Early stopping with a patience of three epochs was used to prevent overfitting and reduce computational cost. Upon completion of the search, the best-performing hyperparameter configuration and corresponding model is selected.

Table 1: Parameters explored during NAS

Parameter type	**Values tested**
Input context: sequence length (timesteps)	298, 384, 480, 596, 672
Model hyperparameter: units per BiLSTM layer	64, 128, 256
Model hyperparameter: number of BiLSTM layers	1, 2
Model hyperparameter: dropout rate	0.0, 0.2, 0.4
Model hyperparameter: learning rate	1×10^{-3}, 5×10^{-4}, 1×10^{-4}

5 Discussion

This work describes a framework for applying Neural Architecture Search with BiLSTM models for multi-step time-series forecasting in the context of energy consumption prediction. By leveraging the Hyperband algorithm, we systematically explored key parameters, including sequence length, number of layers, hidden units, dropout rate and learning rate. This approach allowed us to dynamically adapt the model architecture to the temporal dependencies of the dataset, ultimately identifying a configuration that achieved robust performance on the test set. The study also highlighted the significant computational demands of NAS, especially when run on CPU-only infrastructure. Training large recurrent models with long input sequences (up to 672 timesteps) resulted in trials that could each take over 11 hours in the final stage of Hyperband. With hundreds of parameter combinations to evaluate, the full search can easily span several weeks, making access to GPU or HPC resources essential for practical deployment. Future work will focus on scaling the methodology across thousands of buildings by leveraging knowledge distillation (Teacher-Student networks) [6]. Large BiLSTM models trained on HPC infrastructure will act as teacher models, while smaller, compressed student models will be distilled to maintain accuracy with significantly reduced complexity. This will enable efficient deployment of predictive models on resource-constrained devices, such as embedded chips in smart meters, ensuring scalable, real-time forecasting across large building networks. Future work could also address the limitations of BiLSTMs in handling very long temporal dependencies and their sequential processing bottleneck by exploring Transformer-based architectures. Transformers, with their self-attention mechanism and parallelization capabilities, are better suited for capturing global patterns in long time-series and could further improve forecasting accuracy and scalability.

[2] `https://mlr3hyperband.mlr-org.com/reference/mlr_tuners_hyperband.html`

6 Acknowledgments

The authors' research was co-funded by the Republic of Slovenia, the Ministry of HE, Science and Innovation and the Slovenian Research and Innovation Agency of the European Union - NextGenerationEU through the DIGITOP project and by the Slovenian Research and Innovation Agency (ARIS) through the annual work program of Rudolfovo. The work is also funded through the business experiment Advanced Multi-objective Optimization of Energy Management - AIMED-HPC, which received funding through the FFplus project, which is financed by the European High-Performance Computing Joint Undertaking (JU) under the funding agreement No. 101163317. The JU receives support from the Horizon Europe program of the European Union.

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KERNELIZATION-BASED ITERATIVE REFINEMENT OF UPPER BOUNDS FOR MAXIMUM CLIQUE PROBLEM

Aljaž Krpan*‡, Janez Povh †,‡

*University of Ljubljana, Faculty of Mathematics and Physics, Slovenia
†University of Ljubljana, Faculty of Mechanical Engineering, Slovenia
‡,Rudolfovo Institute, Novo Mesto, Slovenia
email: krpan.aljaz@gmail.com, janez.povh@rudolfovo.eu

Abstract
The Maximum Clique Problem (MCP) is a central challenge in combinatorial optimization and graph theory, with numerous applications in areas such as computational biology, social network analysis, and coding theory. Due to its NP-hardness, practical approaches often employ preprocessing techniques—particularly kernelization—to reduce the problem size prior to applying exact or heuristic algorithms. In this paper, we propose a novel kernelization method for the Maximum Clique Problem that introduces a new edge elimination strategy, enhancing the effectiveness of existing reduction techniques and further improving overall preprocessing performance. Additionally, we demonstrate how kernelization can be leveraged to iteratively refine existing upper bounds for MCP. Experimental results confirm the effectiveness of our approach, highlighting its potential as a powerful tool for advancing large and complex instances of the Maximum Clique Problem.

Keywords: Maximum Clique Problem, kernelization, graph theory, combinatorial optimization, NP-hard problems, clique number, preprocessing techniques, upper bound refinement, graph reduction, quantum annealing

1 INTRODUCTION

The *Maximum Clique Problem* (MCP) is one of the most studied problems in graph theory and combinatorial optimization. Let $G = (V, E)$ be a simple, undirected graph with $|V| = n$ vertices and $|E| = m$ edges. A subset $S \subseteq V$ is said to form a *clique* if every pair of distinct vertices in S is connected by an edge in E, that is, the induced subgraph $G[S]$ is a complete graph. The *clique number* of G, denoted by $\omega(G)$, is defined as the cardinality of a largest clique in the graph. The *Maximum Clique Problem* (MCP) seeks to compute the value of $\omega(G)$ and, optionally, to identify a corresponding subset of vertices that realizes this maximum. Due to its NP-hardness [5], MCP has attracted considerable interest both in theory and in practice. The problem arises in a variety of domains, including bioinformatics (e.g., protein interaction networks) [11], social network analysis (e.g., community detection) [3], and computational chemistry (e.g., structure alignment) [7].

Given the computational intractability of the Maximum Clique Problem (MCP), modern solvers frequently employ preprocessing techniques to simplify the input graph prior to applying exact or heuristic algorithms [10, 8]. One particularly powerful such technique is *kernelization*, which transforms the original instance into a smaller, equivalent instance known as a *kernel* [1]. When effective, kernelization can significantly reduce both the search space and computational time, particularly in the case of large, sparse graphs [2, 4].

In this paper, we introduce a novel kernelization method for the Maximum Clique Problem, incorporating new edge reduction technique that enhances preprocessing effectiveness. In addition, we propose a mechanism for iteratively refining known upper bounds on the clique number—a strategy that proves beneficial for tightening bounds within branch-and-bound frameworks or other exact search procedures. Our contributions are substantiated through a

comprehensive empirical evaluation on standard benchmark instances, demonstrating that the proposed method has the potential to yield tighter upper bounds and to enhance the efficiency of solution procedures.

2 KERNELIZATION AND UPPER BOUNDS

Our first reduction technique is based on the concept of the k-core of a graph, originally introduced by Seidman [9]. This technique exploits the fact that every vertex belonging to a clique of size k must have degree at least $k-1$.

Definition 1 *Let the graph $G=(V,E)$. The induced subgraph G' is the k-core of the graph G if for every vertex $v \in V'$, $|N(v)| \geq k$, and it is the largest subgraph with this property.*

Proposition 1 *Let the graph $G=(V,E)$ and $K \subseteq V$ of size k. Then K is a clique in the graph G if and only if K is a clique in the $(k-1)$-core of the graph G.*

By computing the k-core of a graph, it is possible to reduce the graph while still preserving all cliques of size at least k.

The next well-known result, although difficult to attribute to a specific original source, is widely recognized within the mathematical community. Specifically, if an edge $\overline{uv}$ is part of a clique of size k, then it must hold that $|N(u) \cap N(v)| \geq k-2$, where $N(u)$ and $N(v)$ denote the neighborhoods of vertices u and v, respectively. Motivated by this observation, we introduce a definition and corresponding proposition analogous to that of the k-core.

Definition 2 *Let the graph $G=(V,E)$. The subgraph G' is the k-edge-core of the graph G if for every edge $\overline{uv} \in E'$, $|N(u) \cap N(v)| \geq k$, and it is the largest subgraph with this property.*

Proposition 2 *Let the graph $G=(V,E)$ and $K \subseteq V$ of size k. Then K is a clique in the graph G if and only if K is a clique in the $(k-2)$-edge-core of the graph G.*

The main contribution of this paper is the introduction of the definition and corresponding proposition for the (k,d)-*edge-core*, which represents a natural generalization of the well-established concept of the k-edge-core.

Definition 3 *Let $G=(V,E)$ be a graph, $\overline{uv} \in E$, and $k,d \in \mathbb{N}$ such that $0 \leq d \leq k$. Then define the condition $P_{G,k,d}(u,v)$ as:*

1. *If $d=0$:*

$$P_{G,k,0}(u,v) \;:=\; |N(u) \cap N(v)| \geq k$$

2. *If $d \geq 1$:*

$$\begin{aligned} P_{G,k,d}(u,v) \;:=\; & \exists w_1 \in N(u) \cap N(v), \\ & \exists w_2 \in N(u) \cap N(v) \cap N(w_1), \\ & \exists w_3 \in N(u) \cap N(v) \cap N(w_1) \cap N(w_2), \\ & \qquad \vdots \\ & \exists w_d \in N(u) \cap N(v) \cap N(w_1) \cap \ldots \cap N(w_{d-1}), \\ & \qquad |N(u) \cap N(v) \cap \ldots \cap N(w_d)| \geq k-d \end{aligned}$$

Definition 4 *Let the graph $G = (V, E)$. The subgraph G' is the (k, d)-edge-core of the graph G if for every edge $\overline{uv} \in E'$, $P_{G,k,d}(u, v)$ holds, and it is the largest subgraph with this property.*

Proposition 3 *Let the graph $G = (V, E)$, $K \subseteq V$ of size k, and $0 \leq d \leq k$. Then K is a clique in the graph G if and only if K is a clique in the $(k-2, d)$-edge-core of the graph G.*

It is evident that by setting $d = 0$, the proposed definition reduces to the classical k-edge-core. The introduction of the parameter d provides a mechanism to trade computational time through increasing values of d for potentially stronger reduction results.

The preceding observations can be directly employed to reduce the graph. This leads to the following reduction procedure:

$$\begin{aligned}
H_0 &:= G \\
H_1 &:= (k-1)\text{-core of } H_0 \\
H_2 &:= (k-2, d)\text{-edge-core of } H_1 \\
H_3 &:= (k-1)\text{-core of } H_2 \\
H_4 &:= (k-2, d)\text{-edge-core of } H_3 \\
&\vdots \\
H_i &:= (k-1)\text{-core of } H_{i-1} \\
H_{i+1} &:= (k-2, d)\text{-edge-core of } H_i \\
&\vdots
\end{aligned}$$

The process is terminated at the smallest value of l satisfying the conditions:

$$|V_{H_l}| = |V_{H_{l+1}}| \quad \text{and} \quad |E_{H_l}| = |E_{H_{l+1}}|.$$

We denote this reduction process by $R_{k,d}(G) := H_l$. By applying Propositions 1 and 3, we observe that:

Observation 1

$$K \text{ is a clique of size } k \text{ in } H_i \;\Leftrightarrow\; K \text{ is a clique of size } k \text{ in } H_{i+1}.$$

This implies that the reduced graph $R_{k,d}(G) := H_l$ preserves all cliques of size at least k.

To the best of our knowledge, the following approach has not been previously introduced, as we have not encountered it in any of the reviewed literature. By leveraging the proposed reduction techniques, we iteratively refine the existing upper bound for the maximum clique in a graph. In particular, using Observation 1, it follows that:

$$\begin{aligned}
\omega(H_1) < k &\Rightarrow \omega(G) < k \\
\omega(H_2) < k &\Rightarrow \omega(H_1) < k \\
\omega(H_3) < k &\Rightarrow \omega(H_2) < k \\
&\vdots \\
\omega(H_{l-1}) < k &\Rightarrow \omega(H_{l-2}) < k \\
\omega(H_l) < k &\Rightarrow \omega(H_{l-1}) < k,
\end{aligned}$$

Combining these results yields the following observation:

Observation 2

$$\omega(H_l) < k \;\Rightarrow\; \omega(G) < k.$$

Based on this observation, we propose the following algorithm for iteratively improving the upper bound on the clique number:

1. Compute an initial upper bound using any suitable algorithm. Denote this initial value by $k := \omega^{ub}(G)$. Initialize the depth parameter by setting $d = 0$.

2. Compute the reduced graph $H = R_{k,d}(G)$. Using Observation 2, the following holds:

$$\begin{aligned} \omega^{ub}(H) < k \;&\Rightarrow\; \omega(H) < k \\ &\Rightarrow\; \omega(G) < k \\ &\Rightarrow\; \omega(G) \leq k - 1. \end{aligned}$$

3. If in Step 2 it is determined that $\omega^{ub}(H) < k$, this implies that the upper bound can be improved to $k - 1$. In this case, update $k := k - 1$ and repeat the process from Step 2. Otherwise, increment d by one and proceed again to Step 2.

The algorithm terminates either when the lower and upper bounds coincide, or when the allotted computation time is exhausted.

3 COMPUTATIONAL RESULTS

In this section, we apply the method introduced in the previous section to compute upper bounds for the graphs described in [6]. The selected instances are restricted to graphs containing at most 125 vertices. This selection allows for a clearer presentation of the main ideas, as the results are preliminary. Both the methodology and its implementation require further refinement to enhance efficiency and scalability for application to larger and more complex graphs.

The algorithms were implemented in `C++17`, and all computations were performed on an `Intel(R) Core(TM) i7-9750H CPU @ 2.60GHz`. For the initial upper bound, we adopt the simple choice $\omega^{ub}(G) = |V(G)|$, allowing for a transparent observation of how the upper bound improves as the parameter d increases. The computations were terminated either when the upper and lower bounds coincided or when the runtime reached the one-hour limit.

The value reported in the $\omega(G)$ column corresponds to the best-known lower bound, which, given the small size of the graphs, also represents the exact clique number in all cases considered. The computational results are summarized in Table 1. The large central block of the table presents the best computed upper bounds for each value of d. Cases where the lower and upper bounds coincide are highlighted in bold. The graphs are ordered by edge density, defined as the ratio between the number of edges and the total number of possible edges in the graph.

As the results indicate, the proposed method exhibits diminished effectiveness on denser graphs. In contrast, for graphs with edge density below 0.56, the method consistently succeeded in reaching the exact clique number, demonstrating its potential for application to sparser graph instances.

4 CONCLUSIONS AND FUTURE WORK

The objective of this study was to present preliminary results on graph reduction using the (k, d)-edge-core and to demonstrate how kernelization techniques can be employed to iteratively improve upper bounds for the Maximum Clique Problem. The results indicate that

Graph G	Vertices	$\omega(G)$	Best upper bounds for different d								Edge density	Run time
			0	1	2	3	4	5	6	7		
hamming6_4	64	4	8	**4**							0.35	<1s
dsjc125.5	125	10	22	16	13	11	**10**				0.50	45s
paley61	61	5	16	9	6	**5**					0.50	2s
paley73	73	5	19	11	7	**5**					0.50	7s
paley89	89	5	23	13	9	**5**					0.50	25s
paley97	97	6	25	15	9	7	**6**				0.50	44s
paley101	101	5	26	15	8	**5**					0.50	51s
johnson8_2_4	28	4	8	**4**							0.56	<1s
johnson16_2_4	120	8	68	48	32	31					0.76	1h
johnson8_4_4	70	14	38	30	24	22	22	**14**			0.77	9m 4s
evil_s3m25x5	125	20	91	89	87	81	79				0.89	1h
C125.9	125	34	86	83	81	78	76				0.90	1h
hamming6_2	64	32	52	48	45	43	42	42	39	37	0.90	1h
dsjc125.9	125	34	88	86	83	81	77				0.90	1h
evil_chv12x10	120	20	94	92	88	85					0.92	1h
MANN_a9	45	16	37	36	34	33	31	29	28	26	0.93	1h
evil_myc11x11	121	22	97	94	90	87	101				0.93	1h
evil_myc5x24	120	48	105	104	103	102	101				0.97	1h

Table 1: Results of the proposed upper bound algorithm with computation time limited to one hour. For each value of d, the corresponding improvement in the upper bound is reported. Instances where the upper and lower bounds coincide are highlighted in bold.

the proposed method shows potential to contribute to clique-related computations; however, further work is required to enhance its practical effectiveness.

As part of future work, the first step will involve optimizing the existing algorithms to minimize the reliance on recursive calls when computing the (k, d)-edge-core. Following this, efforts will be directed toward parallelizing the computation of the (k, d)-edge-core to improve scalability and runtime performance. Once these improvements are achieved, the method will be evaluated in combination with other established reduction techniques to assess whether it can contribute to state-of-the-art results. In addition, it may be worthwhile to explore the development of heuristic, approximation, or probabilistic algorithms for computing the (k, d)-edge-core, as the current approach becomes prohibitively slow for larger values of d.

Moreover, once the proposed algorithmic refinements have been implemented, these reduction techniques offer a promising application beyond the computation of upper bounds—namely, as a preprocessing stage for quantum annealing. One of the main bottlenecks of current quantum annealers is the limited size of the input problem, which stems from both the inherent noisiness of quantum devices and the sparsity of the underlying qubit topology. As a result, embedding dense or large graphs onto a sparse qubit topology introduces additional noise, since multiple physical qubits must be chained together to represent a single vertex or edge as described in [12]. In other words, the denser the graph, the smaller it must be to be suitable for embedding on a quantum annealer. This bottleneck can be mitigated by reducing the size of the input graph. Consequently, an effective graph-reduction phase could enable quantum hardware to handle significantly larger and denser instances, in a manner analogous to the approach presented in [6].

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The 18th International Symposium on
Operational Research in Slovenia

SOR ’25

Bled, SLOVENIA
September 24 - 26, 2025

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MAKING SENSE OF RATIONALITY I: PROS AND CONS.

Drago Bokal
Faculty of nat. sc. and mathematics, University of Maribor, Slovenia,
Institute of mathematics, physics, and mechanics, Ljubljana, Slovenia,
DataBitLab, d.o.o., Maribor, Slovenia
Špela Kajzer
Faculty of organisational sc., University of Maribor, Slovenia,
Faculty of nat. sc. and mathematics, University of Maribor, Slovenia
Ernest Petrič
European Faculty of Law, New University, Ljubljana, Slovenia

Abstract:
From behavioral economics it is known that individuals in general do not behave rationally, and thus game theoretical results have limited predictive power. We use the Pirate game to demonstrate the conflict between utility-based and agency-based definitions of rationality. We apply Snowden's Cynefin framework to demonstrate sense-making process of observing and applying concepts that lead to agent's internal goals as an alternative to merely maximizing externally imposed utility of the agent. We illustrate the process with examples from legislative theory and practice and conclude with a list of applications of a the new approach to rationality modelling.

Keywords: decision making, social-choice, rationality, Cynefin, Pirate game.

1 INTRODUCTION

Snowden [8] has proposed an elementary sense-making device, Cynefin, which is a taxonomy of decision contexts. It passes the paper napkin test in the sense that it can be sketched on a paper napkin over lunch and then used reasonably well by the listeners. Thus, it has potential for a wide-spread application and acceptance, akin to concepts like Ockham's razor and 80-20 rule of time usage estimation.

In Cynefin, the decision making contexts are classified in two dimensions. First according to whether decisions have predictable consequences or not, and second, whether rational analysis plays a role in decision making or not. *Simple contexts* are those where consequences of decisions are known and best practices have been established. Rational analysis is no longer required, and best practice is followed, as circumstances are perceived, categorised, and decision according to the known recipe is implemented. Similarly in *complicated decision contexts*, consequences are understood but best practices are still emerging. An expert analysis is therefore required to yield a proposed decision. In *complex contexts*, consequences of the decisions are no longer predictable, yet once the decisions are implemented, the consequences can be explained and future behaviors can be adapted. In *chaotic contexts*, the consequences cannot be predicted, and there is insufficient understanding of consequences to yield their verifiable explanation. While observation can lead to understanding and adaptation in complex situations, in chaotic circumstances, action to establish order is a prerequisite for any explanations to become verifiable and then for good and best practices to emerge. Finally, *disorder* denotes the circumstances in which insufficient information is available to categorise them in one of the four domains.

Social choice is a clear instance of decision making circumstances. It has been modelled by game-theoretic tools which, frequently relying on strong assumptions of rationality, proposed best practices in analysed contexts [1]. However, behavioral sciences have established [5] that frequently, these best practices are not observed amongst decision makers.

In the next section, we present the pirate game [7, 19], a multiplayer version of the ultimate game [9]. We provide critique of the best practices produced by pure rational analyses of seemingly simple decision making circumstances, arguing the contra aspect of our contribution. We continue with an overview of chaotic, real life legal contexts in which imposing a rational model is beneficial if not required to reach a favorable decision, thus arguing the pro aspect. Finally, we conclude by listing some chaotic decision making contexts that would benefit from better understanding of concept-based rationality.

2 VARIANTS OF THE PIRATE GAME

The original Pirate game (yet generalized according to the number of pirates) states that n pirates have captured g golden coins and need to distribute them according to the following rules. The most senior pirate proposes a split, and then they vote about his proposal. If the proposal is supported by at least half of the votes, then it is accepted, the coins are distributed and the game is over. Otherwise, the proposer is thrown overboard and the next most senior pirate becomes the proposer for another attempt at the distribution. Pirates have the following preferences regarding the outcome: first, each wants to survive. Second, they each want to maximize the number of coins obtained. Subject to the above, they prefer to throw another overboard. Finally, they distrust each other and will not make any deals outside of these assumptions.

Under the above assumptions, we can prove using mathematical induction that whenever there are sufficiently many coins, ie. $2n \leq g$, then the proposer gets $g - (n - 1)$ coins and each pirate with the same parity as the proposer (assuming indexing from the least senior towards the most senior) gets a single coin. For $2n > g$, in most cases the proposers ger discarded one after another, unless $g = 2n + 2^k$ for some integer k. For such numbers, the desire to survive convinces the most senior 2^{k-1} proposers to accept the proposal allocating them nothing, while attributing one coin to any set of n pirates with seniority index at most $2g + 2^{k-1}$.

Social experiments with this game have observed way different behavior [11], implying actual humans do not respect the stated priorities at their decision making. Nevertheless, the game can be used in thought experiments for reasoning about decision and policy making. This direction explores interesting aspect of sense-making in the interplay between chaotic decision context (conflict of interest for the coins) and the simple decision context (setting pirate's preferences, analysing the game and determining the outcome). For instance, the inclusive variant of the game in which the failed proposer is added to the end of the (circular) queue rather than thrown overboard, changes the rational outcome of the game so that they obtain equal shares of the spoils (if g is divisible by n) or the game under assumptions is played indefinitely (each hoping to obtain one of the extra coins remaining after a perfectly equal split). The latter issue can be avoided if infinite divisibility of the coins is assumed, or if the pirates value time and do not distinguish between outcomes that are not significantly different.

3 BEYOND GAME-THEORETIC RATIONALITY

In applying operations research models, results, and solutions to real world problems, one faces the issues of liability, which is a formalized form of accountability. Both are based on responsibility that is rooted in rationality and free will. The Pirate game illustrates that when internal, subjective, free-will based rationality (understood as freedom to choose between options) is replaced by objective external criterion (such as wealth in the Pirate game or game theory in general), the agency of the agent is transferred to these external sources and free-will is constrained by external influences based on assumed structure of the decision process. It is therefore highly debatable, whether rational agents in the pirate game have their rationality based on free will, or is their rationality in fact devoiding them of free will and enforcing their determinism.

However, a rational decision maker (with his rationality based on internal agency of free will, rather than some external factor, such as game-theoretic utility) can have a richer structure of preferences, depending on concepts beyond just plain utility.

An example with several analogies to the Pirate game is the legal doctrine of excluding evidence that is fruit of the poisonous tree, ie. obtained through illegal or otherwise improper procedures. By the Cynefin categorization, persecution process is an example of chaotic decision circumstances resulting from conflict of interest between the persecutor and the persecuted. The persecutor is through its government mandated role in a position analogous to the senior pirate, and is in coalition with the victims of the crime. The persecuted, however, is in a position analogous to the pirates receiving nothing in the distribution. A strictly game-theoretically rational legislator would, analogously with the pirate game, disproportionally empower the persecutor against the persecuted. However, this would open possibilities for inquisition-style legislation. Therefore, the aforementioned doctrine was invented, resolving the issue with a simple decision context of excluding the tainted evidence. Still, the doctrine is not absolute as shown for instance in [22]: while the criminal may not be penalized for the robbery should the state attempt to prove it by unacceptable procedures, he may still be devoid of the benefits of the robbery using unexplained wealth laws, as explained for instance in [10]. Similarly, if there is a criminal case with a victim, excluded evidence from the criminal case can sometimes be used in civil case to establish the right to compensation of the victim. An example is the notorious case of O. J. Simpson, who was found not guilty [16], but liable [6]. Furthermore, in some jurisdictions, exclusion is not automatic, but rather case dependent and at the discretion of the court [13, 21].

The above can be seen as examples of rationality in chaotic multi-criteria-decision-making context steering away from utility maximization towards either maintaining agency of the court or ideating new concepts and developing procedural mechanisms supporting them. Integrating these aspects into the pirate game is an interesting open problem to be modelled in a pirate-game like setting, thus building beyond the artificial-neural-network models that are already capable of reproducing the game-theoretic rationality of the learned agents [14].

4 DISCUSSION OF POSSIBLE APPLICATIONS

The above discussion supports desire for a more complex model of the Pirate game, that would inherit didactic applications from the original game, yet allow to reason about the issues identified above. Ideally, emerging concepts would be result of iterative interaction among the players, who would be able to introduce new concepts and rules. The more detailed desiderata for such a model is beyond the space limiations of this controbution, however, it would provide opportunity for applying operations research techniques, such as stochastic processes, in investigating social choice and building on existing research [2, 20]. Moreover, there are some specific social-choice related problems that could benefit from the models, as

follows:

Continuing from the legal contexts in the previous section, an interesting application of shared resources that has established global legal guidelines is atmosphere. While countries have sovereignty over the space above their territory, the matter contained in that space is common to all countries and is formalized both as a common concern of humankind, as well as a common responsibility [12]. A trivial adaptation of the pirate game that may model this social choice is that all the pirates get executed if the proposal is rejected. A slightly more interesting one would prevent the pirates from breathing for a given period of time, and the pirates may have different stochastic endurances of surviving without air.

Not-invented-here problem. A new pirate proposes an improvement to the system, but other pirates elude its application, as the proposer has (not yet) gained their trust. How is the extent of the evasion dependent on the parameters of the model?

General acceptance of innovations. Which parameters govern acceptance of innovations? To what extent could we predict upfront, how would a specific change in processes proliferate among a community of potential users? Attempts at understanding this problem have been investigated in [17, 18].

Circumstances-dependent voting threshold. Already in established democracies, for certain changes a qualified majority needs to be reached. What insights would be gained from implementing a circumstances dependent voting threshold? Specifically, would law-accepting majority dependent on the per capita GDP induce a more cooperative society?

Qualified majority for major societal changes. In recent history, several peaceful major changes have been proposed and implemented. Slovenian independence vas voted for by 89.31 % of eligible voters and 35 years later, we may claim the change was a success [15]. Brexit vas voted for by 51.89 % of participating voters, but only 37.47 % of eligible voters. Today, Google Scholar lists 10.700 papers on “Brexit regret,” which was measured at 28.7% [4]. Finally, the Catalan independence referendum succeeded by 92.01 % voting in favour, but that was only 39.59 % of eligible voters. The independence movement stirred the politics, but failed to implement any change.

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DATA-DRIVEN OPTIMIZATION OF INDUSTRIAL DRYING PROCESS FOR ENERGY USAGE AND EMISSIONS REDUCTION

Elena Čokor[1,2] , Nikola Čokor[1,4] , Anja Šketa[1,2,3], Andraž Čevka[5], Dominik Jagodic[5] , Drago Bokal[1,2,3]

elena.cokor@student.um.si, nikola.cokor@student.um.si, anja.sketa@databitlab.eu, andraz.cevka@exoterm.si, dominik.jagodic@exoterm.si, drago.bokal@um.si

[1]DataBitLab d.o.o., Perhavčeva ulica 13, 2000 Maribor, Slovenia

[2]University of Maribor, Faculty of Natural Sciences and Mathematics, Koroška cesta 160, Maribor, Slovenia

[3]Institute of Mathematics, Physics and Mechanics, Jadranska ulica 19, 1000 Ljubljana, Slovenia

[4]University of Maribor, Faculty of Electrical Engineering and Computer Science, Koroška cesta 46, 2000 Maribor, Slovenia

[5]EXOTERM-IT d.o.o., Struževo 66, 4000 Kranj, Slovenia

Abstract

Industrial drying is energy-intensive and often uses uniform cycles, ignoring product-specific needs. This wastes energy and increases CO_2 emissions. We present a data-driven optimization of the Garnex drying process, using real-time product characteristics for strategic batch planning. Grouping products with similar requirements and adjusting drying durations improved oven efficiency without extending operating time. The optimized method increased production capacity by 8% and cut energy use and CO_2 emissions per product by over 7%, all without equipment changes. This shows how operational intelligence can support climate goals. The model also enables future integration of production data into real-time simulations for further optimization. This case confirms that adaptive process control can improve both environmental and economic performance.

Keywords: process optimization, energy efficiency, CO_2 emissions, industrial drying, production scheduling, sustainability, smart manufacturing

1 INTRODUCTION

Industrial drying processes are among the most energy-intensive operations in manufacturing [1]. Traditionally, drying cycles are applied uniformly, regardless of product-specific needs, resulting in unnecessary energy use and elevated CO_2 emissions.

This project addresses this challenge by developing a data-informed approach that dynamically adapts drying times based on real-time production parameters [2]. Through simulations of different loading and scheduling scenarios in two industrial ovens, we identified opportunities to shorten drying duration, reduce energy consumption per unit, and cut emissions — all without compromising product quality.

The study focuses on thermal insulation products manufactured by Exoterm, where drying cycles were previously uniform despite varying product-specific requirements. By using real-time data and targeted simulations, we show how dynamic timing adjustments reduce energy waste and emissions while maintaining quality. This approach represents a step toward more sustainable industrial operations by combining process efficiency with measurable environmental benefits.

2 OPTIMIZATION AND ANALYSIS OF THE DRYING PROCESS

The optimization targets thermal insulation products, which exhibit varying optimal drying times due to differences in material composition, size, or moisture content. While some products require shorter drying cycles, others need extended durations. The core improvement introduced by this project lies in the strategic grouping of products based on their optimal drying duration [3]. Instead of loading the oven randomly with a mix of products, products with similar drying time requirements are now grouped and processed together. This allows each batch to be dried for an optimal duration — neither under- nor over-dried — which significantly reduces unnecessary drying time and avoids energy waste.

By aligning product characteristics with the drying schedule, the process becomes more time-efficient, energy-efficient, and ultimately less carbon-intensive. The effectiveness of this approach is demonstrated in Table 1, which summarizes relative improvements achieved through optimization in two industrial ovens. The optimization was evaluated by comparing the number of product trolleys processed within a fixed observation period before and after the new scheduling approach. Energy consumption was assumed constant across both scenarios, and from this baseline we calculated the average energy consumption per trolley. CO_2 emissions per trolley were estimated proportionally, based on an established electricity emission factor. Additionally, we monitored the average drying time per batch, which enabled us to quantify process acceleration. The resulting percentage improvements clearly show the benefits of aligning drying duration with product-specific needs.

Table 1. Percentage improvements for key indicators after drying process optimization (Oven 1 and Oven 2).

Indicator	**Oven 1**	**Oven 2**
Increase in number of products	8.14%	7.97%
Reduction in energy per product	-7.53%	–7.38%
Reduction in CO_2 emissions per product	–7.53%	–7.38%
Reduction in average drying time	–20.24%	–26.08%
Change in total operating time	0%	0%

For both ovens, the total operating time over the observation period remains unchanged. This means that the energy consumed is also constant — the ovens are working the same number of hours. However, due to process optimization, the average drying time per product batch was reduced significantly for Oven 1, drying time dropped by 20.24% and for Oven 2, the reduction was even more substantial, reaching 26.08%. This reduction in drying time

enabled more product batches to be processed within the same time frame Oven 1 processed 8.14% more product units, Oven 2 processed 7.97% more units.

Since energy input remained constant but the number of products increased, the energy consumption per product unit decreased by 7.53% for Oven 1 and by 7.38% for Oven 2. This improvement directly translates to lower CO_2 emissions per unit, following the same percentage reductions as energy use — a result of the proportional relationship between electricity consumption and emissions. Specifically Oven 1 reduced emissions by 7.53% and Oven 2 achieved a reduction of 7.38%.

These results highlight the strategic value of process monitoring and optimization. By avoiding rigid, one-size-fits-all drying cycles and instead tailoring drying time to actual product requirements, it is possible to increase output capacity without any increase in operating time or energy input. In practical terms, this means that: more products are processed with the same amount of energy; each product has a smaller energy and carbon footprint; equipment is used more efficiently, with reduced idle or overprocessing time.

The approach supports broader sustainability goals and aligns with European Green Deal [4] targets by demonstrating a scalable method for reducing industrial CO_2 emissions without costly equipment upgrades.

3 ENVIRONMENTAL AND OPERATIONAL IMPACT

The optimization of the drying process contributes to climate neutrality goals not through technological overhaul, but through smarter use of existing equipment. By improving how production is scheduled and how batches are formed, the process reduces waste — not just of energy, but of time and capacity.

One of the most important environmental outcomes is the lower carbon footprint per product, achieved without increasing total energy. This highlights the potential of operational adjustments as a viable alternative to expensive retrofitting, particularly in factories with long equipment lifecycles.

From an operational standpoint, the benefits are equally significant. Higher product throughput in the same operating window means better return on energy, labor, and machine time. This effectively raises the productivity of the kilns, allowing the factory to respond more flexibly to demand fluctuations, tight delivery schedules, or energy price volatility — all while maintaining consistent product quality.

What makes this approach especially scalable is its simplicity: it requires no new hardware or complex digital infrastructure. Instead, it relies on process awareness, basic data, and a change in planning logic — all of which can be extended to other production stages or factories with similar characteristics.

The alignment between operational efficiency and emission reduction illustrates a core principle of sustainable manufacturing: that ecological and economic objectives need not be in conflict, and that optimization at the process level can generate meaningful environmental outcomes.

4 CONCLUSION

This project demonstrates that even small-scale adjustments in production planning — such as aligning drying durations with product-specific requirements — can yield measurable environmental and operational benefits. Without increasing total energy use or changing kiln infrastructure, the factory achieved lower energy consumption per product, reduced CO_2 emissions, and improved equipment utilization.

The approach is practical, scalable, and based on accessible production data. It shows that sustainability and productivity can be addressed simultaneously through intelligent process design.

Looking forward, there are several opportunities to further enhance this model simulation with real production data that would enable calibration of the system to actual operating conditions, allowing for more accurate predictions and decision-making. By integrating order flows into the simulation model, it would be possible to optimize not only drying efficiency but also inventory planning — ensuring that products are dried just in time and in the right quantities. Such integration could support the optimization of stock levels, minimize overproduction, and reduce storage-related energy and material losses.

Ultimately, expanding the simulation framework toward a real-time decision-support tool could empower factories to respond dynamically to changing demands, reduce their environmental footprint, and align with broader climate neutrality goals.

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FORESIGHT IN THE DANUBE REGION: AWARENESS, APPLICATION AND TRAINING NEEDS

Janja Jerebic
University of Maribor, Faculty of Organizational sciences, Kidričeva cesta 55a, Kranj, Slovenia
E-mail: janja.jerebic@um.si

Špela Kajzer
University of Maribor, Faculty of Organizational sciences, Kidričeva cesta 55a,
Kranj & Faculty of Natural Science and Mathematics, Koroška cesta 160, Maribor, Slovenia
E-mail: spela.kajzer1@um.si

Alenka Brezavšček
University of Maribor, Faculty of Organizational sciences, Kidričeva cesta 55a, Kranj, Slovenia
E-mail: alenka.brezavscek@um.si

Abstract: This paper presents the main findings of a survey conducted as part of the ForeDanube project to assess the awareness, application and training needs related to foresight in the Danube region. Although foresight is not yet widely applied, its strategic importance is well recognised, especially in the areas of planning, innovation and technology. The organisations show great interest in practical and flexible training formats. Based on the results, we recommend the development of a dedicated educational platform, possibly through micro-certificates within master's programmes, and the establishment of a regional foresight competence centre. These efforts could strengthen strategic planning, encourage innovation and support long-term cross-border co-operation.

Keywords: strategic foresight, Danube region, organisational planning, capacity building, foresight training, Interreg

1 INTRODUCTION

With increasing economic uncertainty and growing competitive pressure, strategic foresight is becoming a crucial competence for companies that want to remain resilient and future orientated. Defined as a structured process of observing, interpreting and responding to early signals of change, strategic foresight enhances dynamic capabilities across a range of strategic activities (Iden et al., 2017; Cook et al., 2014). In a case study with several large organisations, Vecchiato (2012) shows that foresight practises, such as horizon scanning and scenario planning, support the coordination of insights to improve decision-making under uncertainty. This link to decision quality is particularly important in highly competitive and rapidly evolving environments, where recognising weak signals early can provide a strategic advantage.

The increasing availability of digital tools such as artificial intelligence, big data and machine learning has further strengthened the role of foresight by enabling the identification of trends in real time and the creation of predictive models (Mühlroth & Grottke, 2020). These developments underline the growing importance of institutionalising foresight practises in both public and private sector organisations.

The ForeDanube project (Interreg Danube, n.d.) directly addresses the existing foresight gap in the Danube region by promoting capacity building and institutional support for forward-looking planning. In this paper, we present the results of a survey conducted as part of the project to assess the level of awareness, practical application and training needs in relation to foresight methods among organisations in the Danube region. A more detailed presentation of the survey results is available in the ForeDanube internal report (Jerebic & Brezavšček, 2025).

2 METHODOLOGY

The aim of the survey was to gain an insight into the current use of foresight in strategic planning and innovation processes, to identify gaps in knowledge and implementation and to assess the interest of organisations in further education and training. In addition to descriptive analysis, selected statistical tests were conducted to examine patterns and differences between sectors, organisational sizes and ownership structures.

The survey was developed using the 1KA online platform and sent by email to over 1,000 contacts across the Danube region. It was available in three languages (English, Slovenian and German) to ensure broad accessibility and was open from 11 March to 19 May 2025.

3 RESULTS

3.1 Survey sample overview

The survey collected 68 valid responses, including 51 fully and 17 partially completed questionnaires, from 422 link clicks (response rate: 16.1%). The relatively high number of clicks compared to the number of valid responses indicates that many recipients may not have been familiar with the topic of foresight and therefore decided not to continue with the survey. This suggests that the concept of foresight is still relatively unknown or unclear to a large proportion of the target audience in the Danube region.

The respondents represented a diverse mix of organisation types, with 69.1% from the private sector, 23.5% from the public sector and 4.4% from mixed-ownership companies. In terms of size, medium-sized organisations were the most represented (33.8%), followed by small (26.5%), large (25.0%) and micro (14.7%) organisations.

Geographically, most responses came from Slovenia (39.7%) and Serbia (38.2%), with smaller proportions from Germany, Romania, Croatia, Austria and Montenegro. Most organisations (63.2%) indicated a strategic planning horizon of 2–5 years.

The sample contains a high proportion of highly qualified staff: 43.3% of organisations indicated that 76–100% of employees have a Master's or PhD degree. In terms of the role of respondents, 26.5% were top managers, 27.9% were middle managers and 27.9% were experts, suggesting that the majority were well positioned to think about forward-looking practises. Most respondents (mean=3.74 at the 5-point Liker t scale) are frequently involved in strategic planning, with managers indicating the highest level of involvement.

3.2 Knowing and using foresight

Only 40% of respondents reported familiarity with the term foresight, and the average self-assessed understanding was 2.77 on a 5-point scale, indicating limited familiarity. Most respondents rated their understanding as poor to average, while 22% of respondents did not specify, likely due to additional uncertainty.

In terms of use within the organisation, only 5.9% indicated that it is used regularly and 29.4% that it is used occasionally, while 27.9% indicated that foresight is not used and 14.7% were not sure. Interestingly, medium-sized organisations reported the most frequent use, while large organisations had the highest level of uncertainty.

Foresight is used most frequently in the areas of trend recognition (mean=3.92), product/service development (3.79), strategic planning (mean=3.71) and technological innovation (3.71). It is used less frequently in personnel management (mean=3.04) and strategy development (mean=3.08).

The main obstacles to the introduction of foresight include a lack of internal knowledge (76%) and a lack of qualified personnel (60%). Other barriers cited include time, financial resources and a lack of external consultants.

Qualitative methods are more widespread than quantitative methods. For example, 85.4% of respondents use a SWOT analysis, while only 52% use data-based forecasts. More advanced methods such as simulation, big data or AI are used less frequently.

To complement the descriptive results, selected statistical tests were conducted to evaluate the predefined research questions related to the understanding and use of foresight. The results are summarised in Table 1.

Table 1: Overview of statistical test results for RQ1–RQ6: Understanding and use of foresight

RQ	Research question	Test	Result	Conclusion
RQ1	Is average understanding of foresight lower than 3 (in the 5-point Likert scale?	One-sample *t*-test	p=0.058	Not significant, but borderline
RQ2	Is understanding of foresight related to organisation size?	One-way ANOVA	p=0.063	Not significant, but borderline
RQ3	Is understanding of foresight different between public and private sectors?	Independent samples *t*-test	p=0.803	No significant difference
RQ4	Is there a correlation between respondents' understanding of foresight and their involvement in strategic planning?	Pearson correlation test	r=0.343 p=0.006	Significant positive correlation
RQ5	Is use of foresight related to organisation size?	Chi-square test	p=0.717	No significant association
RQ6	Is the use of foresight related to the organisation ownership and funding?	Chi-square test	p=0.031	Statistically significant difference: higher usage in private sector

3.3 Importance of foresight methods for organisations

Respondents generally consider foresight to be a strategically relevant tool. The average rating of its importance for organisational development and competitiveness was 3.65 on a 5-point scale (s=0.93), which indicates a strong positive orientation. Almost two thirds of respondents rated foresight as "important" or "very important", with only 2.9% considering it unimportant.

The perception of importance was consistent across both the public and private sectors and across different organisation sizes. While medium-sized organisations rated foresight slightly higher, these differences were not statistically significant.

To investigate these patterns further, we conducted statistical tests on three research questions related to the perceived importance of foresight. The results are shown in Table 2.

Table 2: Overview of statistical test results for RQ7–RQ9: Perceived importance of foresight

RQ	Research question	Test	Result	Conclusion
RQ7	Is the average perceived importance of foresight methods significantly higher than the neutral midpoint 3 (on a 5-point Likert scale)	One-sample *t*-test	p<0.001	Statistically significant: foresight is seen as important
RQ8	Does perceived importance differ by organisation size?	One-way ANOVA (Welch & Brown-Forsythe)	p=0.083/ 0.168	Not significant
RQ9	Does average perceived importance differ by ownership sector?	Independent samples *t*-test	p=0.857	No significant difference between public and private sectors

3.4 Interest in additional training or education in the field of foresight

The survey results indicate a strong interest in additional proactive training. The average agreement with the statement "*If your organisation had the opportunity, it would definitely send its employees to additional foresight training*" was 3.57 on a 5-point scale (s=0.76). More than 65% of respondents agreed or strongly agreed.

Interest was consistently high in organisations of all sizes, but respondents from the public sector expressed a significantly higher level of interest than those from the private sector. Favourite training formats include practical workshops (mean=4.12), webinars (mean=3.70) and individual coaching or mentoring (mean=3.55). Formal university programmes received the lowest average score (mean=2.76), suggesting that companies prefer flexible, short-term and applied learning formats.

To confirm these findings, statistical tests were conducted for the research questions that focused on training interest and differences between organisational types. Table 3 summarises the results.

Table 3: Overview of statistical test results for RQ10–RQ12: Interest in foresight training

RQ	Research question	Test	Result	Conclusion
RQ10	Is average interest in foresight training significantly higher than neutral 3 (on a 5-point Likert scale)?	One-sample *t*-test	$p<0.001$	Statistically significant: clear overall interest
RQ11	Does interest differ by organisation size?	One-way ANOVA	$p=0.534$	No significant difference
RQ12	Does interest differ between public and private sectors?	Independent samples *t*-test	$p=0.006$	Statistically significant: higher interest in public sector

4 CONCLUSION

The results of the survey show that although foresight is not yet widespread in companies in the Danube region, its strategic importance — particularly for planning, innovation and technology — is certainly recognised. Decision-makers generally consider it important. Private sector organisations report more frequent use, whereas the public sector shows a greater interest in training.

This study also points to directions for future research. The large discrepancy between clicks on the survey link and completed responses suggests a need to investigate barriers to the use of foresight. Further studies could investigate how funding appeals drive foresight adoption, particularly in academia, and test the hypothesis that medium-sized organisations rely on foresight to drive their growth, while large organisations may be more stable and less inclined to structured foresight. Comparative or longitudinal studies could shed light on these dynamics.

There is a clear need for capacity building, especially through practical and flexible formats such as workshops, webinars and mentoring. The ForeDanube project is well positioned to promote foresight competencies by developing a dedicated educational platform, possibly with micro-certificates within master's programmes, and by establishing a regional foresight competence centre. Such efforts could strengthen strategic planning, promote innovation and support long-term cross-border cooperation in the Danube region.

Acknowledgements
This research was conducted as part of the ForeDanube project, co-funded by the European Union through the Interreg Danube Transnational Programme. We would like to thank all participants of the survey for their contribution.

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MAKING SENSE OF RATIONALITY II: EMERGING CONCEPTS AND PRACTICES

Špela Kajzer
Faculty of organisational sc., University of Maribor, Slovenia & Faculty of nat. sc. and mathematics, University of Maribor, Slovenia
Anja Šketa
Institute of mathematics, physics, and mechanics, Ljubljana, Slovenia & DataBitLab, d.o.o., Maribor, Slovenia
Alenka Brezavšček
Faculty of organisational sc., University of Maribor, Slovenia
Drago Bokal
Faculty of nat. sc. and mathematics, University of Maribor, Slovenia, Institute of mathematics, physics, and mechanics, Ljubljana, Slovenia & DataBitLab, d.o.o., Maribor, Slovenia

Abstract: Cynefin is Snowden's sense making device that categorizes decision contexts into chaotic (unpredictable, unexplainable consequences), complex (unpredictable, explainable consequences), complicated (predictable consequences, emerging good practices) and simple (predictable consequences with best practices). In a parallel contribution, we have demonstrated the pros and cons of addressing chaotic decision contexts with rigorous mathematical tools suitable to simple contexts. In this contribution we demonstrate an alternative path, where modelling addresses the path through introducing stochastic tools. The sound mathematical fundament still allows for rational reasoning, but due to its stohastic nature, flexibility is not lost in trivialized complex relationships and their stability rather emerges through evolutionary processes.

Keywords: stochastic processes, perception, Markov kernel, Markov antimatroids, Cynefin

1 Introduction

In 1997, Olav Kallenberg's work on stochastic processes provided the mathematical formalism needed to describe systems evolving under uncertainty [8]. The view that perception is fundamentally shaped by evolutionary pressures and uncertainty was further developed in Donald Hoffman's Interface Theory of Perception [7]. Hoffman posits that perceptual systems evolved not to represent reality accurately, but to construct adaptive interfaces optimized for survival [7]. While Kallenberg formalized uncertainty mathematically, Hoffman reframes the purpose of perception biologically, suggesting that what we perceive is filtered through an evolutionary lens prioritizing utility over truth. Mark et al. [11] used evolutionary games to analyze under what circumstances natural selection favors veridical perceptions. Results from their research state that veridical perceptions of the world have little if any chance of surviving evolutionary competition, which favors utilitarian interface-like perceptions, optimized to perceive the utility derived from the actions in a given environment [7]. In his later work, Prakash expands upon the agent, defined in the interface theory of perceptions, and proposes a Conscious Agent Thesis (CAT): Any process of consciousness can be modeled by a 7-tuple $\langle W, X, G, P, D, A, N \rangle$ explained below.

In our contribution, we highlight that the work of Fic and Bokal [4], casts a shadow on CAT. Specifically, we address the claim that evolution favours utilitatistic rather than veridical perceptions. While we provide additional arguments for this claim, in the process we propose a Markov antimatroid as a generalization of Prakash's conscious agent that better models cognitive processes. This does not disprove Prakash's thesis, but offers embedded agent model, studied by Uexküll [14], Ay, and Löhr, [1] as a better model of conscious agent. We further

use their model to argue for explicit presentation of both veridical and utilitaristic roles in the evolutionarily reinforced ontology learning process.

2 Existing models

In their work, Mark et al. [11] use the setting of territorial games to define veridical and utilitarian perceptual strategies of the amount of resource r in the environment. Agent with veridical perception either observes the exact amount of resource in an environment (*truth* strategy) or n distinct categories of the amount of resource (*critical realist* strategy), while the agent with utilitarian perception observes the environment in terms of utility (*interface* strategy).

The agents are later formalised by Prakash, who introduces a Conscious Agent, denoted CA, as a 7-tuple $\langle W, X, G, P, D, A, N\rangle$, where W denotes the states of the world, X the perceptions of the agent and G the actions of the agent on the world. W, X and G are probability spaces. N is an integer that counts the number of perception-action events after some arbitrary start. The act of perception is represented by a Markov kernel $P : W \times X \to X$, the decision by a Markov kernel $D : X \times G \to G$ and the action by a Markov kernel $A : G \times W \to W$ [12].

In his work, he proposes a Conscious Agent Thesis: Any process of consciousness can be modeled by a 7-tuple $\langle W, X, G, P, D, A, N\rangle$ as above.

Independently to Prakash, Ay and Löhr propose a formal model of an embodied agent based on a causal description of the sensorimotor loop [1]. A central feature of this framework is the formalization of the agent's *Umwelt*—a concept introduced by Uexküll [14] to describe the subjective world experienced by an organism. The Umwelt is formalized as a σ-algebra $\mathcal{W}_{\text{int}} \subseteq \mathcal{B}(W)$, which captures the distinctions between world states that are accessible to the agent through its ongoing perception and action. In contrast, the extrinsic perspective of an external observer is represented by a typically larger σ-algebra $\mathcal{W}_{\text{ext}}$. Ay and Löhr show that, under continuity and compactness assumptions, a minimal model of the world can be constructed that is observationally equivalent to the agent's experience, even without full access to the external dynamics.

We address Prakash's thesis using the Cynefin framework, which is a conceptual framework used to aid decision-making by assessing the agent's context, proposed by Snowden [13] and introduced to Operations Research by French [5]. Cynefin offers five decision-making domains: simple, complicated, complex, chaotic and a center of disorder. The simple domain has a clear causal connection between cause and effect while the behaviour in a simple domain is stable. In a complicated domain, the problem is also identified and resolved, but the process producing a solution to a given problem is not a simple categorization. Rather, it requires expert analysis that substantiates the known causal connection and finds an appropriate solution that is not commonly known in such a domain and therefore represents a (new) good practice. A complex domain differs from the previous ones in the abundance of relevant concepts that influence the processes and their results, or in the fact that the conceptual apparatus for functioning in them has not yet been sufficiently understood. A chaotic domain is identified by blurred cause-and-effect relationships. This ambiguity prevents their post-festum analysis. The center of disorder is the last domain that represents situations where there is no clarity about which of the other domains apply. In the center of disorder, the primary goal is to gather more information, so that an agent can identify a specific domain of complexity and adhere to its practices.

Connecting the Cynefin framework with the agents, described by Mark et.al. [11] and later formalised by Prakash [12], the world is assumed to be chaotic. The process needed to shift from chaos to complex realm, by French, is acquiring knowledge through scientific methods. If a veridical agent in our model is capable of observing the situation and seeking out patterns that can help him introduce new concepts and thus innovate, such an agent can become competitive and long term, even more fit than the interface agent.

Intuitively, if the world provides the agent enough resources, the agent that distinguishes between 'too little', 'enough' and 'too much', has the time to observe, learn and innovate, moving first to a complex, and then to a complicated realm. If the agent has veridical perception, it can innovate based on veridical perceptions of the environment to move to a complicated realm, so that through predictability, good practices can emerge and be, upon further usage and utility-based innovation, translated to best practices rendering the final agent, who combines both veridical and utilitarian perceptions in their decision making, the most fit for survival in the long term.

Where agents from Prakash et al. and Ay et al. have a predefined Markov kernel to use for every agent's state, we provide a new structure: Markov antimatroids, which allows us to shift between kernels as necessary to optimise the utility function of the model. We present some of the theory on Markov antimatroids in the next section.

3 MARKOV ANTIMATROIDS

In this section, we use known definitions on antimatrodis, as defined in [10] and known definitions on Markov kernels, as defined in [9]. We avoid restating them in this paper due to the length constraints.

In what follows, different sequences of learning will play an important role. We formalize them as:

Definition 1 (Learning path) *Let $\mathcal{F} \subseteq 2^C$ be an antimatroid on a finite set of concepts C. For $n \in \mathbb{N}$, let $(c_i)_{i=1}^n$ be a sequence of concepts, ie. $c_i \in C$, $i = 1, \ldots, n$. This sequence is a **learning path**, if and only if for each $i = 1, \ldots, n.$, c_i is accessible from the set $\{c_k\}_{k=1}^{i-1}$.*

We use the following Theorem to combine sequences of Markov kernels, cf. [9, Corollary 14.24].

Theorem 2 *Let $n \in \mathbb{N}$ and let $(X_i, \mathcal{A}_i)$, $i = 0, \ldots, n$ be measurable spaces. For $i = 1, \ldots, n$, let κ_i be a Markov kernel from $\left(\times_{k=0}^{i-1} X_k, \bigotimes_{k=0}^{i-1} \mathcal{A}_k\right)$ or from $(X_{i-1}, \mathcal{A}_{i-1})$ to $(X_i, \mathcal{A}_i)$.*

Then the recursion $\kappa_1 \otimes \ldots \otimes \kappa_i := (\kappa_1 \otimes \ldots \otimes \kappa_{i-1}) \otimes \kappa_i$ defines a Markov kernel $\bigotimes_{k=1}^{i} \kappa_k := \kappa_1 \otimes \ldots \otimes \kappa_i$ from $(X_0, \mathcal{A}_0)$ to $\left(\times_{k=0}^{i-1} X_k, \bigotimes_{k=0}^{i-1} \mathcal{A}_k\right)$.

Definition 3 *Let C be a finite set of* concepts*, and let $\mathcal{F} \subseteq 2^C$ be an antimatroid on C. Let $(O, \mathcal{O})$ be a fixed measurable space called the* ontology space*, and let each concept $c \in C$ be associated with a measurable space $(Y_c, \mathcal{B}_c)$, called* the refinement by c*, a measurable space $(X_c, \mathcal{A}_c)$, called* the input of c*, the measurable space $Z_c := X_c \times Y_c$, called* the output of c*, and a Markov kernel $\kappa_c : X_c \rightsquigarrow Z_c$.*

*We define a **Markov antimatroid** as the tuple: $(C, \mathcal{F}, O, \{\kappa_c\}_{c \in C})$, subject to the following conditions:*

1. ***Feasibility-Dependent Input Spaces (FDIS):** For any concept $c \in C$, define its* input prerequisite set $\mathrm{IPr}(c) \subseteq \mathrm{Nec}(c)$ *and its prerequisite space P_c as the product of measurable refinement spaces:*

$$P_c := O \times \prod_{c' \in C} Z_{c', \mathrm{IPr}(c)}, \text{ where for } c' \in C, S \subseteq C, Z_{c',S} := \begin{cases} Y_{c'} & \text{if } c' \in S, \\ \varnothing & \text{if } c' \notin S. \end{cases}$$

   Here, $\varnothing$ is treated as a measurable space with σ-algebra $\{\varnothing\}$, so that the product remains well-defined. We require that, for each $c \in C$, X_c is measurably isomorphic to P_c.

2. ***Initial Transitions from $\varnothing$ (ITFE):** For any kernel κ_c such that $\{c\} \in \mathcal{F}$, its input space is $X_c = O$, the ontology space.*

Definition 4 (Markov learning space) *A Markov antimatroid $(C, \mathcal{F}, O, \{\kappa_c\}_{c\in C})$, is a Markov learning space, if it satisfies* ***commutativity of accessible concepts (CAC)****, ie. whenever $c, c' \in C$ are concepts and $S \in \mathcal{F}$ is a feasible set such that both c, c' are accessible from S, then for any learning path P ending in S and the kernels κ resulting from the path Pcc' and κ' resulting from the path $Pc'c$ are the same.*

Markov antimatroids can be used to generalize stochastic (decision) processes and Interface theory of perception. We omit the proofs due to length restrictions.

Theorem 5 *Let $\mathfrak{S} = \{X_t\}_{t\in\mathbb{N}_0}$ be a stochastic process on a Polish space S, equipped with its Borel σ-algebra $\mathcal{S}$, and suppose the process satisfies the Markov property. Then, there exists a Markov kernel κ learning space $\mathfrak{M}$, such that the Markov kernel associated with $\mathfrak{S}$ is the same as the Markov kernel associated with $\mathfrak{M}$.*

Theorem 6 *Let $\mathfrak{S} = (S, \mathcal{S}, A, \mathcal{A}, \rho, \pi, \kappa)$, be a stochastic decision process. Then, there exists a unique Markov learning space $\mathfrak{M}$ that yields an equivalent stochastic process as $\mathfrak{S}$.*

In a similar manner, we prove that Markov learning spaces generalize also Interface theory of perception and Embedded agents introduced by Ay et al. [1]. In fact, due to a natural kernel associated with the Markov learning space, each of these theories can be interpreted as the previous one by identifying a new concept to be learned. Thus, we do not disprove the CAT, but rather refine it: any conscious process can be modelled through a more or less refined Markov antimatroid, and the learning process of the conscious process alters the concepts-kernels of the cognitive process to a more and more predictable understanding of reality. We leave to future research the quantifications of this learning.

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DETECTING GNSS INTERFERENCE ATTACKS WITH FUZZY LOGIC MODELS

Polona Pavlovčič Prešeren*, Staš Dolinšek, Boštjan Batagelj** and Aljaž Blatnik****
University of Ljubljana
*Faculty of Civil and Geodetic Engineering, Jamova cesta 2, 1000 Ljubljana
** Faculty of Electrical Engineering, Tržaška cesta 25, 1000 Ljubljana
E-mail: polona.pavlovcic-preseren@fgg.uni-lj.si, sd4973@student.uni-lj.si, bostjan.batagelj@fe.uni-lj.si, aljaz.blatnik@fe.uni-lj.si

Abstract: This article presents an approach for detecting deliberate interference in global navigation satellite system (GNSS) measurements, focusing on the effects of jamming and spoofing. Any kind of GNSS interference can threaten the integrity of GNSS, resulting in positioning and time synchronization failures with catastrophic consequences. Jamming is a total loss of signal and spoofing is when the receiver is misled as to its actual understanding of position using an artificial GNSS signals as though they were from satellites. In response, we have proposed a generic fuzzy logic technique to evaluate the quality of the GNSS signal and estimate the extent to which it has been jammed or spoofed. The research developed several indicators of jamming, including carrier-to-noise ratio (C/N_0), position dilution of precision (PDOP), and a simple time-series analysis of position coordinates to indicate when GNSS signals were likely to be jammed or spoofed. The best thing about this fuzzy classification model was the way it treated different indicators as one, as it performed fuzzy classification using many sources of expert knowledge in the form of fuzzy rules in the risk assessment of the detected interference. The simulation results show that fuzzy logic provides a practical, flexible and easily interpretable approach for the development of interference detection methods – especially in stochastic or interference-prone GNSS environments and especially for receivers that do not have spectrum monitoring capabilities.

Keywords: GNSS interference, spoofing, jamming, fuzzy logic, PDOP, signal integrity, positioning and navigation

1 PROBLEM DESCRIPTION

Global Navigation Satellite Systems (GNSS) – including GPS**,** GLONASS**,** Galileo, and BeiDou – underpin the modern capabilities of positioning, navigation, and timing (PNT) applications. Beyond providing geospatial positioning and enabling navigation, GNSS plays a critical role in time synchronization, which is vital for the reliable operation of various infrastructure systems and technologies. This includes communication networks, digital protocols, and the coordination of power generation and distribution within energy systems. As GNSS-based PNT capabilities have expanded – bringing increased efficiency, accuracy, and global reliance – so too have the vulnerabilities associated with their use. Disruptions, signal interference, spoofing, and system dependencies pose growing risks to the stability and security of GNSS reliant services. Specific threats include purposeful introduction of radio frequency interference, (i.e., jamming or spoofing of GNSS signals) which can affect the reliability, integrity or safety of GNSS services and systems.

Jamming involves transmitting radio signals that overpower or disrupt legitimate GNSS signals, causing loss of tracking. Spoofing creates fake GNSS signals to mislead receivers with false position or timing data, making it a more dangerous, often undetected threat to civilian and critical infrastructure. Regrettably, some GNSS receivers lack the function of signal spectrum monitoring. This feature is often absent even in certain geodetic-grade receivers. However, some models – including low-cost ublox ZED-F9P (see Figure 1), do provide limited detection of jamming and spoofing by spectrum monitoring.

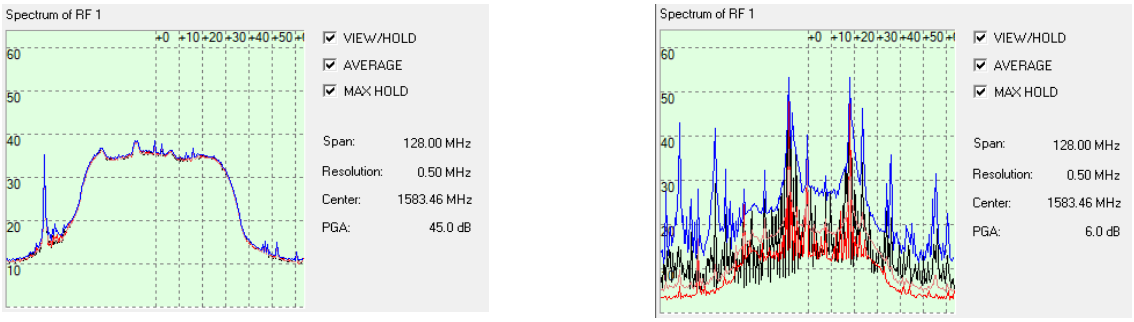


Figure 1: Spectrum of GNSS signals in the L1 band, acquired by a ublox ZED-F9P receiver under: (a) normal (quiet) conditions, and (b) conditions of deliberate interference.

Nevertheless, there are still several indicators that can suggest problematic positioning caused by falsified signals. These include unusually high dilution of precision (DOP) values, low or unstable carrier-to-noise density ratios (C/N_0), and sudden shifts in position that place the receiver far from its actual location, as used in several studies [1–3]. Such anomalies can serve as warning signs even when spectrum monitoring is not available.

Building on earlier studies exploring fuzzy logic in the sphere of GNSS quality assessment, see for example [5–7], this paper introduces a proposed model and conceptual framework for detecting signal interference – in both real-time and post-processed positioning – for receivers that lack spectrum monitoring capabilities. The proposed model will assist field practitioners and promote confidence in GNSS-based positioning and time synchronization when confronted with signal degradation and uncertainty.

Fuzzy logic represents a good application area since it allows for approximated reasoning based in terms of natural language and expertise, as opposed to a binary classification. Given these challenges, this paper will assess fuzzy logic as an adaptive, rule-based reasoning for GNSS interference detection.

1.1 Measurement conditions and signal quality-monitoring

GNSS measurement quality depends on environmental conditions. Monitoring factors like DOP, C/N_0 time series, and spectral analysis reveals signal disturbances such as interference, multipath, or degradation [8–9]. Sudden changes or abnormalities often indicate jamming or spoofing attacks.

Signal spectrum monitoring is essential for confirming interference indicated by other metrics. By analyzing power distribution across frequency bands over time, it helps detect signal issues and enhances GNSS positioning accuracy. Monitoring spectrum anomalies supports measurement quality checks within integrated GNSS frameworks, especially under dynamic conditions like satellite visibility changes, environmental factors, or receiver motion. Advanced spectral analysis is crucial for reliable quality assessment and early detection of threats to GNSS performance.

2 PROPOSED FUZZY-LOGIC SOLUTION

The fuzzy logic method for detecting GNSS signal anomalies involves four main steps: (1) defining the output (positioning accuracy) and inputs (PDOP and C/N_0), (2) setting membership functions, (3) creating the rule base, and (4) simulating the system to generate weighted ratings under various conditions. Expert knowledge on GNSS positioning quality was encoded through fuzzy rules, following these guidelines:

- If PDOP is high (above 3), the solution is considered poor and given a low weight (first membership function); if PDOP is moderate with acceptable coordinate accuracy, the solution is moderately reliable and assigned a medium weight (second membership function).
- Additionally, if the C/N_0 is low, the signal quality is considered poor → a low weight is assigned. Conversely, if C/N_0 is high, indicating strong and stable signal reception, the solution is of good quality → a high weight is assigned.
- If coordinate accuracy is high – in our case, by using carrier-phase double difference positioning, for 3D kinematic positioning – the solution is of good quality → high weight (second membership function).

Table 1: Results of the fuzzy logic system for GNSS signal quality assessment based on PDOP, average C/N_0, and positioning accuracy. The output includes the assigned weight and linguistic classification.

Trial No.	***PDOP***	***Avg. C/N0 [dB-Hz]***	***Accuracy [m]***	***Weight***	***Fuzzy Output (linguistic)***	***Interference Risk (linguistic)***
1	8.5	62	120.0	0.1	Very low	Very High
2	7.1	22	0.03	0.1	Very low	Very High
3	2.2	41	0.06	0.8	Medium-high	Low
4	5.0	28	0.12	0.3	Low	High
5	6.2	25	0.35	0.2	Very low	Very High
6	4.5	30	0.90	0.4	Low	High
7	3.2	35	0.65	0.6	Medium	Moderate
8	2.5	40	0.05	0.7	Medium-High	Low
9	1.8	44	1.20	0.9	High	Low
10	9.3	65	150.0	0.1	Very low	Very High
11	***2.0***	***58***	***0.10***	***0.8***	***High***	***Low → HIGH!***
12	2.3	20	0.04	0.7	Medium-High	Low

The results in Table 1 demonstrate that the fuzzy logic system generally interprets combinations of signal quality indicators effectively to identify potential interference scenarios, although it does not yet capture all cases perfectly. A notable exception is Trial 11, which highlights a significant challenge for the model and underscores the need for improvement, which has since been addressed.

For example, Trials 1 and 10 illustrate situations where PDOP values are high and C/N_0 values are unusually high, yet the positioning accuracy is poor. These cases were correctly assigned low weights and classified with a "Very High" interference risk, showcasing the fuzzy logic system's ability to detect suspicious or degraded positioning quality – even when some individual indicators, such as high C/N_0, might appear favourable if considered in isolation.

The fuzzy logic system in its initial form failed to flag certain spoofing cases as high interference risk (see Trial 11) because it lacked specific rules or thresholds to detect unrealistically high C/N_0 values – an important indicator of spoofing attacks. Recognizing this limitation, we enhanced the model by introducing an explicit constraint that caps the maximum allowable C/N_0 at 54 dB-Hz, which reflects the practical upper limit achievable by typical GNSS receivers under ideal conditions. By enforcing this threshold, the system can better differentiate genuine signals from spoofed ones that attempt to mimic strong, reliable satellite signals with artificially inflated C/N_0 values.

3 CONCLUSIONS

This study shows that fuzzy logic enables practical risk assessment using multi-parameter GNSS data, detecting subtle anomalies missed by traditional methods. When combined with

spectrum monitoring, it enhances GNSS integrity by cross-validating signal and positioning inconsistencies.

Regular updates to the fuzzy rule base – based on new threats, research, and field data – are vital for resilience against evolving interference. Collaboration across sectors and real-world testing on various receivers further strengthen the model's reliability.

It should be noted that the model is not intended to characterize the particular type of interference, but rather to identify disturbances that impact GNSS signals. While specific categorization and mitigation techniques are left to auxiliary systems or additional research, this focus allows for the prompt identification and warning of any signal problems.

Acknowledgement

This research was carried out within the framework of the research project Care4SIGNAL supported by the Slovenian Research and Innovation Agency (ARIS) under grant number V2-2342: Risk assessment of GNSS interference in the Slovenian continuously operating reference station network SIGNAL.

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BAT INKUBATOR: A CASE STUDY OF APPLYING THE MATURITY MODEL FOR CLIMATE NEUTRALITY AND BUSINESS PROCESS OPTIMIZATION

Klemen Tršinar[1], Melani Potrč[1], Špela Kajzer[1], Špela Korpič[4], Simon Ravnič[4], Anja Šketa[1,2,3], Drago Bokal[1,2,3]

klemen.trsinar@student.um.si, melani.potrc1@um.si, spela.kajzer@student.um.si, spela.korpic@panvita.si, simon.ravnic@panvita.si, anja.sketa@databitlab.eu, drago.bokal@um.si

[1] University of Maribor, Faculty of Natural Sciences and Mathematics, Koroška cesta 160, Maribor, Slovenia

[2] DataBitLab d.o.o., Perhavčeva ulica 13, Maribor, Slovenia

[3] Institute of Mathematics, Physics and Mechanics, Jadranska ulica 19, Ljubljana, Slovenia

[4] Panvita d.o.o., Lendavska ulica 5, Rakičan, p.p. 106, Murska Sobota, Slovenia

Abstract: The Climate Neutrality Maturity Model presented at SOR 2023 has been extended to the BAT Incubator process of using companies "internal data and processes" information systems to enrich the research, development and innovation processes of a large enterprise. In our contribution, we present a summary of the process and a case study of applying the model to food production at Panvit. Specific focus is on identifying Best Available Techniques (BAT) opportunities, where the company can offer demonstration opportunities for state-of-the-art research in improving its environmental, social, and governance performance indicators. We put these improvements into the context of a company's ranking by the Climate Neutrality Maturity Model and conclude with proposals for in-depth research on the topics.

Keywords: Climate Neutrality, Maturity Models, Business Process Optimisation, Environmental, Social, and Governance standards, Agro-Food industry.

1 INTRODUCTION

Industrial activities play a crucial role in the economy at the national, European, and global levels [12]. They contribute to job creation, economic growth and the improvement of living standards for millions of people [11]. However, they are also a major source of environmental pollution, affecting air, water and soil quality, with potentially serious consequences for human health, animals and ecosystems [4].

In response, companies are increasingly required to align sustainability goals with business efficiency. One innovative approach to addressing this challenge involves a systematic review of Best Available Techniques (BAT) and BAT Reference Documents (BREF). BAT refers to the most effective and advanced methods for reducing emissions and minimising the environmental impacts of industrial activities. The BREFs, developed by the European Commission through the BREF (or Seville) Process, are dynamic documents that are regularly updated to reflect new techniques and an expanding range of environmental considerations [1,9].

In this case study, conducted at the prominent Slovenian agri-food company Panvita, we used the Incubator framework [2] to analyse the organisation's maturity in climate neutrality, business process optimisation, digitalization, and the implementation of BAT. The findings underscore the importance of a comprehensive approach that integrates both technological and organisational changes, informed by European environmental policies and legislation.

2 METHODOLOGY

2.1 Presentation of the Participating Company

The Panvita, headquartered in Murska Sobota, is one of the leading agri-food companies in northeastern Slovenia, operating across the Pomurje region and beyond. It comprises nine

interconnected companies that span all stages of the agri-food chain, from farm to market. The Group is committed to sustainable and environmentally responsible food production, following the highest environmental standards and objectives. The principles of the circular economy and responsible environmental management guide Panvita's operations. With its comprehensive sustainability strategy, Panvita serves as a regional leader and role model, making a significant contribution to food security and rural development in Slovenia [13].

2.2 Structured Interview

A structured interview with a Panvita Management Board member was conducted in April 2025, based on the maturity model for climate neutrality and business process optimisation [10]. The interview also included additional questions concerning the knowledge and implementation of BAT and BREFs relevant to the company's operational field. Based on the responses, the company's maturity level was determined using the aforementioned maturity model. Simultaneously, key BAT parameters were analysed to identify strengths and areas needing support to meet climate targets. Additionally, lists of researchers, institutions, and academic programs capable of supporting BAT development and implementation were compiled through the BAT Incubator.

3 RESULTS

As part of this research, Panvita was compared against 35 anonymous companies included in the study by Potrč et al. (2023). Maturity was assessed across five areas: Strategies, Digitalization, Environment, Data Analysis, and LCA Analysis. Each company was evaluated on a five-level scale from Beginner, Follower, Expert, Leader, to Influencer [10]. Figure 1 shows the distribution of maturity levels for all areas and all the companies that participated in the interview. The results show that Panvita ranks above average in all five domains, achieving the Leader level overall. Notably, it reaches the Influencer level in LCA analysis and climate neutrality and sustainability.

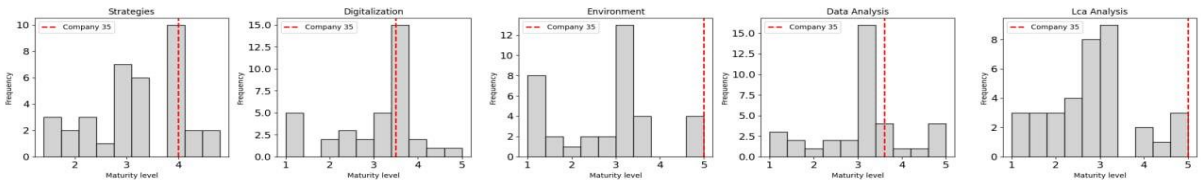


Figure 1: Maturity levels across all observed categories compared to the Panvita (red dashed line).

Panvita exhibits a high level of awareness and implementation of BAT, as evidenced by its active pursuit of new technologies and continuous investment in research and development aimed at achieving advanced environmental goals. The effectiveness of BAT is regularly monitored and reviewed every three to five years. The highest level of BAT awareness is particularly evident in the use of various air purification filters, anaerobic waste digestion, and the utilisation of decomposition products as biofuels. It is also reflected in the adoption of regulated feed compositions and nutritional strategies, which include techniques such as reducing crude protein content, multi-phase feeding, and the controlled addition of essential amino acids. However, areas like bioremediation remain less developed.

4 DISCUSSION AND OPPORTUNITIES

BAT and BREF could form the basis for sustainable and environmentally responsible operations in all activities of the Panvita. BAT includes the most effective and advanced technologies that are economically and technically feasible, while BREF provides reference data for their implementation. As seen in Figure 1, most opportunities for improvement can be found in areas of digitization and data analysis. By integrating the Universal Process Model [3] into a digitalization strategy, companies can harness process-generated data more effectively.

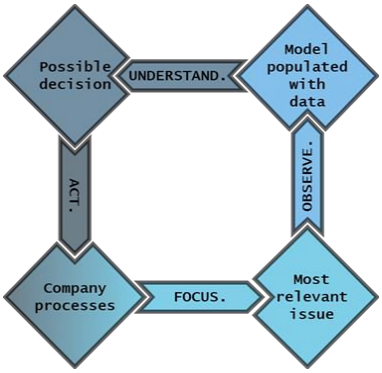


Figure 2: Universal process model. Based on Bokal and Fic (2019).

For each targeted business process as seen on Figure 2, attention is directed toward its most critical issues. Through careful observation, a digital twin of the process can be created and enriched with relevant data. Once the digital twin is fully understood, it becomes possible to define a range of decision options and simulate their impact. By analysing how these decisions influence the digital twin, companies gain insight into their real-world consequences. This approach enables informed decision-making, allowing the implementation of strategies that yield the greatest positive impact on business performance. All of the data analysis needed in applying the universal process model can be done with the comprehensive framework [5] for the development and deployment of analytical processes in response to business demands. It guides the company through a clearly defined lifecycle, from initial request and conceptualisation to prototyping, validation, demonstration, and finally, routine operational use. The framework reduces risk and accelerates implementation by incorporating early-stage prototyping, validating edge conditions, and supporting iterative refinement. It also fosters evidence-based decision-making through the creation of digital twins, enabling the simulation of various decision outcomes and thorough evaluation of their impact. As a result, only analyses that demonstrate measurable value are deployed at scale and replicated across the organisation, leading to more consistent and impactful use of data-driven insights. In essence, the framework empowers the company to systematically unlock value from data, increase operational efficiency, and make well-informed, high-impact decisions based on proven analytical results.

Preliminary analysis confirms that Panvita has integrated various Best Available Techniques (BAT) into its operational processes. Within the BAT Incubator framework, energy efficiency was selected as one of the key parameters for achieving climate neutrality. In this context, two research groups were identified as relevant contributors: the Institute of Energy Technology at the Faculty of Energy Technology, University of Maribor, and the Laboratory for Power Engineering at the Institute of Electrical Power Engineering. Potential collaborators include the Energy Efficiency Centre at the “Jožef Stefan” Institute, the Innovation and Development Institute of the University of Ljubljana, and Pomurje Technology Park could also be considered. Significant academic contributions in this field are found in the works of Košnjek

et al. (2024), Hrovatin et al. (2016), and Kim et al. (2024), offering practical insights for improving energy efficiency indicators.

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The 18th International Symposium on
Operational Research in Slovenia

SOR ’25

Bled, SLOVENIA
September 24 - 26, 2025

Special Session 7:

Research and Application of MCDM Methods and Decision Systems in Cybersecurity

SYSTEMATIZATION AND APPLICATION OF MCDM METHODS IN CYBERSECURITY

Andrej Bregar
Informatika d.o.o., Vetrinjska ul. 2, 2000 Maribor, Slovenia
E-mail: andrej.bregar@informatika,si

Abstract: In the paper, we review and systematize the application of multi-criteria decision-making methods in cybersecurity. We first identify potential cybersecurity areas where we can effectively use these methods, make a mapping between suitable methods and problems, and propose a taxonomy of approaches. We provide some recommendations, good practices, and scenarios. We demonstrate multi-criteria decision-making on a use case from the electricity domain by applying it in the time-dependent mitigation selection process. The study utilizes our methodology, which facilitates stakeholders from different organizational levels in selecting remediation measures and strategies against cyberattacks. It shows the benefits of using multi-criteria decision-making methods in cybersecurity.

Keywords: multi-criteria decision-making, decision support systems, cybersecurity, case studies

1 INTRODUCTION

Our society is becoming increasingly exposed to cyberattacks due to the geopolitical situation and a strong reliance on information and operational technology (IT/OT). Organizations must select and implement efficient mitigation actions, encourage collaboration between different organizational levels and stakeholders, and facilitate the exchange of Cyber Threat Intelligence (CTI). Current trends prioritize cyber risk management through systematic decision-making, continuous management of threats and vulnerabilities targeted at achieving high availability of corporate assets, and situational awareness that provides necessary information to make well-informed decisions [7]. Therefore, cybersecurity technology should be supported with decision modules and AI/ML to optimize and ease incident detection, analysis, and response.

Multi-criteria decision-making (MCDM) methods and decision technologies are gaining relevance in cybersecurity to cope with the above challenges. They have been traditionally applied for risk and vulnerability assessment [4, 5, 6]. However, they also have potential for various other tasks and problems [1, 2], including incident detection and severity assessment, prioritization of incident response procedures and actions, selection of cybersecurity controls and mitigation measures, cost-benefit analysis, CTI analysis, infrastructure and application security analysis, stochastic analysis of time-dependent threats, prioritization of cybersecurity strategies and policies, capability maturity assessment, regulatory compliance assessment, etc.

This paper reviews, systematizes, and applies MCDM methods for cybersecurity problems. Section 2 proposes a taxonomy and systematization of available methods. Section 3 describes some applications of MCDM methods. It presents a case study, showcasing the use of MCDM methods in cybersecurity. It also introduces a centralized decision support system that enhances the efficiency of cybersecurity technology. Finally, Section 4 concludes the paper.

2 TAXONOMY AND SYSTEMATIZATION

Based on the literature review [1, 2, 4, 5, 6, 8], a variety of MCDM methods can be used for cybersecurity purposes, such as AHP, TOPSIS, ELECTRE, PROMETHEE, additive value and utility models, fuzzy and stochastic models, time series, Bayes and Markov analyses, attack trees, regression methods, risk and threat simulations, qualitative models, CVSS, Delphi, etc. Their use depends on the properties of the decision process and the cybersecurity problem to be solved. Figure 1 defines the taxonomy of MCDM methods for cybersecurity.

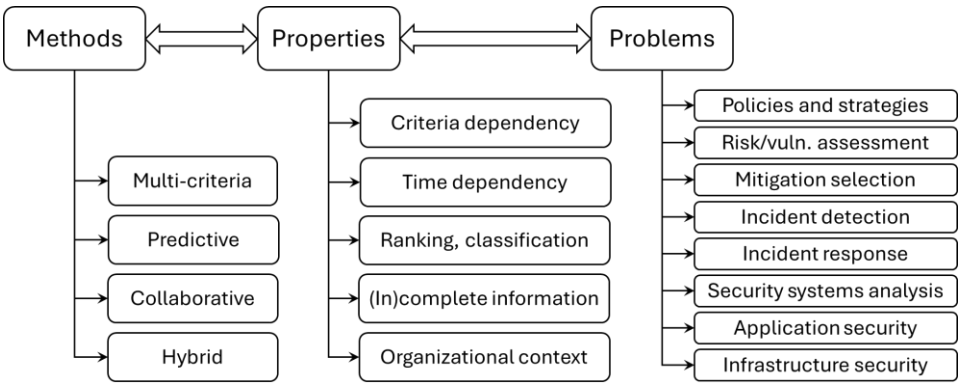


Figure 1. Taxonomy of MCMD methods for cybersecurity.

We use different multi-criteria, predictive, collaborative, or hybrid methods based on potential criteria inter-dependencies, time dependencies of threat or risk severity, the completeness of decision-making information, the need to make decisions on different organizational levels or from the perspective of several stakeholders, and the decision-making problematics (ranking, sorting, classification, or choice). Table 1 introduces a systematization of MCDM methods in the scope of cybersecurity. It recommends the most suitable use of methods according to five key characteristics defined by the taxonomy. Due to length restrictions, we omit references to methods. These methods can address various cybersecurity problems as specified in Figure 2.

Table 1. Systematization of MCDM methods for cybersecurity.

Class of methods	***Methods***	***Recommended use***
Ratios – hierarchical methods	AHP, MACBETH	Independent criteria, ranking, broad use
Ratios – referential point	SMART, SWING	Independent criteria, prioritization, risk
Network methods	ANP, attack trees	Dependent criteria, incomplete/uncertain information, ranking, prediction
Utility/value function	MAUT, VIKOR, TOPSIS, CVSS	Independence, all problematics, broad use
Pairwise comparisons	ELECTRE, PROMETHEE	Incomplete information, all problematics
Fuzzy preferences	OWA, Choquet, DEMATEL	Uncertain information, advanced use
Stochastic analysis	Bayes/Markov analysis, SMMA	Missing data, time-dependent, prediction
Ordinal/qualitative methods	DEX, ZAPROS, CVSS	Classification, risk assessment, rules
Interactive and visual methods	Pareto and sensitivity analysis	Problem structuring, robustness
Regression-based methods	UTA, UTADIS, ELECTRE, ML	Inference of models and decisions
Group/collaborative methods	Delphi, consensus, distance	Consolidation, structuring, organizations

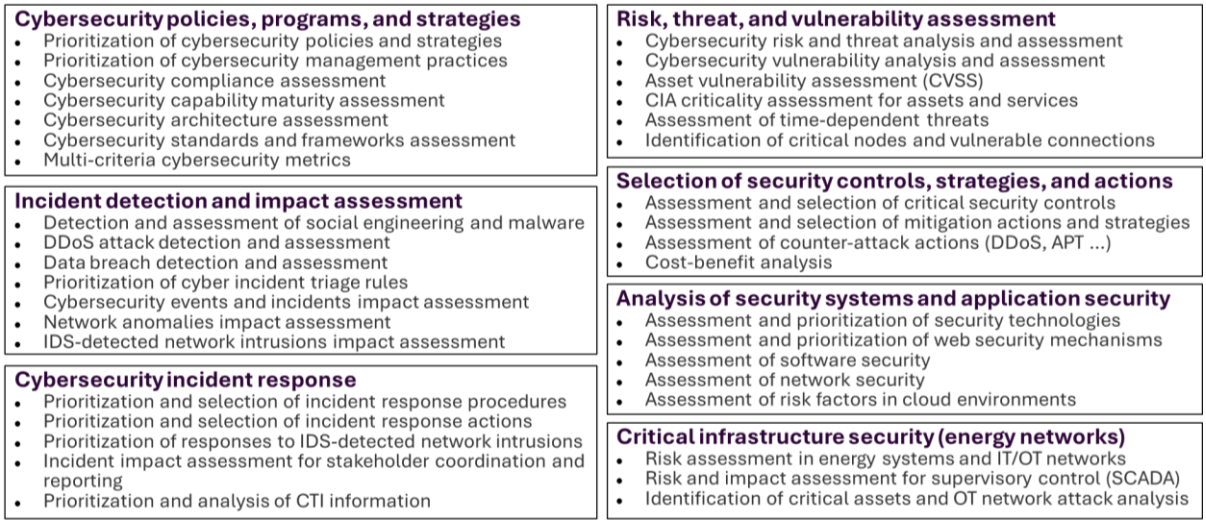


Figure 2. Systematization of cybersecurity problems solved by MCDM methods.

According to the literature, AHP is the most common MCDM method in cyber and information security [5]. It can be used for risk assessment, capability maturity assessment, compliance assessment, selection of critical security controls, prioritization of policies and defense actions, CIA (Confidentiality, Integrity, Availability) analysis, IT/OT and network security assessment, ranking of incidents and alarms, etc. Similarly, other methods have their specific purposes.

In cybersecurity, it is a common practice to combine quantitative and qualitative scores. Hence, CVSS (Common Vulnerability Scoring System) maps numerical evaluations to ordinal classes. Our cyber-attack mitigation methodology also adopts this uniform scoring system [3].

Several standard criteria are usually applied in cybersecurity decision models. They include technical, financial, social, and environmental impacts, RTO (Recovery Time Objective), data loss, system downtime, attack severity, required expertise to respond, etc. These criteria must be aligned with the established standards, frameworks, and recommendations. For example, our cyber-attack mitigation methodology utilizes criteria based on ISO 15408, ISO 27001, NIST 800-39, and NESCOR standards [3].

3 APPLICATIONS OF MCDM METHODS

3.1 Application of the group cyber-attack mitigation methodology

The group decision-making process for cyber-attack mitigation is presented in Figure 3. Based on the uniform scoring system, it aligns the quantitative and qualitative approaches. The basic preference and aggregation model applies the numerical value function (MAVT), but it can be transparently substituted with a qualitative method, such as DEX. An iterative Delphi process allows for group decision-making that involves experts from different organizational levels. It is independent of the preference format, supporting quantitative and qualitative statistics. The group decision-making process includes the intelligence, design, choice, and implementation phases. It incorporates the incident impact assessment and mitigation selection models.

In (3), the numerical MAVT-based approach combines the compensatory weighted additive aggregation in (1) with the non-compensatory veto function in (2) to prevent the selection of inefficient countermeasures related to the impacts of mitigated cyber incidents. Key formulas are provided below. The interested reader should refer to our previous paper [3] for details. Some demonstrative results of the applied methodology are summarized in Table 2, showing the convergence of Delphi statistics from (4) for the selected mitigation strategy MS_2.

$$s^I(A_l) = \sum_{j=1}^{n} w_j s_j^I(A_l) \tag{1}$$

$$v^E(M_k) = \gamma \frac{s^E(M_k) - \text{MIAT} \cdot s^I(A_l)}{(1-\text{MIAT}) s^I(A_l)} + (1-\gamma) \frac{s^E(M_k) - (C_{max} - C_\delta)}{C_\delta} \tag{2}$$

$$S^E(M_k) = s^E(M_k) + v^E(M_k)\left(10 - s^E(M_k)\right) \tag{3}$$

$$\forall\left(CSE_i, A_j\right): \left(s_{\min}^I, s_{\text{avg}}^I, s_{\max}^I\right) \text{ for } \{DM_1, \dots, DM_m\} \tag{4}$$

Table 2. The convergence of Delphi statistics for the selected mitigation strategy.

	MS_2 – 1st round			***MS_2 – 2nd round***		
	Min	***Average***	***Max***	***Min***	***Average***	***Max***
Information system	3.87	5.51	7.32	3.26	4.69	6.33
Business process	1.34	4.00	6.26	1.34	3.17	5.08
Overall quantitative score	2.60	4.76	6.79	2.30	3.93	5.70
Overall qualitative score	Low	Medium	Medium	Low	Low	Medium

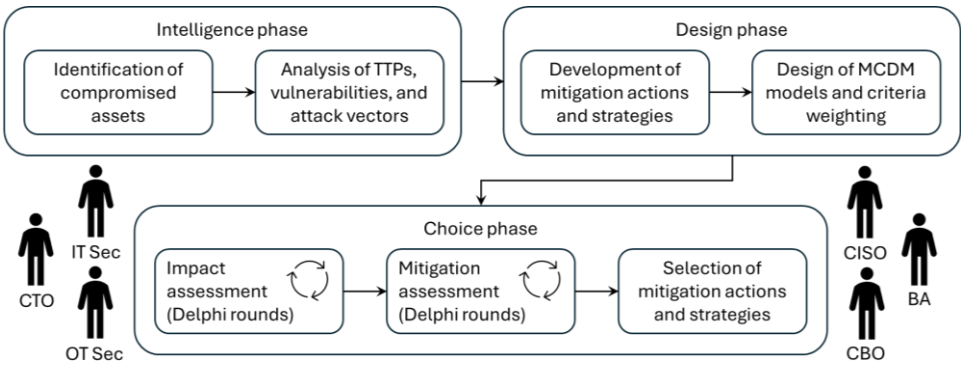


Figure 3. Group decision-making process for cyber-attack mitigation.

3.2 Centralized decision support system

MCDM methods can be used directly to solve cybersecurity problems identified in Section 2. However, they can also be implemented into a decision system utilized in a comprehensive cybersecurity ecosystem or platform. In Figure 4, we propose a model in which the decision support system is a central block integrating the traditional cybersecurity components.

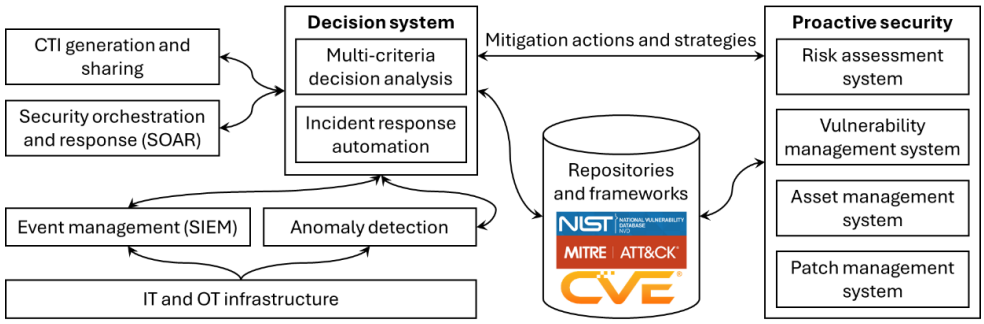


Figure 4. Centralized decision support system for an enhanced cybersecurity ecosystem.

4 CONCLUSIONS

In this paper, we systematized the use of MCDM methods in cybersecurity. We aimed to show why these methods are important. Cybersecurity threats increase drastically if we do not make timely decisions and act. According to our studies, more than 90% of assets become vulnerable.

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EXTENDED SUSTAINABLE DEVELOPMENT LEVEL MEASURING – CASE OF POLAND

Helena Gaspars-Wieloch
Poznan University of Economics and Business/
Department of Operations Research and Mathematical Economy
Al. Niepodległości 10, 61-875 Poznan, Poland
E-mail: helena.gaspars@ue.poznan.pl

Kacper Zielak
Poznan University of Economics and Business/
Department of Operations Research and Mathematical Economy
Al. Niepodległości 10, 61-875 Poznan, Poland
E-mail: kacper.zielak@ue.poznan.pl

Abstract: Nowadays, lots of countries tend to maintain sustainable development since it's very beneficial in terms of numerous aspects. Nevertheless this goal is quite difficult to reach, among other things, because of social inequalities. The objective of this research is to analyse the aforementioned phenomenon in Poland and to generate a ranking of Polish voivodships. When assessing sustainable development three orders (areas) are usually taken into account, but in this research a fourth dimension is considered. The most frequent way of sustainable development level measuring consists on computing a synthetic indicator covering diverse factors. Nevertheless, in this paper we use an extended hybrid approach, i.e. we apply SAW procedure (to take different criteria into consideration and create an aggregated measure) and the modified max-min method (to assign better ranking positions to voivodships with relatively high minimal objective values). The choice of such a methodology results from the essence of sustainable development. The research shows the strengths and weaknesses of particular voivodships in Poland.

Keywords: sustainable development, Polish voivodships, SAW, modified max-min rule, ranking.

1 INTRODUCTION

Sustainable development constitutes a goal of numerous countries, but this target is difficult to achieve due to diverse factors. Resource limitations, social disparities among regions as well as political and economic instability are examples of phenomena which usually disrupt sustainable development.

Within the following research we would like to concentrate on the case of Poland. The objective of the paper is to analyse sustainable development in Poland and to create a ranking of Polish voivodships. Voivodships are administrative divisions equivalent to cantons in Switzerland or departments in France. Poland is divided into 16 voivodships.

The social order, the economic order and the environmental order are commonly recognized in the sustainable development evaluation process [1], [2], [5], [8], but in this investigation we tend to take an additional order into account, i.e. the institutional-political dimension. Such an approach allows us to complement the dimensions aforementioned and to obtain a broader perspective on the phenomenon discussed. The relation between the three orders already used and the fourth dimension suggested is very visible. For example, if the administration is well-trained, uncorrupted and supported by non-profit organizations, decisions concerning the environmental protection, the economic indicators and the situation of the society can be more rational.

Usually the assessment of sustainable development is associated with some aggregated measures [6], [7]. In our investigation we partially follow this approach, i.e. we apply SAW (simple additive weighing method) as the basic procedure, but due to the specificity of the analysed phenomenon we enrich the study by combining it with the modified max-min rule.

On the one hand, sustainable development requires the comparison of numerous criteria and the creation of one measure covering all these factors (and that is why SAW is used), but on the other hand the notion "sustainable development" implies the necessity to evenly develop all its areas in a given region (and therefore the modified max-min rule can constitutes a valuable supportive tool).

2 METHODOLOGY

The assumptions made in the study are as follows:

1. The research concerns 16 Polish viovodships (D – Dolnośląskie, KP – Kujawsko-Pomorskie, LE – Lubelskie, LU – Lubuskie, Ł – Łódzkie, MP – Małopolskie, MZ – Mazowieckie, O – Opolskie, PK – Podkarpackie, PL – Podlaskie, PM – Pomorskie, ŚL – Śląskie, ŚW – Świętokrzyskie, WM – Warmińsko-Mazurskie, WL – Wielkopolskie, Z – Zachodniopomorskie).
2. The investigation is based on data from 2022.
3. The analysis involves 4 dimensions (social, economic, environmental and institutional-political). Each dimension is represented by 5 factors, which means that 20 variables are used in the whole study.
4. There are diverse methods enabling the unification of criteria expressed by different units and scales, but in this research the following unitarization procedure is applied (equations (1), (2), (3) and (4) are for stimulants – S, destimulants – D, point nominants – PN, interval nominants – IN, respectively):

$$z(\mathrm{S})_{ik} = \frac{x_{ik} - x_k^{min}}{x_k^{max} - x_k^{min}} \tag{1}$$

$$z(D)_{ik} = \frac{x_k^{max} - x_{ik}}{x_k^{max} - x_k^{min}} \tag{2}$$

$$z(PN)_{ik} = \begin{Bmatrix} \frac{x_k^{max} - x_{ik}}{x_k^{max} - d_k} & if\ x_{ik} > d_k \\ 1, & if\ x_{ik} = d_k \\ \frac{x_{ik} - x_k^{min}}{d_k - x_k^{min}} & if\ x_{ik} < d_k \end{Bmatrix} \tag{3}$$

$$z(IN)_{ik} = \begin{Bmatrix} \frac{x_k^{max} - x_{ik}}{x_k^{max} - d_k^{max}} & if\ x_{ik} > d_k^{max} \\ 1, & if\ x_{ik} \in [d_k^{min}, d_k^{max}] \\ \frac{x_{ik} - x_k^{min}}{d_k^{min} - x_k^{min}} & if\ x_{ik} < d_k^{min} \end{Bmatrix} \tag{4}$$

where x_k^{max} (x_k^{min}) denotes the maximum (minimum) value of the k-th criterion and z_{ik} means the unitarized value of the k-th variable for the i-th object (viovodship). Symbol d_k (d_k^{min}, d_k^{max}) represent the desired (min., max.) value of criterion *k*.

5. The factors are selected from the report prepared by the Statistical Office in Katowice [9]. This document contains a huge number of criteria, but not all of them are suitable for regional level analysis where comparable and complete data across voivodships are required.
6. The social dimension includes: X_1(D) – at-risk-of poverty rate after considering social transfers in income (%), X_2(D) – road traffic fatalities per 100 000 population (persons), X_3(S) – number of housing units put into use per 1000 population aged 24-

35 (units), X_4(S) – share of adults participating in education or training aged 24-65 (%), X_5(S) – number of doctors (personnel working in total) per 10 000 population (persons). The economic dimension is represented by: X_6(S) – labor productivity in the industrial sector (thousand PLN per employee), X_7(S) – voivodship budget revenues per capita (PLN), X_8(S) – share of railway lines adapted for speed of 120 km/h and above in the total length of operational railway lines (%), X_9(D) – registered unemployment rate (%), X_{10}(S) – investment expenditures per capita (PLN). The environmental order includes: X_{11}(S) – share of renewable energy in total electricity production (%), X_{12}(S) – forest cover (%), X_{13}(D) – annual water consumption per capita (m^3), X_{14}(D) – municipal waste generated per capita (kg), X_{15}(S) – share of legally protected areas in the total area (%). And the institutional-political area is represented by: X_{16}(S) – share of public administration units providing training for employees in telecommunications and information technology (%), X_{17}(S) – number of active non-profit organisations per 10 000 population (units), X_{18}(D) – number of corruption crimes per 100 000 population (units), X_{19}(IN) – number of public administration employees per 10 000 population (persons), X_{20}(PN) – share of women in the legislative bodies of local government units (%). Symbols S, D, PN and IN are used to show the character of the variables.

7. The ranking is created on the basis of a hybrid approach combining SAW with the modified max-min rule. When applying SAW equal weighs are used for each dimension and for each factor within a given dimension. Such a choice (i.e. weigh equal to 1/20 for each criterion) results from the nature of sustainable development. The SAW measure constitutes the sum of 20 products involving particular unitarized criteria values and their weighs. The original ranking is generated according to the descending SAW values. The essence of the modified max-min rule for multi-criteria decision analysis has been described in detail in [3], [4]. In this study we do not use this approach as a separate method, but we apply this technique within each set of voivodships with a similar level of SAW measure.

3 RESULTS AND CONCLUSIONS

Table 1 represents unitarized values (we assume that [103.5; 117.5] is the desired interval for variable X_{19} and 50 is the desired level for X_{20}), the SAW measure, the initial rank of each voivodship, the minimal unitarized value, the second-worst unitarized value and the revised ranks of each object. There are lots of voivodships with quite similar SAW indices. Therefore, before generating the final ranking, it's recommended to divide the objects into smaller sets according to the decreasing SAW values (e.g. set 1: LU, MZ, PM; set 2: D, KP, MP, WM, WL, Z; set 3: Ł, PK; set 4: LE, O; set 5: PL, ŚL, ŚW) and to refer to the modified max-min rule by checking the 1st and then the 2nd-worst unitarized value within particular groups. Such an approach enables revising the ranking by assigning higher ranks to regions with relatively higher first and second-worst unitarized values. The regions aforementioned should be rewarded since they better tend to maintain a decent level for all criteria. Symbol ↑ (↓) means that the original position in the ranking is raised (lowered) by one level.

This paper briefly presents the whole study, but we look for an opportunity to describe the research in details in another publication. The supportive tool applied in the investigation (i.e. the modified max-min rule) is only one of the possible ways to reward voivodships trying to maintain a suitable level for each factor. The approach used in the article could be replaced with the standard deviation or the range of values, however those techniques are not so efficient since they may easier assign a relatively high rank to objects with zero unitarized values, which can be considered as contrary to the idea of sustainable development.

Table 1: Unitarized values, SAW measure, initial and revised ranks, first and second-worst unitarized values

Voiv.	*D*	*KP*	*LE*	*LU*	*Ł*	*MP*	*MZ*	*O*	*PK*	*PL*	*PM*	*ŚL*	*ŚW*	*WM*	*WL*	*Z*
X_1	0,92	0,37	0,05	0,65	0,60	0,63	0,85	0,72	0,38	0,00	0,62	1,00	0,55	0,18	0,58	0,63
X_2	0,54	0,46	0,23	0,13	0,00	1,00	0,44	0,03	0,69	0,23	0,69	0,85	0,31	0,21	0,21	0,59
X_3	0,85	0,49	0,36	0,60	0,51	0,60	0,87	0,00	0,36	0,45	1,00	0,21	0,08	0,49	0,76	0,64
X_4	0,53	0,71	0,30	0,99	0,97	0,29	0,00	0,87	0,85	0,49	0,77	0,21	0,68	1,00	0,85	0,82
X_5	0,66	0,33	0,76	0,00	0,92	0,61	1,00	0,10	0,25	0,76	0,55	0,85	0,43	0,08	0,28	0,38
X_6	0,43	0,15	0,10	0,22	0,18	0,20	1,00	0,17	0,03	0,22	0,31	0,28	0,08	0,00	0,20	0,17
X_7	0,28	0,90	0,51	0,64	0,23	0,42	1,00	0,69	0,73	0,43	0,20	0,00	0,42	0,53	0,35	0,94
X_8	0,56	0,67	0,79	0,48	0,79	0,49	1,00	0,35	0,33	0,00	0,55	0,34	0,46	0,45	0,92	0,25
X_9	0,73	0,25	0,14	0,75	0,56	0,75	0,76	0,49	0,00	0,31	0,71	0,86	0,17	0,03	1,00	0,36
X_{10}	0,61	0,22	0,04	0,26	0,25	0,31	1,00	0,30	0,08	0,41	0,34	0,40	0,00	0,15	0,46	0,61
X_{11}	0,05	0,54	0,51	0,33	0,01	0,17	0,02	0,00	0,49	0,83	0,65	0,04	0,17	1,00	0,52	0,63
X_{12}	0,30	0,08	0,08	1,00	0,00	0,26	0,07	0,19	0,60	0,35	0,54	0,39	0,25	0,37	0,16	0,51
X_{13}	0,95	0,96	0,94	1,00	0,96	0,96	0,57	0,95	0,98	1,00	0,99	0,99	0,00	1,00	0,74	0,54
X_{14}	0,00	0,36	0,93	0,13	0,37	0,47	0,36	0,22	1,00	0,70	0,22	0,16	0,82	0,57	0,23	0,06
X_{15}	0,00	0,29	0,09	0,41	0,02	0,75	0,24	0,19	0,57	0,28	0,31	0,08	1,00	0,61	0,24	0,07
X_{16}	0,39	0,69	0,19	0,45	0,35	0,39	0,29	1,00	0,29	0,00	0,53	0,73	0,21	0,34	0,62	0,35
X_{17}	0,44	0,46	1,00	0,61	0,66	0,77	0,84	0,67	0,84	0,55	0,39	0,00	0,87	0,67	0,82	0,45
X_{18}	0,87	1,00	0,84	0,93	0,71	0,95	0,78	0,47	0,36	0,18	0,94	0,00	0,67	0,93	0,92	0,70
X_{19}	0,59	0,93	1,00	1,00	1,00	0,00	0,00	1,00	1,00	0,99	0,48	0,23	1,00	1,00	0,15	1,00
X_{20}	0,41	0,33	0,25	0,38	0,35	0,18	0,37	0,32	0,00	0,17	0,32	0,33	0,27	0,46	0,31	0,54
SAW	0,51	0,51	0,45	0,55	0,47	0,51	0,57	0,44	0,49	0,42	0,56	0,40	0,42	0,50	0,52	0,51
In.rank	8	6	12	3	11	7	1	13	10	15	2	16	14	9	4	5
Min(1)	0,00	0,08	0,04	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,20	0,00	0,00	0,00	0,15	0,06
Min(2)	0,00	0,15	0,05	0,13	0,00	0,17	0,00	0,00	0,00	0,00	0,22	0,00	0,00	0,03	0,16	0,07
Revisions	↓	↑		↑			↓↓				↑			↑		↓

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CONSUMER SENSITIVITY TO CREDIT CARD MISUSE AND DEBT ACCUMULATION: INSIGHTS FROM THE CROATIAN MARKET

Mario Pepur, PhD
University of Split, Faculty of Economics, Business and Tourism
Cvite Fiskovica 5, 21000 Split, Croatia
E-mail: mpepur@efst.hr

Goran Dedic, PhD
University of Split, Faculty of Economics, Business and Tourism
Cvite Fiskovica 5, 21000 Split, Croatia
E-mail: mpepur@efst.hr

Lucija Skopljanac Macina
E-mail: lpleti01@live.efst.hr

Abstract: This study examines credit card misuse among young Croatian consumers, focusing on financial literacy and the psychological "credit card effect." Using the SCCMI scale, data from 140 credit card holders aged 21–35 were analyzed. Exploratory factor analysis confirmed a one-factor structure (48.85% variance, $\alpha = 0.733$) reflecting irrational credit card use. Financial dependence was significantly linked to higher susceptibility scores. The findings validate the SCCMI scale in the Croatian context and emphasize the need for targeted financial education and policy interventions to promote responsible credit behavior among youth.

Keywords: credit card misuse, indebtedness, financial literacy, young consumers

1 INTRODUCTION

Credit cards, introduced in 1950, have become a global payment standard, offering 24/7 access to funds and reducing the risks of carrying cash (Delamaire et al., 2009). Widely accepted in retail and service settings, they are now deeply integrated into daily life through online transactions, ATMs, and point-of-sale systems (Devlin et al., 2007).

Yet, this convenience presents challenges. Easy access to credit often leads to overspending and financial mismanagement, especially among young adults. Many users experience psychological detachment from spending, only realizing the impact when bills arrive (Awanis & Chi Cui, 2014). This “invisible money” effect contributes to irrational behaviour and long-term financial vulnerability, particularly in the 21–35 age group.

Despite these risks, credit cards remain central to modern consumer finance. Promoting responsible use requires greater financial literacy, awareness, and sound spending habits (Arnold et al., 2012). In this context, the present study aims to validate the Susceptibility to Credit Card Misuse and Indebtedness (SCCMI) framework in Croatia, focusing on young adults to identify psychological and behavioural drivers of financial vulnerability and inform targeted education and policy efforts.

2 THEORETICAL BACKGROUND

In recent years, rising credit card use among young consumers has revealed a growing gap between their willingness to spend and their ability to manage the financial consequences. This is especially evident among those aged 21–35, where psychological factors like the “credit card effect” encourage overspending and reduce self-control. As a result, they often underestimate past spending and overestimate available funds, leading to excessive debt and financial instability.

While the concept of SCCMI (Susceptibility to Credit Card Misuse and Indebtedness), originally developed by Awanis and Chi Cui (2014), provides a framework for understanding consumer vulnerability to credit card misuse, its application has yet to be empirically validated within the Croatian context. There is a pressing need to examine the relevance and validity of SCCMI among young Croatian credit card users, identify the behavioural and psychological variables contributing to financial vulnerability, and explore differences in debt repayment strategies.

Given the unique financial behaviours and educational needs of younger population, particularly in transitional economies, this research seeks to address a critical gap by providing evidence-based insights that can inform targeted interventions, consumer education initiatives, and policy measures aimed at fostering responsible credit card use and long-term financial well-being.

Previous research on payment methods suggests that the form of payment significantly influences consumer behaviour. Notably, credit cards have been found to encourage higher spending compared to cash, due to the psychological decoupling of payment from consumption (Soman, 2001; Prelec & Simester, 2001). This phenomenon, often referred to as the credit card effect, is characterized by increased willingness to spend, diminished memory of past expenditures, and an overestimation of available funds (Awanis & Cui, 2014; Soman & Cheema, 2002).

Empirical studies have consistently shown that consumers using credit cards are more likely to engage in impulsive and excessive spending (Shimp & Moody, 2000; Wang et al., 2011), particularly when payment is deferred and transaction transparency is reduced. Students and younger consumers, lacking financial experience, are especially susceptible to these effects, often interpreting their credit limits as indicators of future income (Warwick & Mansfield, 2000).

In response to such findings, regulatory bodies—most notably in the U.S.—have implemented policies aimed at restricting credit card marketing to minors and enhancing consumer protection (e.g., CARD Act of 2009). Similar initiatives exist within the European Union and Croatia, where legal constraints include minimum income and age requirements, as well as annual taxes on credit card services.

Credit card usage also reflects broader socioeconomic dynamics. Research indicates that men generally have higher credit limits than women, and that marital status and education level are significant predictors of credit limit allocation (Sun et al., 2023; Wang et al., 2011). In Croatia, the proliferation of credit cards and digital payment platforms has been accompanied by increased transaction volume and a gradual shift from cash to card payments, particularly for higher-value purchases.

Despite growing awareness, many consumers remain vulnerable to the psychological mechanisms that underlie credit card usage, reinforcing the need for further investigation and consumer education on responsible credit management. Gaps persist in operationalizing and measuring individual susceptibility to the credit card effect, representing a valuable avenue for future research (Shimp & Moody, 2000; Khan & Belk, 2024).

Financial sophistication is a concepts which is instrumental in determining the extent of financial literacy and ability to manage their own finances. As s concept, it incorporates the ability to assess and manage financial risks, plan personal finances, and utilize financial tools to ensure long-term financial stability (Vehovec et al., 2015).

A lack of financial sophistication refers to individuals' limited understanding of financial products, services, and fundamental economic principles. This includes inadequate knowledge of interest rates, credit terms, investment basics, and the inability to make informed and rational financial decisions (Robb &Sharpe, 2009).

In Croatia, the lack of financial sophistication presents a growing challenge, especially given the increasing complexity of the financial market and the expanding array of financial products. Research conducted by the Croatian National Bank indicates that a significant portion of the population lacks understanding of basic financial concepts such as inflation, risk diversification, and compound interest. Consequently, individuals are more prone to making poor financial decisions, including irresponsible credit card usage, taking on unfavourable loans, or mismanaging their personal finances (Megušar, 2022).

Individuals with low financial literacy are often unaware of the total cost of borrowing or the long-term implications of failing to repay debt, which can lead to debt accumulation and deteriorating financial well-being. Moreover, unfamiliarity with investment principles exposes consumers to higher risks, especially during economic downturns. According to the Croatian Financial Services Supervisory Agency (HANFA, 2024), Croatian citizens tend to rely primarily on basic banking products like savings accounts, while more complex instruments such as investment funds or stocks remain underutilized. This limited engagement suggests that insufficient financial sophistication restricts individuals from capitalizing on wealth-building opportunities and optimizing financial well-being.

To address these challenges, there is a growing need to enhance financial literacy in Croatia through targeted educational programs and initiatives. These should encompass both foundational and advanced financial topics such as investment strategies, retirement planning, and risk management. The Organisation for Economic Co-operation and Development (OECD) defines financial literacy as a process through which financial consumers and investors improve their understanding of financial products and concepts by accessing information, advice, and guidance. This process equips them with the skills and confidence necessary to make informed financial decisions and seek support when needed (Lusardi & Mitchell, 2011).

Recent international research coordinated by the OECD in 2022–2023 found that Croatian citizens scored 62 out of 100 in financial literacy, surpassing the average across participating countries (60%) but slightly below the OECD average (63%). Croatia ranked highest in financial knowledge, with a score of 70%, exceeding the OECD average of 67%. While mostly at or below OECD averages, these results suggest a relatively strong foundation for further improvement.

3 METHODOLOGY AND RESULTS

To investigate consumer susceptibility to credit card misuse and indebtedness, a cross-sectional descriptive survey was conducted using the 1KA online platform. The study targeted a purposive sample of young individuals in Croatia, who personally own at least one credit card. Data was collected through an anonymous online questionnaire distributed via social media platforms such as WhatsApp, Facebook, and Instagram. To enhance response rates, recruitment was conducted via multiple calls on social media, and support was available during questionnaire completion.

A total of 198 respondents participated, of which 193 completed the survey. For the final analysis, data from 140 participants were retained based on a screening question confirming the use of at least one credit card. Respondents who used only debit cards or were unable to distinguish between debit and credit cards (n = 53) were excluded from further analysis.

The questionnaire consisted primarily of closed-ended questions and was divided into two sections: (1) demographic information (gender, age, education level, student/employment status, and monthly income), and (2) a section designed to measure the observed constructs using validated and widely used measurement scales. To assess susceptibility to credit card misuse and indebtedness, a validated five-item scale (SCCMI) developed by Awanis and Chi

Cui (2014) was used. An additional five-item scale by the same authors was used to assess young credit card users' behaviours related to covering incurred expenses.

Three thematic constructs were analysed: (1) the *credit card effect*, referring to the inability to treat credit as equivalent to cash; (2) *lack of financial sophistication*, reflecting weak budgeting and expense-tracking behaviour; and (3) *debt normalization*, indicating the tendency to rationalize indebtedness. Responses were rated on a seven-point Likert scale (1 = Strongly disagree, 7 = Strongly agree). Data were analysed using SPSS Version 24. The survey was conducted during June and July 2024.

This section presents a structured overview of the empirical findings, focusing on the demographic characteristics of the surveyed population. The analysis aims to provide a clear understanding of the sample composition, serving as a basis for interpreting subsequent results. Key demographic variables include gender, age, education level, student and employment status, and total monthly income.

The gender distribution within the sample was notably imbalanced, with female respondents comprising 80% of participants, while males accounted for the remaining 20%. In terms of age structure, the majority of respondents (52.1%) were under the age of 25, indicating a predominantly younger sample. Participants aged between 30 and 40 represented 27.1% of the total, while those between 25 and 30 accounted for 20.7%.

Educational attainment data indicate that 51.4% of respondents had completed secondary education. A significant proportion (47.9%) held either a bachelor's or master's degree, whereas only one individual reported holding a postgraduate or doctoral qualification. Regarding student status, 56.4% of respondents were not enrolled in any form of formal education at the time of the survey. Of those who were students (43.6%), a majority (33.6%) studied full-time, while 8.6% were part-time students. Only two participants were enrolled in postgraduate studies.

Employment status analysis revealed that 56.4% of respondents were employed full-time. An additional 25% of the sample, primarily full-time students, reported holding part-time or occasional employment. Unemployed individuals constituted 17.1% of the sample, some of whom also reported student status. A marginal proportion (1.4%) were self-employed. Monthly income levels varied across the sample. Approximately 15% reported earnings below €500. The largest income bracket, comprising 32.1% of respondents, reported earnings between €801 and €1.100. Additionally, 24.3% earned between €1.101 and €1.500, 16.4% between €501 and €800, while only 12.1% reported monthly income exceeding €1.500.

Finally, analysis of credit card ownership patterns revealed that a substantial majority (74.3%) used only one credit card registered in their name. A further 21.4% used two credit cards, and a small minority (4.3%) reported possession of three or more cards.

This study followed the methodological guidelines outlined by Pallant (2010) for conducting exploratory factor analysis (EFA), including assessment of data suitability, factor extraction and rotation, and interpretation. The analysis was based on responses from 140 participants who reported using at least one credit card registered in their name. The sample size met the recommended ratio of at least 10 participants per item, justifying the application of EFA.

Descriptive statistics revealed that the item "*When I use a credit card, it does not feel like spending my own money*" had the highest mean score (M = 3.58), suggesting limited awareness of the psychological effects of credit card use. Other items reflected moderate levels of self-reported financial monitoring and responsibility. Notably, the lowest scoring item (M = 2.72) indicated some concern over repayment, contradicting assumptions of normalized debt among youth.

Reliability analysis showed that the SCCMI scale had satisfactory internal consistency (Cronbach's $\alpha = 0.733$), exceeding the 0.70 threshold. Although this was slightly lower than in

the original SCCMI study (α = 0.849–0.895), average item-total correlations fell within the recommended range (r = 0.357), and no item deletion improved alpha, indicating that all five items contributed meaningfully to the scale's reliability.

Factorability was further confirmed via KMO measure (0.754) and Bartlett's Test of Sphericity (χ^2 = significant at $p < 0.001$). Principal component analysis (PCA) extracted a single factor with an eigenvalue >1 (2.443), explaining 48.85% of the variance. Scree plot inspection supported a one-factor solution.

All items loaded strongly on this factor (ranging from 0.589 to 0.792), suggesting a coherent latent construct representing irrational credit card use. The item "*I postpone thinking about repayment until I receive the bill*" had the strongest loading (0.792), while "*I don't think it's necessary to track my credit card expenses frequently*" had the lowest (0.589). Communality values ranged from 0.347 to 0.627, all exceeding the 0.3 threshold. The results confirmed the measurement model of the SCCMI proposed by Awanis and Chi Cui (2014), consisting of a single factor and five items, as the most appropriate structure of the SCCMI for use among younger populations.

A one-way ANOVA examined differences in SCCMI scores based on how respondents financed their credit card expenses. Three groups were identified: those who paid fully from their own income (72.9%), partially with external help (23.6%), and fully reliant on others (3.6%). Significant group differences were observed ($F = 9.626$; $p < 0.001$), with higher SCCMI scores among those receiving external financial support. These findings align with Awanis and Chi Cui's (2014) assertion that financial independence among young adults is associated with more rational credit card use.

4 DISCUSSION AND CONCLUDING REMARKS

This study provides empirical validation of the SCCMI scale in the Croatian context, confirming its psychometric reliability (α = 0.733) and relevance for assessing irrational credit card behaviour among young adults. The findings highlight the psychological decoupling of spending from perceived monetary loss - known as the credit card effect - as a key factor contributing to debt accumulation without full consumer awareness.

Respondents who relied on external financial support demonstrated significantly higher susceptibility to misuse, underscoring financial independence as a protective factor. Despite self-reported financial literacy, behaviours such as poor budgeting and delayed repayment reveal a gap between knowledge and practice. The study also identified three distinct repayment profiles, reflecting different levels of financial autonomy.

Factor analysis confirmed the unidimensional structure of the SCCMI, supporting its applicability in transitional economies. The results align with prior international research and emphasize the behavioural and emotional dimensions of financial decision-making.

However, the use of a purposive, non-random sample of 140 participants limits generalizability. Additionally, the lack of prior national studies limits comparative analysis. Despite these limitations, the study offers novel insights into the challenges of credit card usage among younger consumers in Croatia, laying a foundation for future comparative studies.

The implications of this research are clear: enhancing financial education must extend beyond knowledge transfer to address motivation, emotional regulation, and real-life applications. Policymakers and educators should prioritize experiential learning, promote early financial independence, and integrate diagnostic tools such as the SCCMI into intervention strategies. The underlying importance of such efforts lies in empowering young consumers to navigate digital financial environments responsibly, which is essential for building long-term financial resilience in emerging European economies.

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The 18th International Symposium on
Operational Research in Slovenia

SOR ’25

Bled, SLOVENIA
September 24 - 26, 2025

Session 1:

Econometric Models and Statistics

THE ROLE OF MONOTONICALLY DECREASING DISTRIBUTIONS IN CRYPTOCURRENCY MARKETS: COMPARING BENFORD'S LAW AND STIGLER'S LAW

Ivana Cunjak Mataković
University of Applied Science Baltazar
Vladimira Novaka 23, 10290 Zaprešić, Croatia
E-mail: icunjakmatakovci@bak.hr

Ksenija Dumičić
DOBA Business School
Prešernova 1, 2000 Maribor, Slovenia
E-mail: kdumicic.prof@gmail.com

Abstract: Benford's Law is a discrete logarithmic distribution that describes the expected patterns of digits in numerical data. Its use in detecting financial market manipulation is well established and has also been applied to identify anomalies in cryptocurrency markets. Stigler's Law, a lesser-known discrete harmonic-based distribution, similarly outlines expected digit patterns in numerical data. In this paper, the applicability of both Benford's and Stigler's Laws is demonstrated using selected cryptocurrencies to assess their effectiveness in detecting anomalies. The empirical results indicate that cryptocurrencies conform more closely to Benford's Law than to Stigler's Law.

Keywords: Benford's distribution, Stigler's distribution, Goodness-of-Fit test, cryptocurrencies' market, forensic data analysis, wash trading

1 INTRODUCTION

According to [14], the phenomenon of financial fraud has not yet been sufficiently investigated, despite recognition that it is shifting from the fringes of financial market activity to the financial industry as a whole. In [18], the authors argue that cryptocurrency fraud has shown a growing trend in reported global losses and that the potential of cryptocurrencies for fraudulent activity has not been systematically examined. The European Union has recognised the potential of crypto-assets and, in May 2023, adopted Regulation (EU) 2023/1114 of the European Parliament and of the Council on markets in crypto-assets [13]. The Regulation places particular emphasis on ensuring trust and market integrity within the crypto-asset space. Consequently, it includes provisions aimed at combating abuse within these markets. As data distributions play an important role in forensic data analysis, deviations in the distribution of the first digits of cryptocurrency trading volumes are examined using the theoretical Benford and Stigler distributions. A literature review shows that few studies have explored atypical values in cryptocurrencies exposed to market manipulation using Benford's distributions. This paper analyses the detection of manipulation in the trading volume of cryptocurrencies that are not exposed to the wash trading technique. Wash trading is a market manipulation technique in which an investor or group of investors artificially inflate the trading volume of a cryptocurrency by buying and selling the same crypto assets, creating the illusion of market activity. As argued in [1], in situations where direct evidence of transactions subject to market manipulation is unavailable, indirect estimates are commonly used. However, the accuracy of such estimates is questionable, particularly when the sample used may be affected by various manipulation strategies. Therefore, [1] emphasises the importance of comparing indirect estimation methods with direct evidence. For the selected cryptocurrencies, the Blockchain Transparency Institute reports that the percentage of fake trading volume is below 5%, which supports the assumption that the distributions of the first digits of trading volumes should conform to the theoretical distributions. The research hypothesis to be investigated in this

study is that the distribution of the first digits in the trading volumes of cryptocurrencies not exposed to market manipulation does not deviate in a statistically significant way from the Benford and Stigler distributions.

2 PREVIOUS RESEARCH

In [6], the authors analysed whether the first digits of asset prices and yields of companies listed in the S&P 500 index conform to Benford's Law. Their research found that the empirical distribution of yields aligns more closely with Benford's Law than the distribution of prices. In [16], financial statements of 20 Fortune 500 technology companies, including Amazon, Google, IBM, Apple, eBay, Microsoft, Oracle, and Xerox, are examined, confirming that the observed data closely followed Benford's Law. Further, Benford's Law is used in [3] to analyse hospital financial data, identifying the missing digits in the first and second positions, and the authors concluded that further research is necessary to recognize the fraud. The study conducted in Slovenia [12], aimed to investigate whether the financial statements of Slovenian companies listed on the Ljubljana Stock Exchange comply with Benford's Law. The analysis included 44 companies listed on the Ljubljana Stock Exchange and having an audit opinion on their financial statements. The period of observation was three years (2011-2013), and the analysed data were first- and second-digit tests according to Benford's Law. [19] presented, one of the first studies to explore the application of Benford's Law in the blockchain domain for detecting anomalies in cryptocurrency transactions. This study covered transactions of major cryptocurrencies from the initial day of confirmation in the blockchain ledger through the end of 2018. The findings confirmed the applicability of Benford's Law in this emerging field. The authors noted that Benford's Law can serve as a preliminary research technique in machine learning and data exploration for identifying anomalies. In [15], one-day rates of return for Dow Jones Industrial Average were analysed, following the methodology presented in [10], and indicated that, depending on the data characteristics, the distribution of first digits may correspond more closely to Stigler than to Benford distribution. In [7], the authors investigated the compatibility of Benford's Law with the Stigler distribution of the probability of the appearance of the first digits and concluded that the two distributions are statistically significantly different when sample sizes exceed 700 observations.

3 RESEARCH METHODOLOGY

The frequency of first-digit occurrence in randomly selected numerical data is not uniform. Simon Newcomb, a Canadian-American astronomer and mathematician, was the first to observe this phenomenon. He formulated a Law stating that number 1 appears as the first digit in any number with a probability of $\log_{10}2 \cong 0{,}301$, the number 2 as the first digit with a probability of $\log_{10}(3/2) \cong 0{,}176$ and so on, decreasing monotonically until the number 9, for which the probability of appearing as the first digit in a number is 0.046 [8]. Unaware of Newcomb's earlier discovery, the American physicist Frank Benford came to similar conclusions 57 years later. Benford found that the actual proportion of the number 1 as the first digit was almost equal to the logarithm of 2 (or 2/1). This logarithmic pattern continued up to the number 9. Benford then derived the expected probability of the first digits occurring, which is known as Benford's Law [11], described by the equation (1).

$$P(D_1 = d_1) = log_{10}\left(1 + \frac{1}{d_1}\right)\ d_1 = \in \{1,2,\dots,9\}, \tag{1}$$

where D_1 stands for the first digit, P denotes the probability of occurrence, and log stands for the logarithm to the base 10. The Benford distribution of the first digits is shown in Table 1.

Table 1: The Benford distribution of the first digits

Digit	d	1	2	3	4	5	6	7	8	9
Probability of occurrence	**P(d)**	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046

Source: adapted from [11].

The Benford distribution is widely used in fraud detection. However, not all datasets conform to Benford's Law; some appear to follow Stigler's Law instead [9]. George Stigler, a Nobel laureate in economics, argued that a combination of uniform and non-uniform distributions leads to an inconsistent distribution of maximum values. Consequently, he proposed an alternative first-digit distribution, known as the Stigler distribution, which is less skewed toward lower digits compared to Benford's distribution [17]. The Stigler distribution is based on the assumption that each digit from 1 to 9 occurs with equal probability (1/9) as the leading digit in a number. Furthermore, it assumes that all subsequent (smaller) numbers also follow a uniform distribution of first digits. Stigler examined the Newcomb-Benford Law and suggested that the average frequency of the digit d as the leading digit in a number can be calculated using the expression shown in equation (2).

$$F_d = \frac{d\ln(d) - (d+1)\ln(d+1) + m}{9}, \tag{2}$$

where m is a constant equals 3.558428, which can be calculated using the equation (3).

$$m = \frac{\sum_{i=1}^{9} i^2 \ln(d_i) - d_i d_{i+1} \ln(d_{i+1})}{9 - \sum_{i=1}^{9} d_i} = 1 + \frac{10}{9}\ln(10) = 3.558428. \tag{3}$$

According to the above equation (2), the Stigler distribution of the probability of occurrence of the first digits is as shown in Table 2.

Table 2: The Stigler distribution of the first digits

Digit	d	1	2	3	4	5	6	7	8	9
Probability of occurrence	**P(d)**	0.241	0.183	0.145	0.117	0.095	0.076	0.060	0.047	0.034

Source: adapted from [9].

To assess whether an observed dataset significantly deviates from the Benford distribution, several statistical tests can be employed, including the Z- test, the chi-square test, the Kolmogorov–Smirnov Z-test, and the mean absolute deviation test [11]. The chi-square test statistic (χ^2) is defined in equation (4).

$$\chi^2 = \sum_{i=1}^{k} \frac{(f_i - e_i)^2}{e_i}, \tag{4}$$

where f_i is the actual number of the first digit in the pattern and e_i is the expected number of the first digit in the pattern. The null hypothesis is rejected at the 5% significance level if the calculated χ^2 value exceeds the critical value of 15.507 with 8 degrees of freedom. The Kolmogorov–Smirnov Z-test is a non-parametric statistical test that measures the maximum distance between the empirical distribution of the sample and a specified theoretical distribution, in this case, the Benford distribution [11]. The Kolmogorov–Smirnov test statistic is defined according to [11], as given in equation (5).

$$K - S = \frac{1.36}{\sqrt{N}}, \tag{5}$$

where 1.36 is a constant at a 5% significance level, and N is the number of observations.

4 EMPIRICAL RESULTS

This paper analyses the trading volumes of cryptocurrencies not exposed to the wash trading market manipulation technique, according to [4]. For the selected cryptocurrencies, data were collected from the first day of trading up to 30 June 2019, as available in [5], allowing a comparison with the Blockchain Transparency Institute's report from 30 March 2019, [4], which presents the percentage of fake trading volumes for cryptocurrencies involved in wash trading. The Blockchain Transparency Institute developed an algorithm for monitoring trading volumes on cryptocurrency exchanges. According to the report [4], global wash trading had decreased by 35.7% since the algorithm's implementation. [2] provides the analysis of trading volumes and information asymmetry in Bitcoin transfers on the blockchain, referring to the Blockchain Transparency Institute as an industry-leading initiative for providing "clean and wash-free data".

In this study, a descriptive exploratory analysis was conducted to examine the statistical characteristics of the trading volumes of the selected cryptocurrencies.

Table 3: Results of the descriptive statistics of the trading volume of selected cryptocurrencies (in thous. USD)

Cryptocurrency	Groestlcoin	Navcoin	NoLimit Coin	Okcash	Salus	Singular	Viacoin
Period	11. 04. 2014 - 30. 06. 2019	12. 06. 2014 - 30. 06. 2019	12. 09. 2016 - 30. 06. 2019	14. 12. 2014 - 30. 06. 2019	22. 01. 2016 - 30. 06. 2019	03. 10. 2016 - 30. 06. 2019	18. 07. 2014 - 30. 06. 2019
Sample size (no. of trading days)	1,907	1,845	999	1,660	1,253	1,001	1,809
Arithmetic mean	3,135.318	920.279	69.964	4,873.602	850.980	1,187.036	763.142
Median	1.440	56.899	27.154	739.288	6.811	445.691	25.000
Variance	310,375,478.329	9,277,327.485	15,166.459	90,442,850.629	44,900,151.808	10,050,256.102	33,729,381.912
Standard deviation	17,617.476	3,045.870	123.152	9,510.144	6,700.757	3,170.213	5,807.700
Coefficient of variation	562.904%	330.972%	176.022%	195.136%	787.416%	267.070%	761.024%
Minimum	0.000	0.000	0.000	1.640	0.000	0.070	0.000
Maximum	414,276.150	52,815.900	1,385.552	55,669.974	127,001.001	5,5473.200	222,739.000
Interquartile	1,127.700	494.349	76.412	4,645.820	66.145	840.527	508.000
Asymmetry	14.220	7.957	4.235	2.768	15.810	10.481	32.321
Kurtosis	253.146	93.250	26.220	7.684	277.322	146.983	1,196.151

Source: Authors' calculation

The data presented in Table 3 shows that the coefficient of variation for some cryptocurrencies exceeds 700%, indicating an extremely high level of dispersion. The wide range between the minimum and maximum trading volumes further suggests the potential presence of outliers.

The results of the selected goodness-of-fit (GoF) tests, as here applied chi-square test and Kolmogorov–Smirnov Z-test, which assess the conformity of the observed distributions of the first digits of trading volumes with the Benford distribution, are presented in Table 4. The results of the chi-square test indicate that the distributions of the first digits of the trading volumes for the cryptocurrencies Groestlcoin, Navcoin, OKCash, Salus, and Viacoin are consistent with the Benford distribution at the 5% significance level.

To assess conformity with the Stigler distribution, the same GoF tests were applied to the observed distributions of the first digits of trading volumes. The results are presented in Table 5.

Table 4: GoF tests for the Benford distribution: Results of the chi-square and Kolmogorov-Smirnov Z-test of the first digits of the trading volume of cryptocurrencies

Test	Statistical values	Cryptocurrency						
		Groestlcoin	Navcoin	NoLimitCoin	Okcash	Salus	Singular	Viacoin
Chi-square test	Empirical value	4.590	12.343	15.129	10.953	9.984	35.901	12.510
	Degrees of freedom	8	8	8	8	8	8	8
	Theoretical value	15.507	15.507	15.507	15.507	15.507	15.507	15.507
Kolmogorov-Smirnov Z-test	Empirical value	0.005	0.023	0.054	0.022	0.016	0.079	0.037
	Theoretical value	0.031	0.032	0.043	0.033	0.039	0.043	0.033

Source: Authors' calculation

Table 5: GoF tests for the Stigler distribution: results of the chi-square and Kolmogorov-Smirnov Z-test of the first digits of the cryptocurrency trading volume

Test	Statistical values	Cryptocurrency						
		Groestlcoin	Navcoin	NoLimitCoin	Okcash	Salus	Singular	Viacoin
Chi-square test	Empirical value	62.814	77.514	72.222	48.681	62.598	14.056	25.509
	Degrees of freedom	8	8	8	8	8	8	8
	Theoretical value	15.507	15.507	15.507	15.507	15.507	15.507	15.507
Kolmogorov-Smirnov Z-test	Empirical value	0.064	0.075	0.107	0.051	0.063	0.027	0.028
	Theoretical value	0.031	0.032	0.043	0.033	0.039	0.043	0.033

Source: Authors' calculation

The results of the chi-square and Kolmogorov–Smirnov Z-tests indicate that, at the 5% significance level, the distributions of the first digits in the trading volumes of the cryptocurrencies Groestlcoin, Navcoin, NoLimitCoin, OKCash, Salus, and Viacoin do not conform to the Stigler distribution.

5 CONCLUSION

The results of the trading volume analysis indicate that most cryptocurrencies conform to the Benford distribution. In contrast, the distributions of the first digits for almost all cryptocurrencies do not conform to the Stigler distribution. The discrepancy in expected conformity with the Stigler distribution is likely due to the statistically significant differences between the Benford and Stigler distributions, which become more pronounced with larger sample sizes [7]. The study presented in this paper investigates the suitability of Benford's Law and Stigler's Law in modelling the distribution of leading digits in cryptocurrency market data. This research confirms that both Laws can be used to analyse the distribution of leading digits in cryptocurrency market data. The goodness-of-fit tests applied to various cryptocurrencies show that Benford's Law generally exhibits stronger conformity, especially in larger, more stable digital assets. However, Stigler's Law provides a better fit in selected cases, indicating its potential as a supportive analytical tool. These results suggest that combining both Laws may improve the detection of irregularities in digital financial data and enhance the statistical assessment of data integrity in cryptocurrency markets.

The results obtained in this research partially support the null hypothesis set. Based on the findings of the research conducted and in comparison with direct evidence of market manipulation provided by the Blockchain Transparency Institute, it can be concluded that cryptocurrencies conform more closely to Benford's Law than to Stigler's Law.

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RETHINKING THE HOUSING WEALTH EFFECT IN EUROPE

Anita Čeh Časni
University of Zagreb, Faculty of Economics and Business, Department of Statistics
10 000 Zagreb, Croatia
E-mail: aceh@efzg.hr

Abstract: This study investigates the dynamic relationship between consumption, income, and house prices in thirteen European countries using a panel vector autoregression (PVAR) model and quarterly data from 1999Q1 to 2024Q4. Motivated by recent inflationary pressures linked to the global financial crisis, the COVID-19 pandemic, and rising house prices, the analysis is grounded in the Permanent Income and Life-Cycle Hypotheses. Results indicate that personal consumption responds positively but temporarily to housing wealth shocks, suggesting a short-lived wealth effect. The findings underscore the relevance of housing-driven consumption in macroeconomic policy, particularly during periods of economic stress and recovery.

Keywords: housing wealth effect, consumption, financial crisis, panel vector autoregression, Europe

1 INTRODUCTION

The macro-financial landscape of the past two decades has seen substantial disturbance, culminating in noteworthy implications for housing markets which is one of the most critical components of household balance sheets. Following a prolonged period of price and monetary stability, the post-pandemic surge in global inflation has triggered an aggressive cycle of monetary tightening. This shift, characterised by a rapid increase in policy interest rates and heightened macroeconomic uncertainty, has raised the cost of capital and imposed statistically significant downward pressures on asset valuations, including residential real estate [5].

In this context, the relationship between housing wealth and household consumption has re- emerged as a central topic of macroeconomic analysis. Rooted in the theoretical foundations of Friedman's Permanent Income Hypothesis [7] and the Life-Cycle Hypothesis (LCH) developed by Ando and Modigliani [2], housing wealth is posited to influence consumption by altering households' perceptions of permanent income and lifetime resources. Since the 2008 Global Financial Crisis, real house prices in many European economies have risen persistently, enhancing household net worth and, by extension, potentially fuelling consumption. However, this trend has recently reversed in light of tighter monetary conditions, stagnating real incomes, and a broad-based cost-of-living crisis. These developments raise concerns over potential contractions in perceived permanent income and long-term wealth, which may impair household consumption and, by extension, macroeconomic stability.

From a statistical perspective, the housing wealth effect constitutes a key transmission channel in the broader interaction between financial markets and the real economy. Residential property represents, on average, more than 55% of household net worth across European Union countries [6]. However, the extent and significance of this channel vary considerably across countries, owing to differences in homeownership rates, mortgage market structures, and policy frameworks [13]. Consequently, house price shocks can lead to asymmetric wealth effects, affecting consumption smoothing and resilience differently across member states.

The empirical literature investigating the housing wealth effect presents a wide spectrum of findings, shaped by differing methodologies, data frequencies, and national contexts. One of the most comprehensive recent contributions is by [5], who employed a panel vector autoregression (PVAR) model using quarterly data from 20 European countries over the period 1980–2023. Their results indicate that real household consumption responds positively and promptly to increases in both real house prices and gross disposable income. However, the

magnitude of the effect varies notably across countries, reflecting divergent housing market dynamics and consumption sensitivities.

Additional country-level analyses reinforce the evidence of heterogeneity. In Spain, household panel data from 2002 to 2011 revealed a marginal propensity to consume (MPC) out of housing wealth of approximately one cent per euro, with a stronger response among lower-wealth households [14]. Moreover, the impact of housing wealth shocks was found to be asymmetric: negative wealth shocks induced larger reductions in consumption than comparable positive shocks. A complementary study [4] focusing on Spain during 2002–2017 distinguished between anticipated and unanticipated changes in housing wealth, finding that both types significantly influenced non-durable consumption following the 2008 crisis thus posing a challenge to the predictive validity of the Permanent Income Hypothesis.

Building on these insights, this study empirically investigates the dynamic interrelationships between housing wealth, household income, and private consumption across 13 European countries over the period 1999Q1–2024Q4. The econometric strategy employs a PVAR framework, which allows for the estimation of bidirectional feedback effects and dynamic adjustment processes among the key macroeconomic variables. By focusing on real house prices, real disposable income, and real personal consumption, the model captures the joint development of these series while avoiding the need for cointegration, as all variables are (first-)difference stationary.

The empirical findings indicate that increases in both real house prices and real disposable income exert a positive effect on real personal consumption, though these effects are short-lived and diminish over subsequent quarters. The use of a PVAR methodology, as outlined by [10] and [11] enables a system-wide assessment of these relationships and accounts for cross-country heterogeneity and contemporaneous feedback effects which is an advantage over traditional single-equation models.

A key contribution of this study lies in its coverage of three major macroeconomic episodes: the 2008 Global Financial Crisis, the 2020 COVID-19 shock, and the inflation surge following the 2021–2022 supply-side disruptions. This broad temporal coverage enhances the robustness of the findings and provides a unique perspective on how the housing wealth effect has evolved across evidently different economic regimes.

The remainder of the paper is structured as follows. Section 2 presents the data sources and details the econometric methodology. Section 3 reports the empirical findings, including impulse response functions and variance decompositions. Section 4 concludes with a summary of key insights and discusses their implications for macroeconomic policy and financial stability in the European context.

2 DATA AND METHODOLOGY

This study employs quarterly data from an unbalanced panel of 13 European economies, namely Austria, Belgium, Czechia, Denmark, Finland, France, Germany, Hungary, Italy, the Netherlands, Portugal, Spain and Sweden, selected based on data availability for the variables of interest.

The dataset includes quarterly indices for personal consumption, household disposable income, and real estate prices. Household final consumption expenditure serves as a proxy for household consumption, while real estate price indices are used as a proxy for housing wealth. Gross disposable income of households is used as a proxy variable for household disposable income.

Since the dataset is collected from different secondary data sources, all series were first transformed into base indices (2015=100), deflated to account for inflation, seasonally adjusted, and expressed in logarithms. Performed unit root tests indicate that the variables are

difference-stationary; hence, the empirical analysis uses first differences of the log-transformed, time-demeaned variables. The available time span varies across countries but covers the period from Q1 1999 to Q4 2024 where data permit. Data sources include Eurostat for consumption and income series, whereas real estate price indices (national level, all dwelling types) are sourced from the Bank for International Settlements (BIS).

The empirical analysis relies on the panel vector autoregression (PVAR) methodology developed by [1], which is particularly suited to modelling the dynamic, endogenous relationships among consumption, income and housing wealth. This approach integrates the traditional VAR framework, where all variables are treated as endogenous, with panel data techniques that account for unobserved heterogeneity through fixed effects.

Model specification begins with determining the optimal lag order using the moment and model selection criteria (MMSC) proposed by [3] which are based on [9] J-statistic for testing over-identifying restrictions in GMM estimation. Based on these criteria, the preferred model is a first-order panel VAR. The following first-order three-variable panel VAR model is specified:

$$Z_{i,t} = \mu_i + \Theta(L)Z_{i,t} + f_i + d_{c,t} + \varepsilon_{i,t} \quad (1)$$

where $\boldsymbol{\mu}_i$ is the vector of constant terms for each variable, Θ(L) is the lag operator and $\boldsymbol{Z}_{i,t}$ signifies a vector of three endogenous variables: real household consumption, real disposable income and real housing wealth. Subscripts *i* and *t* refer to country and time, respectively. Additionally, $\boldsymbol{f}_i$ represents the fixed effect; $\boldsymbol{d}_{c,t}$ signifies the country-specific time dummy and $\boldsymbol{\varepsilon}_{i,t}$ represents the vector of residuals.

In a panel vector autoregression (PVAR) model, it is often assumed that all countries share a common structure by using the same parameters across them. However, this assumption is often unrealistic because countries differ in their policies and economic structures. To account for these differences, empirical model in this paper includes fixed effects for each country.

However, a challenge arises since these fixed effects can be correlated with the explanatory variables, especially when past values of the dependent variable are included. To solve this problem, the model uses the Helmert transformation (or forward mean-differencing), which removes future means from the data. This helps ensure that the transformed variables are not correlated with their instruments. In this paper first four lags were used as instruments.

Even with this transformation, some bias may remain due to simultaneity where regressors are still linked with the error terms. In addition, differences across countries can lead to unequal error variances (heteroskedasticity). To handle these issues and get reliable results, the model uses the generalized method of moments (GMM), using past values of the variables as instruments.

Additionally, the model assumes that the error terms are random and unrelated across countries and over time. However, this is often not true in reality, since the actual errors are usually correlated. To separate the underlying shocks that drive the system, the model uses Cholesky decomposition which is a method that imposes a specific order on the variables based on how exogenous they are supposed to be. In this ordering, variables listed first can immediately affect those listed after them, but not vice versa. Based on prior research, this paper places housing wealth after personal consumption and disposable income in the ordering.

The analysis also uses impulse response functions (IRFs) to show how a shock to one variable affects others over time, while keeping all other shocks constant. These responses are calculated from the estimated model. Finally, forecast error variance decomposition is used to see how much of the changes in personal consumption can be explained by shocks to housing wealth and income. This helps evaluate how important these shocks are in influencing consumption behaviour.

3 EMPIRICAL FINDINGS AND RESULTS

The main results from the panel vector autoregression (PVAR) model are presented in Table 1. These estimates correspond to Equation (1), after removing fixed effects and country-time dummies. Post-estimation diagnostics include the Granger causality Wald test and an assessment of the model's stability.

The Granger causality results indicate that disposable income Granger-causes personal consumption and housing wealth at the 1% significance level. However, housing wealth does not Granger-cause either consumption or income. Also, personal consumption Granger-causes housing wealth at the 1% significance level. To check if the model is stable, the eigenvalues of its companion matrix were calculated. According to standard procedures ([12], [8]), the model is stable if all eigenvalues are inside the unit circle. The results show that the estimated PVAR model meets this stability condition.

Impulse response functions (IRFs), shown in Figure 1, illustrate how shocks to one variable affect the others over time. These responses are derived from the estimated model coefficients, which align with theoretical expectations, confirming the validity of the IRFs.

According to the 1-lag PVAR results, personal consumption responds significantly to changes in disposable income at the 1% significance level. However, the response of consumption to housing wealth is not statistically significant. Similarly, disposable income reacts positively to shocks in consumption and housing wealth, but these responses are not statistically significant. Housing wealth shows a statistically significant negative response to changes in consumption and disposable income at 1% significance level, respectively.

Table 1: Main results of a 1-lag PVAR model

Response of:	***Response to (GMM estimates):***		
	C(t-1)	***DI(t-1)***	***HW(t-1)***
C(t)	***-0.0668*** [0.0198]***	***-7.6782*** [0.5145]***	***0.0071 [0.0087]***
DI(t)	***0.000034 [0.0003]***	***-0.2412*** [0.0166]***	***0.0002 [0.00025]***
HW(t)	***-0.4828*** [0.1414]***	***-56.1914** [4.5239]***	***-0.0561 [0.1124]***

*Note: three-variable VAR model is estimated by GMM, country-specific and fixed effects are removed prior to estimation. Heteroskedasticity and serial correlation robust standard errors are in brackets. First four lags of variables were used as instruments *** denotes significance on 1% significance level.*
Source: calculation of the author.

Figure 1 shows how personal consumption, disposable income, and housing wealth respond to a one-standard-deviation shock in the 1-lag PVAR model. The key focus of this paper is on how personal consumption reacts to a shock in housing wealth (as shown in top right panel). The results show that consumption increases immediately after the shock, decreases in the next quarter, then increases again. This effect fades and becomes minimal after about four quarters.

Variance decomposition from the 1-lag PVAR model offers another way to understand the relationships shown in the impulse response functions (Figure 1). Over a 10-quarter period, shocks to housing wealth explain about 0.048% of the changes in personal consumption and 0.027% of the changes in disposable income. In the same period, disposable income shocks explain 0.323% of the variation in personal consumption.

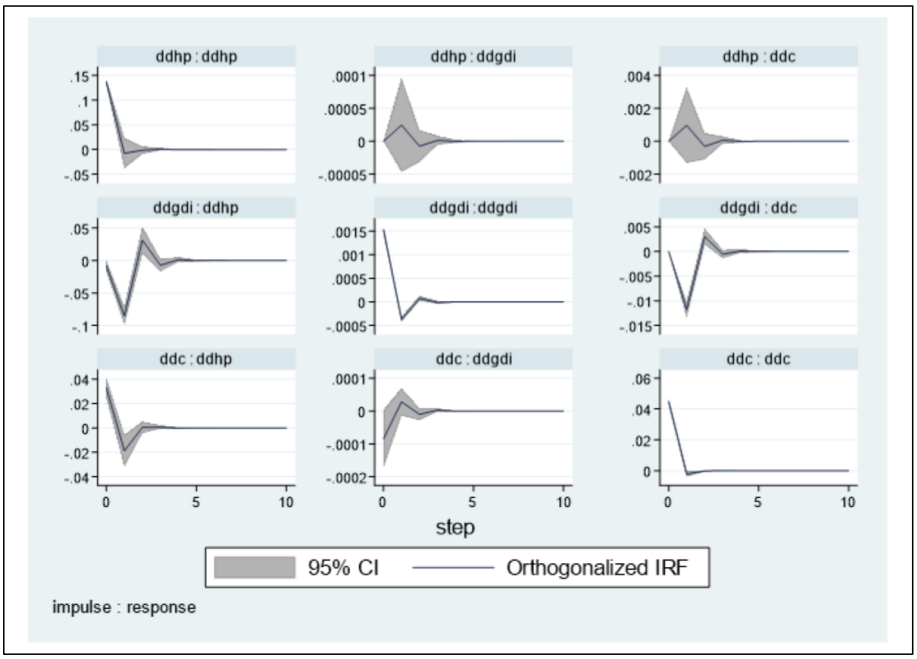


Figure 1: IRF for 1-lag PVAR model (Stata 17)
Note: ddc=consumption, ddgdi=disposable income, ddhp=housing wealth

4 CONCLUDING REMARKS

This paper examines the housing wealth effect on personal consumption across 13 European countries over the period 1999Q1 to 2024Q4, covering three major macroeconomic episodes: the 2008 Global Financial Crisis, the 2020 COVID-19 pandemic, and the recent inflation shock. By applying a panel vector autoregression (PVAR) framework, the study investigates the dynamic interactions among real house prices, household disposable income, and personal consumption, with a focus on how macroeconomic shocks and policy changes affect consumption behaviour. The empirical results provide several important insights. First, the analysis confirms a statistically significant and positive short-term response of personal consumption to changes in disposable income. This finding aligns with both the Permanent Income Hypothesis (PIH) and Life Cycle Hypothesis (LCH), which suggest that households smooth consumption based on expected lifetime income rather than current earnings alone. However, the response of personal consumption to shocks in housing wealth, while directionally positive, is not statistically significant and fades within a few quarters. This suggests that the housing wealth effect, although present, is limited in its ability to sustain household consumption over time. Second, the Granger causality tests indicate strong directional relationships. Disposable income Granger-causes both personal consumption and housing wealth at the 1% significance level, while personal consumption also Granger-causes housing wealth. These results emphasize the feedback mechanisms among key macroeconomic variables, reinforcing the importance of using a system-based estimation approach such as PVAR. Despite these dynamic interactions, housing wealth does not Granger-cause either consumption or income, further underlining the relatively weak role of asset price movements in driving household spending. Third, the variance decomposition and impulse response analysis provide further evidence that shocks to disposable income explain a larger portion of the variability in personal consumption than housing wealth shocks. This suggests that during periods of macroeconomic uncertainty, such as the recent inflationary episode or past crises, household consumption is more sensitive to income developments than to asset valuation changes.

Overall, the findings suggest that policies aimed at stabilizing household income, rather than asset markets, may be more effective in supporting consumption and aggregate demand. As European economies navigate post-pandemic recovery and ongoing inflationary pressures, understanding the limited and short-lived nature of the housing wealth effect is crucial for designing appropriate macroeconomic and fiscal policies.

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HOW TO MAKE FORECASTS IF A DRASTIC DROP OCCURS IN A TIME SERIES: A CASE STUDY

Liljana Ferbar Tratar
University of Ljubljana, Faculty of Economics
Kardeljeva pl 17, 1000 Ljubljana, Slovenia
E-mail: liljana.ferbar.tratar@ef.uni-lj.si

Abstract: The goal of our research is to analyze the impact of a sudden drop in the level and volatility of a time series on forecasts calculated immediately after anomaly on one hand, and on forecasts calculated for a later period during recovery on the other. For the purpose of this research, we examine monthly data on the arrival of Chinese passengers at Ljubljana Airport for the period 2014-2024. We analyze the impact of the pandemic (drop in level and seasonal indexes of the time series in 2020) on forecasts for 2021 (interval immediately after the anomaly) and forecasts for 2024 (recovery interval). For the baseline forecast, we use the multiplicative Holt-Winters (MHW) method and derive three additional variants that incorporate different adjustments within the identified drop interval. To detect the drop/anomaly interval (level and/or trend and/or volatility break), we use percentage deviation. Based on the results of the MHW method and its three variants, we conclude that there is no single best one to be used for short and long term forecasting.

Keywords: forecasting, Holt-Winters method, anomaly detection.

1 INTRODUCTION

Making accurate forecasts is difficult under any circumstances, but forecasting in extreme conditions of uncertainty presents an even greater challenge. This issue was addressed in [3], where various complex models were used to enhance the accuracy of short-term forecasting.

In this paper, we present a simple method for identifying the anomaly interval within the time series that contains sufficient historical data, allowing us to determine a pattern - the level, trend, and seasonality components. Since the MHW method considers the identified pattern for future forecasts, significant deviations of the series from its pattern will naturally lead to incorrect forecasts. For this purpose, we develope three variations of the MHW method, which appropriately adjust the level and trend within the anomaly interval, take into account the percentage of the drop, or forecast using the naive method.

The remainder of the paper is organized as follows. We begin with the description of the forecasting procedures (see Section 2), then in Section 3, we present the data, methodology and results which allow us to compare different variants of forecasting methods and choose the most appropriate one. Finally, in Section 4, after the conclusions of our paper, some further research steps are suggested.

2 FORECASTING METHOD

The Holt-Winters methods of exponential smoothing are based on three smoothing equations: equation for level, for trend and for seasonality. The decision as to which method to use depends on time series characteristics: the additive method is used when the seasonal component is constant and the multiplicative method is used when the size of the seasonal component is proportional to the trend level [2]. Since our data exhibits a multiplicative type of structure, we use the multiplicative Holt-Winters method (MHW) as the baseline forecasting method in our analysis:

$$L_t = \alpha(Y_t/S_{t-s}) + (1-\alpha)(L_{t-1} + b_{t-1}) \quad (1)$$

$$b_t = \beta(L_t - L_{t-1}) + (1-\beta)b_{t-1} \quad (2)$$

$$S_t = \gamma(Y_t/L_t) + (1-\gamma)S_{t-s} \quad (3)$$

$$F_{t+m} = (L_t + b_t m) \cdot S_{t-s+m} \quad (4)$$

where L_t – estimation of variable in time *t*, Y_t – observed value, b_t – trend estimation of time series in time *t*, S_t – estimation of seasonality in time *t*, F_{t+m} – forecast in time *t* for *m* period ahead, α, β, γ – smoothing parameters in the interval [0, 1], *m* – number of forecasted periods, *s* – duration of seasonality (for example, number of months or quarters in a year). The equation (4) is used to calculate the forecast for *m* periods ahead.

To determine initial estimates we need two whole data seasons (that is, 2*s* data):

$$L_s = \frac{1}{s}(Y_1 + Y_2 + \cdots + Y_s) \quad (5)$$

$$b_s = \frac{1}{s}\left(\frac{Y_{s+1}-Y_1}{s} + \frac{Y_{s+2}-Y_2}{s} + \cdots + \frac{Y_{s+s}-Y_s}{s}\right) \quad (6)$$

$$S_1 = Y_1/L_s,\ S_2 = Y_2/L_s, \ldots,\ S_s = Y_s/L_s \quad (7)$$

The method is proven to be (regarding costs and the calculation itself) comparable with more complex methods (for example Box-Jenkins) [4]; in some cases the results gained with Holt-Winters methods were better than those obtained through more complex methods [2].

Models based on pre-break dynamics are often unreliable for predicting post-break periods [6]. For the purpose of improving forecast accuracy in the case of a drastic change in the time series, we derive three variants of the MHW method, which differ from the basic method only in the anomaly interval:

- **Variant 1**: $\boldsymbol{L_t = Y_t}$ and $\boldsymbol{b_t = 0}$,
- **Variant 2**: (additionally reduced seasonal index, $\boldsymbol{k}$ – the number of seasons): $\boldsymbol{L_t = Y_t,\ b_t = 0}$ and $\boldsymbol{F_{t+1} = (L_t + b_t) \cdot S_{t-s+1} \cdot (Y_t/Y_{t-s\cdot k})}$,
- **Variant 3**: $\boldsymbol{F_{t+1} = Y_t}$ (the Naïve method)

3 CASE STUDY

3.1 Data

For research we used monthly data on the arrivals of Chinese passengers at Ljubljana Airport for the period 2014-2024. We acquired the data from SURS [7].

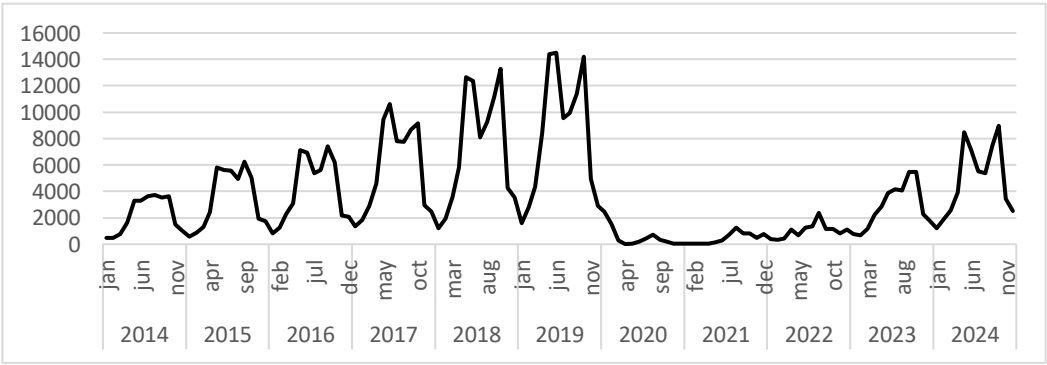


Figure 1: Chinese passanger arrivals in Slovenia from January 20014 to December 2024.

In Figure 1 we observe that the data for the period 2014-2019 corresponds to a multiplicative data model, where we identify a rising trend and seasonal fluctuations that increase along with the level (trend) of the series [1]. There is sudden drop in the number of arrivals in 2020, and up until 2024, the number of arrivals does not return to pre-pandemic levels.

3.2 Methodology

The obtained data was organized into two time series: the first covers data from 2014 to 2021, and the second from 2014 to 2024, allowing us to study the impact of the drop on forecasts in 2021 (immediately after the drop) and on forecasts in 2024 (during the recovery period). Both time series were split into initialization (the first two years, from January 2014 to December 2015), training (period from January 2016 to December 2020 (2023)), and testing subset (period from January 2021 (2024) to December 2021 (2024)). To calculate forecasting values of the testing subset we use a short-term and a long-term forecasting approach for twelve ($m = 12$) months ahead:

$$F_{t+m},\ \ m = 1, 2, \dots, 12 \text{ (monthly forecasting)} \tag{8}$$

where t represents December 2020 in the first time series and December 2023 in the second.

It is well known that there is no "one-size-fits-all" model in anomaly detection [5]. As contextual anomalies require understanding of the data in its given context, such as seasonality or a known pattern, models that can capture these patterns in data are usually more effective.

Since our data exhibits a recognizable pattern, which is accounted for in the smoothing equations of the MHW method, the anomaly interval can be determined in a very straightforward manner. Our proposed anomaly detection algorithm does not require a predefined break point; instead, it alerts there is an anomaly when the ratio of the observed data at time t to the data from the previous season $t - s$ (e.g., the ratio of the value $Y_{jan2021}$ to the value $Y_{jan2020}$) falls below a predefined threshold (e.g., 50%). If $(Y_t/Y_{t-s} - 1) < 0$ and $|Y_t/Y_{t-s} - 1| > 0.5$, $s = 12$.

For the purpose of our study, we used three different anomaly intervals, presented in Figure 2, for which we determined the alerted values according to the requirement:

$$(Y_t/Y_{t-s\cdot k} - 1) < 0 \text{ and } |Y_t/Y_{t-s\cdot k} - 1| > 0.5,\ s = 12,\ k = 1, 2, 3. \tag{9}$$

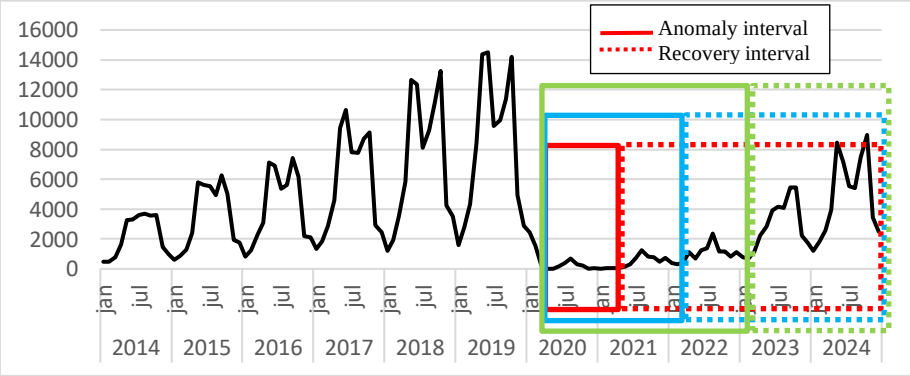


Figure 2: Anomaly and recovery intervals.

For the evaluation of the forecasting methods we applied the following forecasting accuracy measures; the root mean square error (RMSE) and the mean absolute error (MAE):

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(Y_t - F_t)^2} \tag{10}$$

$$\text{MAE} = \frac{1}{N}\sum_{t=1}^{N}|Y_t - F_t| \tag{11}$$

where Y_t represents actual value, F_t forecasted value and N number of samples. Of course, the lower values of RMSE or MAE represent a better forecasting performance.

3.3 Forecasts for 2021

Due to the drastic drop in 2020, the forecasted values for 2021, calculated using the MHW method, are inaccurate. Moreover, the forecasts are negative, as they take into account the series level and trend values from December 2020 (see Equation 4). To facilitate the comparison of all three variants of the MHW method across three different anomaly intervals, we adjusted the negative forecasted values to 0 (denoted as MHW_0). This rule, replacing negative forecast values with 0, was also applied to all other modifications of the MHW method.

Table 1 presents the RMSE and MAE results for the training and test sets (long-term and short-term), obtained using the MHW and MHW_0 methods and all three variants of MHW. We observe that RMSE on the test set for long-term forecasts significantly decreases (by 80%) when using MHW_0 instead of MHW, while there is no difference for short-term forecasts. Similarly, MAE on the test set decreases (by 50%) for long-term forecasting with MHW_0, whereas for short-term forecasting, the error increases.

The table also presents the RMSE and MAE results obtained by using three modifications of the MHW method (see Section 2), where condition (9) considers $k = 1$ for the anomaly interval length. In the notation MHW_k_v, k represents the length of the anomaly interval, while $v = 1, 2, 3$ denotes the variant of the MHW method. We observe that RMSE and MAE decrease with the use of any of the three MHW variants on both the training and test sets for long-term forecasting. The lowest values are achieved with the MHW_1_3 method, where RMSE and MAE on the test set decrease by 38% compared to MHW_0. For short-term forecasting, errors also decrease when using the modified MHW methods, with the lowest RMSE and MAE observed with the MHW_1_1 method, where RMSE decreases by 92% and MAE by 95% compared to MHW_0.

Figure 3 presents graphs of data and forecasts, obtained with three different methods (for RMSE and short-term forecasting).

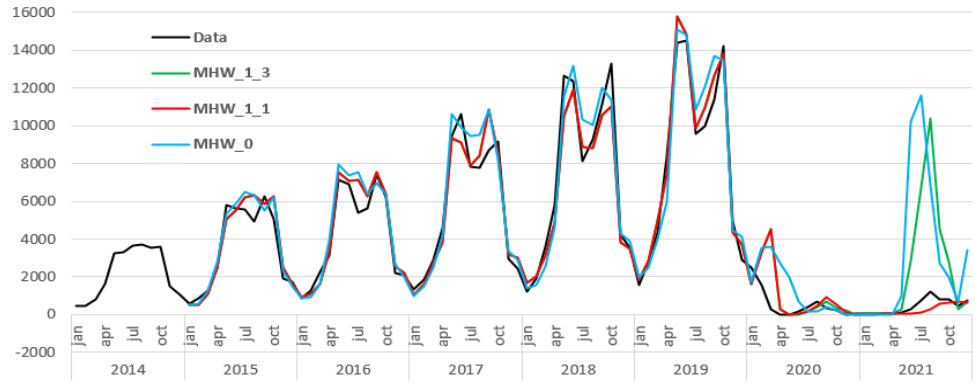


Figure 3: Graphs of data and forecasts, obtained with MHW_0, MHW_1_1 and MHW_1_3.

Table 1: MSE and MAE results for period 2014-2021.

2014-2021	***Training set***		***Test set (2021) Long-term***		***Test set (2021) Short-term***	
	RMSE	***MAE***	***RMSE***	***MAE***	***RMSE***	***MAE***
MHW_0	1.185	848	594	421	4.657	4.523
MHW_1_1	967	637	568	414	349	214
MHW_1_2	968	635	568	421	349	220
MHW_1_3	965	633	366	262	3.459	2.168

3.4 Forecasts for 2024

As seen in Table 2, the RMSE and MAE values decrease with the use of modified variants of the MHW method compared to the MHW_0 method (the only exceptions are four MAE values; at MHW_3_1, MHW_3_2, and MHW_3_3 in long-term forecasting and at MHW_3_3 in short-term forecasting). 5ort he training set, we can summarize that the RMSE and MAE errors are the smallest in modified MHW_*k*_3, with errors further decreasing as the anomaly interval length (*k*) increases (up to 24%).

On the test set, the lowest RMSE and MAE values for long-term forecasting are achieved with the MHW_2_3 method (an improvement of 31% in RMSE and 24% in MAE compared to MHW_0). For short-term forecasting, the lowest RMSE value is obtained with the MHW_2_3 method (a 24% improvement), while the lowest MAE value is achieved with the MHW_1_3 method (a 20% improvement).

Table 2: MSE and MAE results for period 2014-2024.

2014-2024	***Training set***		***Test set (2024) Long-term***		***Test set (2024) Short-term***	
	RMSE	***MAE***	***RMSE***	***MAE***	***RMSE***	***MAE***
MHW_0	1.103	752	2.624	1.721	2.358	1.534
MHW_1_1	929	585	1.919	1.338	1.825	1.324
MHW_1_2	930	584	1.919	1.393	1.824	1.342
MHW_1_3	910	584	1.894	1.394	1.846	1.219
MHW_2_1	982	628	1.971	1.415	1.975	1.312
MHW_2_2	992	657	2.081	1.414	2.038	1.312
MHW_2_3	904	572	1.793	1.299	1.783	1.299
MHW_3_1	953	639	2.503	2.019	1.974	1.401
MHW_3_2	1.021	731	2.502	2.019	1.974	1.401
MHW_3_3	868	570	2.126	2.081	2.121	2.081

3.5 Research findings

Based on RMSE and MAE as performance measures, the MHW_2_3 method is identified as the best forecasting method for Chinese visitor arrivals in Slovenia when we need forecasts (long-term or short-term) for the recovery period (2024). If we need forecasts for 2021 (immediately after the drop), the most suitable method is MHW_1_3 for the training set and test set in long-term forecasting, while for short-term forecasting, MHW_1_1 is preferred. Figure 4 presents data and forecasts for the best-performing methods during the period 2021-2024.

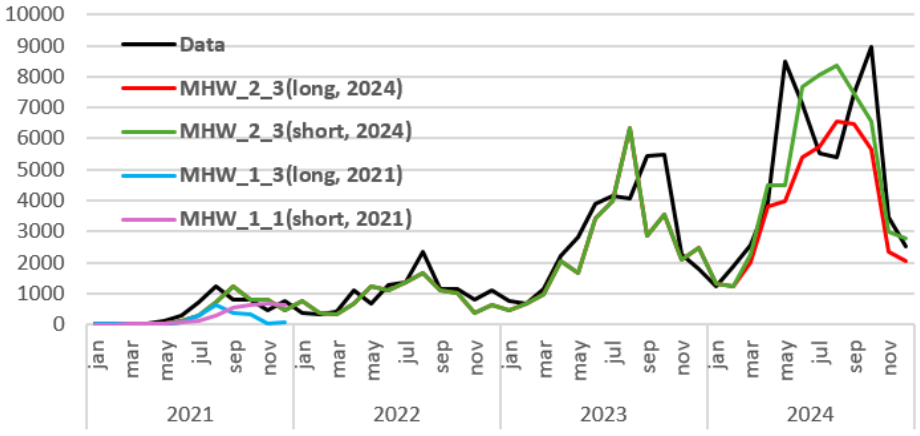


Figure 4: Forecast for Chinese turist arrivals in Slovenia for 2021 and 2024.

4 CONCLUSION

Making accurate forecasts is difficult under any circumstances, but forecasting in extreme conditions of uncertainty presents an even greater challenge. We presented a simple algorithm for recognizing the anomaly interval and derived three variants of the MHW method, which differ from the basic method only in the anomaly interval. We conclude that by applying different variants of the MHW method, forecasting errors - RMSE and MAE - can be reduced. Moreover, as the lenght of the anomaly interval increases, the errors decrease even further.

For further research, we suggest that optimization using the initial parameters is used with all three variations of MHW methods. Additionally, we propose an analysis of the impact of modifying the additive HW method, specifically in cases where the time series exhibits an additive data structure. This investigation could provide deeper insights into potential error reduction and improved forecasting accuracy.

Acknowledgement
The author acknowledges the support of the Slovenian Research and Innovation Agency, program P5–0161.

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HIGH PROFILE CORRUPTION AND TRUST IN POLITICS

Ria Ivandić
Faculty of Economics and Business, J. F. Kennedy Square 6, 10 000 Zagreb
E-mail: *rivandic@efzg.hr*
Velibor Mačkić
Faculty of Economics and Business, J. F. Kennedy Square 6, 10 000 Zagreb
E-mail: *vmackic@efzg.hr*

Abstract: This paper investigates the consequences of corruption charges of Prime Minister of Croatia Ivo Sanader on political preferences and trust. Using three waves of the European Social Survey data in Croatia (treatment) and in Slovenia, Bulgaria, Romania, Slovakia and Hungary (control) we report that the corruption scandal had a significantly large and negative effect on trust in politicians, the serving parliament and politics overall. Difference in differences methodology also yields that this decrease in trust is mainly driven by young voters that are on the Left.

Keywords: difference in differences design, Croatia, corruption, trust

1 INTRODUCTION AND THEORETICAL FRAMEWORK

On the 10th December 2010, Ivo Sanader was arrested in Austria fleeing as the Croatian parliament voted to remove his immunity from prosecution by the Croatian Bureau for Combating Corruption and Organized Crime (USKOK) just the day before. This was less than six months after Sanader unexpectedly announced his resignation as the Prime Minister of Croatia. At that time he was the longest-serving prime minister since independence, holding the office for over five and a half years from 2003 to 2009, following two general elections he won as the leader of centre-right HDZ. Following the arrest, he was charged with corruption[8].

Although, the Sanader corruption scandal was a very public and very high profile case, corruption in the public sector was not uncommon ([2], [5], [13]). As a major political, social and economic phenomenon corruption attracted vast theoretical interest ([1], [9], [11]) with its negative effects on trust in politics being raised by politicians, journalists and the general public [4]. Citizens' absence of trust in political structures can take two forms – mistrust and distrust – [7] with distrust dominating when it comes to political parties and politicians [12].

This leads us to ask how a high case corruption case changes the trust in politicians and politics. Moreover, our research aims to shed light into which voters' political preferences are likelier to be affected by corruption in the public sector.

2 DATA AND METHODOLOGY

The main source of data for this research is the European Social Survey (ESS) – an academically driven cross-national survey that has been conducted across Europe since its establishment in 2001. The survey measures the attitudes, beliefs and behaviour patterns of diverse populations across Europe. Croatia participated in the ESS in round 4 (2008/2009) and round 5 (2010/2011), and later on in round 9 (2018). Ivo Sanader resigned in July 2009 and was arrested in December 2009, exactly in between the first two rounds.
As the control group we include Slovenia, Bulgaria, Romania, Slovakia and Hungary, all countries of the third wave of democratization [10]. The choice of countries in the control group is informed mainly by the similarities in political and economic history [3], and their neighbouring status to Croatia. Serbia, Montenegro and Bosnia and Herzegovina were not part of the ESS until 2018 and hence cannot be included as a control group.

In the ESS, the following question is asked about trust in people: "Generally speaking, would you say that most people can be trusted, or that you can't be too careful in dealing with people? Please tell me on a score of 0 to 10, where 0 means you can't be too careful and 10 means that most people can be trusted." In the Politics and Government section, questions about trust are asked in the following way: "Please tell me on a score of 0 to 10 how much you personally trust each of the institutions I read out, where 0 means you do not trust an institution at all, and 10 means you have complete trust." Firstly, how much do you personally trust i) the parliament ii) the legal system iii) the police iv) politicians v) political parties vi) the European Parliament vii) the United Nations. We also apply a vector of controls. First, basic demographic and socioeconomic indicators are included for gender (base: female), age (years), age squared, religion, education (five categories), employment status (five categories) and income (income percentile in distribution).

For our analysis, we argue that the Sanader arrest and charge was an unexpected shock to the Croatian citizens and their preferences following December 2009. Even when he resigned in July 2009, the majority of press and commentators suspected this was motivated by wanting to run for the president of Croatia rather than any unraveling scandal. Most certainly, when citizens took part in the face-to-face interviews in Croatia (from December 2008 until March 2009 – round 4), they could not have anticipated or known what was about to happen by the end of the year. The main results of this research are obtained using a difference in differences (DiD) research design. DiD research design looks at the differential effect of a treatment by comparing the average change over time in the outcome variable for the treatment group, compared to the average change over time for the control group.

The difference-in-differences (DiD) equation:

$$Y_{i,t} = \beta_1[Treated_i \times Post_t] + \delta_t + \alpha_i + u_{i,t} \tag{1}$$

where $y_{i,t}$is the dependent variable of the analysis, namely the trust in country i at time t; α_i is a dummy for every country i; $Post_t$ is a dummy variable indicating that the year t is equal to 2010; and δ_t is a dummy for every year t.

This method allows causal identification when using observational data by assuming that in the absence of treatment, the unobserved differences between treatment and control groups are the same over time (parallel trends). Treatment induces a deviation from this common trend. The difference in differences research design allows the treatment and control areas to differ in characteristics that effect the treatment, as this difference is captured in the country fixed effect α_i. Moreover, general changes in time or policy that affected all the areas equally are captured in the δ_t that control for unobserved but area-invariant reforms and changes in time. In sum, this strategy punts on comparisons in levels, while requiring the counterfactual trend behaviour of treatment and control groups to be the same.

3 RESULTS AND DISCUSSION

How did the unraveling of the high profile corruption case, at the top of government in Croatia, changed citizens' preferences? We first show the results in Table 1 for the main indicators of trust in political institutions. It shows that Sanader's arrest and prosecution had a highly significant and negative effect on trust in politicians, trust in political parties and trust in the country's parliament. As discussed the survey measures trust on a scale from zero to ten. The high profile corruption case led to a decrease by around one point on this scale, significant under 0.01 percent. Measured as a deviation from the mean of these variables, these decreases amount to around 30 percent reduction in trust across politicians, parties and the parliament. This is a very large effect in magnitude. When we add the vector of individual level socio-demographic controls, the point estimates remain exactly the same, confirming that our research design doesn't suffer from omitted variable bias.

Table 1: Main Results

VARIABLES	(1) Trust in politicians	(2) Trust in political parties	(3) Trust in country's parliament
TreatedPost	-0.98***	-0.94***	-1.36***
	(0.10)	(0.10)	(0.12)
Observations	18,865	18,751	18,824
R-squared	0.06	0.06	0.07
Country Fixed Effect	YES	YES	YES
Year Fixed Effects	YES	YES	YES

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

In additional analysis (available upon request due to page limitations), we also show the results on other outcome variables (trust in the UN, trust in the legal system and trust in the police). As to be expected, there is a decline in trust in the legal system and police. However, we find no significant change in trust in the United Nations which also confirms the plausibility of our research design as Sanader's corruption should not have affected trust in the UN.

We investigate three sources of heterogenous effects, i.e. certain voters being more likely to change their trust in institutions and politics as a result of the corruption case. We test whether the level of education (high versus low educated), age (below 30 or above 30) or ideology (left versus right voters) have a different sensitivity to the case. We find that the corruption scandal did not differently affect the trust in politics of highly educated individuals. When we tested whether the scandal differently affected the trust in politics of young individuals, we find that for young individuals the trust in parliament dropped more than for older voters. The coefficients on trust in politicians and political parties are also negative, yet insignificant.

Table 2: Heterogeneous effects: ideology

VARIABLES	(1) Trust in parliament	(2) Trust in political parties	(3) Trust in politicians
Treated x Post x Left	-0.44***	-0.22*	-0.14
	(0.15)	(0.13)	(0.13)
Left	0.22***	0.24***	0.23***
	(0.05)	(0.05)	(0.05)
Treated x Post	-0.75***	-0.66***	-0.83***
	(0.09)	(0.08)	(0.07)
Observations	18,824	18,751	18,865
R-squared	0.01	0.01	0.01
Country Fixed Effect	YES	YES	YES
Year Fixed Effects	YES	YES	YES

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Finally, we construct an ideological distinction of Left, Right and Center based on response to the question of how close to the right or left respondents feel. We generally see that Left individuals report higher trust in parties, politicians and parliament. Yet after the Sanader scandal in Croatia, Left individuals' trust in the parliament dropped even more sharply than the rest of the electorate as shown in first row in Table 2., a result in line with [6]. In a robustness check (available upon request due to page limitations), we define the control group

including Slovenia, Bulgaria and Romania, but our main results remain unchanged.

4 CONCLUSION

Our analysis indicates that a high profile corruption case significantly lowers trust in politicians, parties and parliament. Applying the DiD method enabled us to draw a clear causal effect between the Sanader corruption scandal and the resulting levels of trust in both the political and legal system and its actors. We also identified a clear mechanism behind these outcomes. Young leftist voters, the ideological opposite of the party of Ivo Sanader, are driving these results. Future research should focus on corruption scandals that succeeded and arrests of ministers serving in the government in Croatia.

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IS IT POSSIBLE TO UNITE (BY MUSIC) WITH THE CURRENT VOTING SYSTEM AT THE EUROVISION SONG CONTEST?

Nikola Kadoić
University of Zagreb Faculty of organization and informatics
Pavlinska 2, Varaždin, Croatia
E-mail: nkadoic@foi.hr

Nikolina Žajdela Hrustek
University of Zagreb Faculty of organization and informatics
Pavlinska 2, Varaždin, Croatia
E-mail: nikolina.zajdela@foi.hr

Bojan Žugec
University of Zagreb Faculty of organization and informatics
Pavlinska 2, Varaždin, Croatia
E-mail: bojan.zugec@foi.hr

Abstract: The permanent slogan of one of the most famous song contests, the Eurovision Song Contest, is *United by music*. However, recent research shows that ESC fans are increasingly dissatisfied yearly due to the voting model. The organiser returned the jurors to the competition to diminish some of the weak points of the only-audience voting model. Juries must ensure objective quality assessment. In this research paper, we investigate the similarity between the juries' votes among different countries in the last two ESC and compare them to the results related to the period between 2017 and 2023. The results indicate that the opinions of jurors from different countries are only weakly correlated – the similarity between them is very low, opposite to the expectations that they are able to evaluate quality, which should result in high correlations. We concluded that, at the practical level, the jury's role remained unclear also in 2024 and 2025 and that the voting system requires some improvements.

Keywords: voting, Eurovision song contest, Eurosong, audience, juries

1 INTRODUCTION

The Eurovision Song Contest (Eurosong) is a hugely popular spectacle that originated in 1956, in the post-war era, with the fundamental goal of achieving unity "through music". From the very beginning, it has been organized by the alliance of public media services, the European Broadcasting Union (EBU), which was established in 1950 in Geneva to enhance radio and television services through cooperation and content exchange, including news and sports (https://www.ebu.ch/about/history). Although established as a prestigious event primarily focused on music, over the years it has also gained recognition as an important platform for "cultural diplomacy" and as a tool of "soft power" for countries. This means that songs and performances can subtly promote national culture and interests, even without direct governmental support [19, 20]. Over time, participation has been extended to countries beyond Europe, and today, live broadcasts via the internet reach audiences worldwide. Additionally, digital communication enables the audience to engage through social media informing themselves, expressing opinions, and interacting—creating a sense of community while also potentially influencing voting preferences [15]. Throughout the years, the voting system of this spectacle has evolved. In certain periods, jury voting predominated; later, public voting gained prominence, and eventually, a "combined voting system" was introduced—50% televoting and 50% jury voting [13]. Numerous studies indicate that various factors influence the voting process and the selection of the winning song, including bloc voting, host country, performance order, linguistic proximity between the song and nation, performer's gender, and even levels of hostility between countries, as well as which voting and scoring system is in use.

This research is a follow-up to previous research related to the objective evaluation of songs by juries [14]. Objective evaluation of songs is a core reason for the' return of juries to Eurovision. However, some previous research concluded that juries see the quality of songs differently. Also, previous research argued about the definition of quality songs and whether juries can see it. In this analysis, we analysed the voting results in 2024 and 2025.

In Section 2, a brief literature review of research published in the last two years regarding the Eurosong is given. In Section 3, the methodology is presented. Section 4 presents the conclusions.

2 LITERATURE REVIEW

A wide range of studies indicates that Eurovision voting is shaped by multiple factors, including social media influence, political and cultural dynamics, and the design of the voting system itself.

Social media plays a significant role in shaping public opinion and voting preferences. Forciniti, Andrea and Magda (2024) applied text mining on Twitter (now X) posts to analyze public sentiment during Eurovision, revealing a strong correlation between online discourse and public voting rankings [9]. Similarly, Mustapić and Dabo (2024) conducted a content analysis of Eurovision's official social media channels, concluding that interactive posts (likes, shares, comments) actively influence user perceptions. Over 58% of surveyed users rated the influence of online communication on their Eurovision opinions as good or very good. Social media thus serves not only as a feedback loop but as a mobilization platform that shapes real-time voter behavior [15].

Contrary to the European Broadcasting Union's (EBU) claim of Eurovision being apolitical, several scholars argue otherwise. Aleksić (2024) and Vuletić (2025) highlight how songs function as instruments of cultural diplomacy, reflecting national identities and geopolitical narratives [2, 16]. Carniel (2024) emphasizes that fans act as political agents through participatory diplomacy, challenging borders and official narratives [4–6]. Studies also show that bloc voting and cultural affinity [18, 19] remain strong influencers. Cultural distance—including language, religion, and historical relations—negatively affects voting outcomes, while cultural diplomacy positively influences public votes but not jury decisions.

Gadea (2024) underlines affective voting factors such as diaspora connections, strategic voting, and bandwagon effects, further politicizing the contest [10]. Zahavi and Ariely (2024) demonstrate that hosting Eurovision can serve as a tool of public diplomacy, citing increased pro-EU sentiment in Israel following its role as host [20].

The concept of "songwashing"—strategically modifying songs to align with acceptable narratives—has also been explored. The 2024 disqualification of Israel's song "October Rain" for political content and its replacement with "Hurricane" [6] illustrates how countries navigate EBU rules to maintain presence and influence. Welslau and Selck (2023) note that while Ukraine used Eurovision for cultural exchange even during conflict, Russia abstained after being banned, highlighting the contest's geopolitical relevance [17].

Eurovision also promotes diversity and inclusivity, often blurring lines between political messaging and humanitarian expression. Some researchers assert that the contest is a platform for normalizing gender and sexual diversity and resisting traditional political norms [3, 7, 11].

Voting behavior is shaped not just by content but by system complexity. Dellis and Kröger (2024) found that the Borda Count, used in Eurovision, encourages more optimal and sincere voting compared to more complex systems like Instant Runoff Voting (IRV). However, voter decisions are still subject to heuristics under pressure [8].

Jury influence has drawn criticism. Kadoić, Žajdela Hrustek & Gligora Marković (2025) found only partial objectivity in jury evaluations, with juries diverging from public preferences

[14]. The controversial 2024 results, where Croatia won the public vote but Switzerland claimed victory, prompted further critique of the voting model. Kadoić (2024) argues for reforms, citing unequal point allocations, unclear song quality definitions, small jury size, and lobbying vulnerabilities [12].

In predictive research, Adamska and Reiss (2025) developed a multimodal model integrating song characteristics, public appeal, and voting data. They confirmed the influence of performance order and persistent bloc voting tied to geography, language, and culture [1].

3 METHODOLOGY

Since this is the continuation of the previous research, we applied the same methodology [14], but on data from 2024 and 2025. The calculations were implemented in Excel. The datasets were taken from the website https://eurovisionworld.com/. During the research, we calculated the Spearman rank correlations (P=public, J=jury, C=country, E=ESC, F=P+J):

- between Eurovision jury and public results for each year (PE-JE),
- between jury and public votes at the national level (same country) for each year (JC-PC),
- between national audiences and European audience for each year (PC-PE),
- between national juries and the European jury for each year (JC-JE),
- among countries (jury JC-JC, public PC-PC and final votes, FC-FC), for each year.

Additionally, at the level of analysis among countries, we counted the number of cases when the correlation between two national public rankings was higher, equal to, or lower than the correlation between two national jury rankings.

The main research question: Is it possible to unite (by music) with the current voting system at the Eurovision Song Contest?

4 RESULTS

Figure 1 presents correlation between public and jury votes. The results show that there is a significant variation in results by public and jury in the dataset ($0.22 < |r| < 0.63$). The average between European public and jury votes was 0.384 in 2017-2023, while in 2024-2025 it was 0.423.

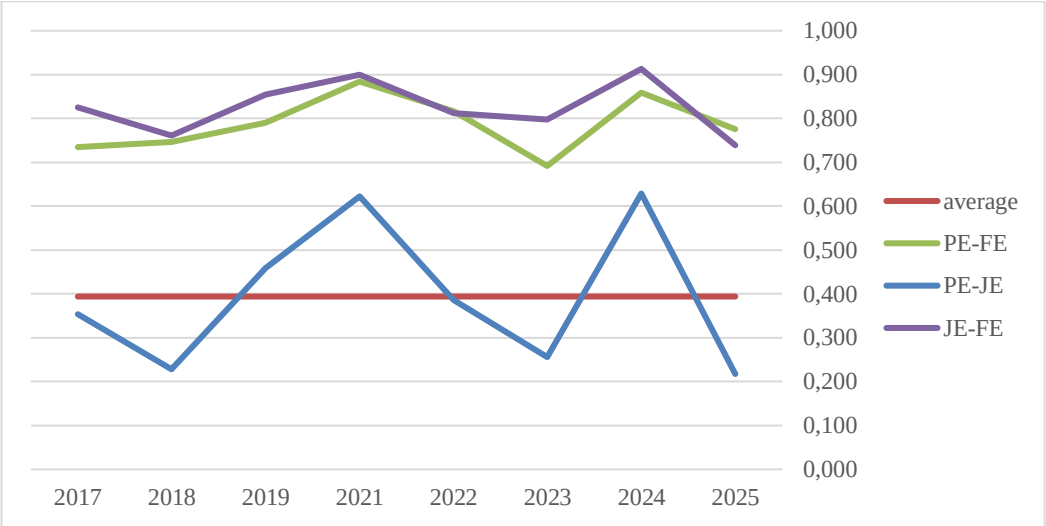


Figure 1: Correlation between public and jury votes per year

Table 1 presents the average, minimum and maximum correlations between jury and public votes at the national level for each year (JC-PC). We can see that in the last two years, we have witnessed very similar results to those in the previous six years (the average is 0.35, similar to 0.336 in the last six years).

Table 2 presents the average, minimum and maximum correlations between national juries and European jury for each year (JC-EC). We can see that in the last two years, we have witnessed very similar results to those in the previous six years (the average is 0.53, similar to 0.52 in the last six years). Table 3 presents the average, minimum and maximum correlations between national audiences and European audience for each year (PC-PE). Again, we can see that in the last two years, we have witnessed very similar results to those in the previous six years (the average is 0.57, similar to 0.61 in the last six years).

Table 1: Analysis of correlations between the public and jury per year

PC-JC	2017	2018	2019	2021	2022	2023	2024	2025
average r	0.41	0.19	0.34	0.41	0.32	0.35	**0.49**	**0.21**
max r	0.81	0.75	0.81	0.76	0.76	0.85	**0.69**	**0.82**
min r	-0.23	-0.31	-0.21	-0.06	-0.12	-0.17	**-0.35**	**-0.07**

Table 2: Analysis of correlations between the national juries and European jury

JC-JE	2017	2018	2019	2021	2022	2023	2024	2025
average |r|	0.52	0.45	0.54	0.54	0.56	0.50	**0.58**	**0.47**
max |r|	0.75	0.74	0.76	0.76	0.78	0.69	**0.72**	**0.76**
min |r|	-0.10	-0.04	0.06	0.21	0.07	0.24	**0.04**	**0.34**

Table 3: Analysis of correlations between the national audiences and the European audience

PC-PE	2017	2018	2019	2021	2022	2023	2024	2025
average |r|	0.69	0.51	0.57	0.63	0.67	0.59	**0.68**	**0.58**
max |r|	0.81	0.78	0.77	0.81	0.82	0.76	**0.77**	**0.83**
min |r|	0.33	0.02	0.26	0.40	0.27	0.10	**0.31**	**0.17**

Table 4: Analysis of correlations between the countries (juries, audiences and final results)

	2017	2018	2019	2021	2022	2023	2024	2025
PC-PC	0.58	0.29	0.38	0.46	0.53	0.44	**0.60**	**0.36**
JC-JC	0.33	0.21	0.30	0.31	0.33	0.29	**0.39**	**0.23**
FC-FC	0.42	0.18	0.29	0.37	0.42	0.35	**0.52**	**0.22**

Table 5: Analysis of the results among the countries

	2017	**2018**	**2019**	**2021**	**2022**	**2023**	**2024**	**2025**	**T**
Number of countries	42	43	41	39	40	37	37	37	
Total number of cases	861	903	820	741	780	666	666	666	6103
The number of cases when the correlation between two national public rankings was higher than the correlation between two national jury rankings	709	529	479	518	570	463	529	421	4218
The number of cases when the correlation between two national jury rankings was higher than the correlation between two national public rankings	149	369	337	217	207	200	134	234	1847
The number of cases when correlation between two national public rankings was equal to the correlation between two national jury rankings	3	5	4	6	3	3	3	2	29

Furthermore, at the level of each year, we calculated correlations (1) among all national juries (JC-JC), (2) among all national audiences (PC-PC), and (3) among all national final results. The results are given in Table 4. The trend established in the 2016-2023 period continued in 2024 and 2025. Finally, we calculated the cases when the correlation between two national public rankings was higher, equal to, or lower than the correlation between two national jury rankings. The results are presented in Table 5.

5 CONCLUSION

The results obtained for the Eurovision Song Contests in 2024 and 2025 confirm findings from previous studies covering the period from 2016 to 2023. If juries are genuinely capable of recognizing musical quality, and if discernible differences in song quality exist, we would expect higher consistency in jury assessments—reflected in higher average correlations between jury components (JC-JE). Conversely, public voting, typically influenced by diaspora and neighborly ties, should demonstrate greater variability due to differing regional contexts, resulting in lower average correlations (PC-PE). However, our data reveal the opposite: jury opinions exhibit significantly greater dispersion than public votes.

This inconsistency raises critical concerns. In 2024, the winning entry ranked only fifth in the public vote; similarly, in 2025, the winner was the audience's fourth choice. Such outcomes challenge the perceived legitimacy of the final results and raise doubts about the unifying role of the contest. While public voting systems are not without flaws—such as potential bias or external manipulation—the jury system, as currently structured, does not mitigate these issues but instead introduces additional complications.

The events of Eurovision 2025, including controversies surrounding Israel's participation and public support, highlight vulnerabilities in the televoting process. Nonetheless, these concerns should not justify maintaining the jury system in its current form. Rather, comprehensive reform is needed. Transparent procedures, clearer criteria for assessing song quality, and a reassessment of the jury-public vote balance are essential steps toward a more equitable and credible voting system.

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INSECTS AS A SUSTAINABLE FEED ALTERNATIVE: CLUSTER ANALYSIS OF SLOVENIAN CONSUMER PERCEPTIONS

Ema Luna Karara Geršak, Lidija Zadnik Stirn, Aleš Kuhar
University of Ljubljana, Biotechnical Faculty, Jamnikarjeva 101, 1000 Ljubljana, Slovenia,
e-mails: emaluna.kararagersak@bf.uni-lj.si, lidija.zadnik@bf.uni-lj.si, ales.kuhar@bf.uni-lj.si

Abstract: Due to the growing demand for more sustainable agricultural systems, insect larvae are increasingly recognized as a promising alternative protein source in animal nutrition. However, wider adoption is limited by several factors, including consumer resistance to the use of insects in the food chain in Western societies. This study investigates how Slovenian consumers perceive insect-based feed, under what conditions they would accept it and the influence of food neophobia and environmental awareness on its acceptance. The data from an online survey was analysed using a cluster analysis, which identified three different consumer profiles with different levels of acceptance. These results provide valuable insights for the development of targeted communication and marketing strategies to promote wider acceptance of sustainable feed alternatives.

Keywords: sustainable agri-food systems, food marketing, insects in feed, consumer perceptions, cluster analysis, consumer characteristics

1 INTRODUCTION

The rapid growth of the global population and the increasing demand for meat are putting considerable pressure on the development of sustainable agricultural and food systems. As the need for livestock feed increases, researchers are looking for alternative sources to reduce dependence on limited resources such as soymeal, fishmeal and cereals [15]. The production of conventional animal feed has become increasingly problematic due to the limited availability of agricultural land and water, the competition between food, feed and energy consumption, and the environmental impact of intensive livestock farming.

In response, there is growing interest across Europe in alternative sources of nutrients such as insect-based feed. Larvae of insects such as the black soldier fly, housefly, and mealworm are seen as promising sustainable substitutes, that provide high-quality proteins and fats while reducing the environmental impact of livestock farming. Larvae can convert organic waste, including agricultural by-products, into valuable nutrients. In addition, the residual material from bioconversion (frass) can be processed into fertilizer. This form of nutrient recycling supports both the resilience of food system and more sustainable livestock farming [6].

The literature on edible insects has expanded considerably in recent years, focusing on the factors that hinder or promote consumer acceptance of this promising new feed innovation. A recent systematic review of consumer attitudes towards insect-based feed identified several consistent barriers, including perceived risks and benefits, food neophobia and concerns about the safety of products made from insect-fed animals. Other influencing factors include attitudes towards insect-based feed, environmental awareness, socio-demographic factors, personality traits, knowledge, pricing and labelling [10].

The aim of our survey-based study was to investigate how food neophobia, environmental awareness, perceived product attributes, and socio-demographic characteristics influence consumers' willingness to accept insect-based feed for livestock. In addition, we aimed to categorize consumers into different groups based on their attitudes towards insect-based feed in order to identify preference patterns and potential market profiles. In line with previous research, we hypothesise that a combination of psychological, perceptual and demographic factors will help to distinguish consumer segments with different levels of acceptance of insects in the food production chain [7].

2 METHODS

2.1 Data collection and analysis

An online questionnaire was created in the open-source platform 1ka and distributed via social media, email, and websites. Informed consent was obtained from all participants prior to their participation. Incomplete responses and participants under the age of 18 were excluded. Of the original 956 responses, a total of 764 valid questionnaires were analyzed. It is important to note that participants were recruited through a convenience sampling. Therefore, sampling bias may limit the generalizability of the results.

The questionnaire included seven sections addressing the key factors influencing consumer acceptance of insect-based animal feed identified in previous studies [10]. To reduce respondent burden and ensure consistency, we mainly used closed-ended questions with a five-point Likert scale, ranging from 1 (strongly disagree) to 5 (strongly agree). The order of items was randomised and some items were reverse-coded to improve measurement accuracy.

Food neophobia was measured using five items from the validated Food Neophobia Scale (FNS), which is widely used in consumer research [11]. Environmental awareness was assessed using ten items from relevant studies [4, 9, 12, 16]. The acceptability of insect-based feed was measured/assessed in three animal types (pigs, poultry, fish) and seven commonly analyzed characteristics of products from insect-fed animals.

Several socio-demographic variables were recoded. Age was categorised into generational cohorts: Baby boomers (1946–1964), Generation X (1965–1977), Generation Y (1977–1994), and Generation Z (1994+) [18]. Place of residence was categorized as urban or rural, and financial status was divided into below average, average, and above average.

2.2 Statistical analysis

All statistical analyses were performed with R (version 4.5.1) in three phases: (i) descriptive statistics, (ii) factor analysis, and (iii) cluster analysis. Descriptive statistics were used to summarize the main characteristics of the sample. Exploratory factor analysis identified latent constructs underlying the observed variables related to food neophobia, environmental awareness, and attitudes towards insect-based feed. Finally, a cluster analysis was conducted to segment respondents based on their acceptance of insect-based feed and associated attitudes to investigate different consumer profiles.

3 RESULTS AND DISCUSSION

3.1 Construct validation using exploratory factor analysis (EFA)

Exploratory factor analysis (EFA) was conducted separately for each construct using maximum likelihood factor analysis (MLFA) and varimax rotation to facilitate interpretation of the factors. Sampling adequacy was confirmed with KMO values ranging from 0.82 to 0.93, and Bartlett's test of sphericity was significant ($p < .001$). Items with low loadings or high uniqueness were excluded (Env1, Env8). Final models explained 54–74% of variance, with high internal consistency (Cronbach's $\alpha = 0.83–0.94$).

3.2 Cluster Analysis

To identify consumer segments, both hierarchical agglomerative clustering (Ward's method) and non-hierarchical K-means clustering were conducted. The analysis was based on standardized composite scores for the variables. Dendrograms from different linkage methods and distances (e.g., Euclidean and Manhattan distances) were evaluated to determine potential

cluster solutions. A three-cluster solution (K = 3) was validated via the elbow method and PCA visualizations (see Fig. 1).

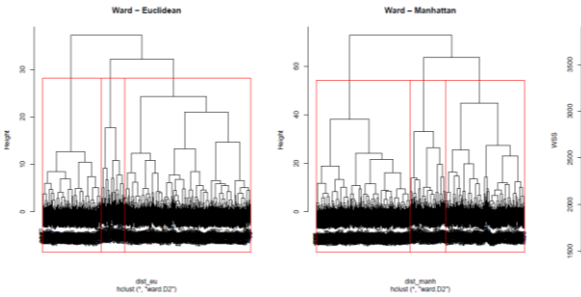


Figure 1: Dendrograms and elbow point for cluster determination

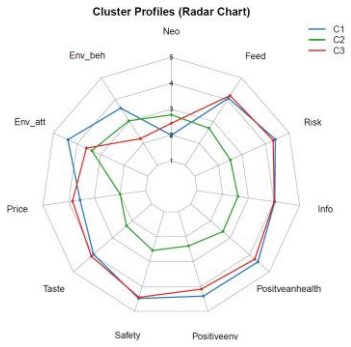


Figure 2: Radar chart showing average acceptance scores per cluster.

As a result, respondents were categorised into three different consumer groups, labelled as Environmetally Engaged Explorers (C1), Skeptical Consumers (C2), and Cautious Adopters (C3). A radar chart (see Fig. 2) shows the average Likert scale scores for each cluster in relation to the most important constructs. Although the three clusters are partially separated in the PCA plot, a degree of overlap is evident. The average silhouette score of 0.21 indicates a modest cluster structure, suggesting that the individual observations do not differ greatly from each other and may share characteristics across cluster boundaries.

3.3 Cross-tabulations and cluster profiles

To analyze the differences in socio-demographic characteristics between all three clusters, we conducted cross-tabulations for five categorical variables: gender, place of residence, education level, financial status, and generational cohort. Frequencies and percentages were calculated for each variable to highlight differences in cluster composition. The results are shown in Fig. 3.

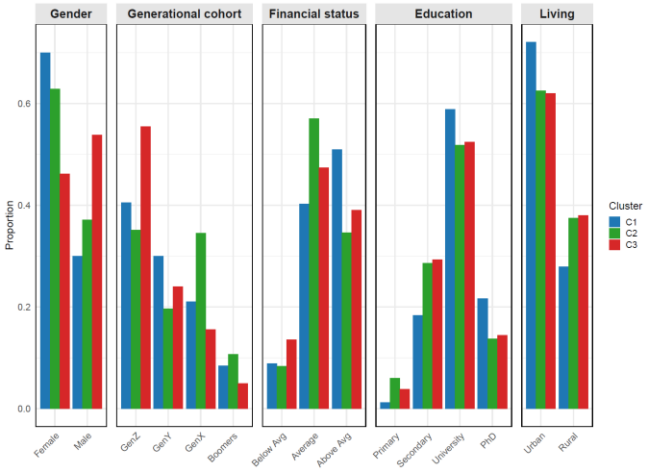


Figure 3: Distribution of socio-demographic variables by cluster.

Cluster 1: A group of respondents, labelled as **"Environmentally Engaged Explorers"**, makes up 44% of the sample. They show the lowest food neophobia and the highest environmental attitudes and behaviors. They are more willing to accept insect-based feed if it is proven to be safe for human consumption and beneficial for both animal health and the

environment. They place less emphasis on taste and price. This cluster includes more urban, highly educated people with above-average financial status and a higher proportion of women.
Cluster 2: The second group, described as "**Highly Skeptical Consumers"**, makes up 22% of the sample. This group has the highest food neophobia and the lowest level of concern for the environment. They are the least accepting of insect-based feed and rate all related factors lower than the other two groups. Older consumers, especially from Generation X and baby boomers, are overrepresented in this cluster.
Cluster 3: The final group of respondents, named **"Cautious Adopters"**, makes up 34% of the sample. People in this group have moderate food neophobia, the lowest environmental behavior, and a moderate environmental attitude. Their acceptance of insect-based feed is more strongly influenced by practical product characteristics such as taste, price and labeling. Compared to the other two clusters, this group is characterized by a higher representation of young consumers (Gen Z) and men.

3.4 Comparison with the relevant literature

Our results show that individuals with lower food neophobia are more open to insect-based feed, which is in line with previous research showing a strong association between food neophobia and rejection of unfamiliar or novel foods [5, 13, 14]. Consistent with previous studies, consumers with higher concern for the environment were also more likely to accept products derived from insect-fed animals [1, 2, 14]. These patterns are clearly reflected in cluster 1.

In all clusters, safety was found to be the most important factor for acceptance, followed by the perceived positive effects on animal health and risk control. This prioritisation is in line with previous research findings [2, 7]. In contrast, taste and price were generally considered less critical, although they were of greater importance to respondents in cluster 3. This observation is consistent with previous studies suggesting that taste and price play an important role in food choice among Generation Z, which is overrepresented in this cluster [3]. In addition, the higher proportion of men in cluster 3 supports previous findings suggesting that men tend to have more hedonistic tendencies in their food preferences compared to women [8].

4 CONCLUSIONS

This study investigated the key factors influencing consumer acceptance of insect-based livestock feed, focussing on food neophobia, environmental awareness, perceived product attributes and socio-demographic characteristics. A cluster analysis was used to identify three different consumer profiles that differ in their openness to food innovation, environmental awareness and perceived risks.

The Environmentally Engaged Explorers (C1) showed strong environmental values and low neophobia, which led to a high level of acceptance of insects in feed. Highly Skeptical Consumers (C2) had the highest neophobia, low environmental concerns and the lowest acceptance. Cautious Adopters (C3) had moderate acceptance, with their decisions influenced more by tangible attributes such as price, taste and labeling.

In this context, consumer segmentation reduces behavioral uncertainty and enables the development of targeted marketing strategies [4, 17]. By identifying different consumer profiles, this study contributes to a clearer understanding of the insect feed market and highlights the need for tailored communication approaches that focus primarily on safety for all groups, animal welfare and sustainability for environmentally conscious consumers, and value and taste for more hesitant, skeptical consumers.

5 LIMITATIONS AND FUTURE RESEARCH

As mentioned in the methods section, this study was based on a random sample and certain groups such as men, older people, people with less education or lower income and people living in rural areas were less represented. Although the results offer valuable insights, they should be considered exploratory and interpreted with appropriate caution, as they may not be generalisable beyond Slovenia. Future research should include cross-national comparisons and consider additional factors influencing acceptance, such as consumer knowledge, emotional reactions (e.g. disgust) and cultural influences. The moderate silhouette coefficient (0.21) shows that the identified clusters are distinguishable but not sharply defined. Nevertheless, the resulting consumer profiles provide a useful basis for the development of mathematical models to support more targeted marketing strategies.

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APPLICATION OF MACHINE LEARNING METHODS IN VOLATILITY FORECASTING: A COMPARATIVE STUDY

Aleš Kresta
VSB - Technical University of Ostrava, Faculty of Economics
17. listopadu 2172/15, 708 00 Ostrava-Poruba, Czech Republic
E-mail: ales.kresta@vsb.cz

Martin Pluskal
VSB - Technical University of Ostrava, Faculty of Economics
17. listopadu 2172/15, 708 00 Ostrava-Poruba, Czech Republic
E-mail: martin.pluskal.st@vsb.cz

Abstract: This paper compares traditional econometric and modern machine learning models for forecasting volatility in financial markets. Using minute-level data from the S&P 500 index, the realised volatility is computed at daily, weekly, and monthly frequencies. The predictive performance of the GARCH(1,1), FIGARCH, EWMA, and HAR models is compared with the ensemble machine learning methods XGBoost and CatBoost. The results suggest that machine learning models can outperform classical econometric models in terms of MAE, RMSE, and MAPE, particularly during volatile periods. The study demonstrates the potential of modern learning-based models in practical risk management and financial forecasting.

Keywords: volatility, GARCH, HAR, XGBoost, CatBoost, realised volatility, financial forecasting

1 INTRODUCTION

Forecasting financial market volatility is a fundamental task in quantitative finance, with broad applications in risk management, option pricing, portfolio allocation, and regulatory compliance. Traditional econometric models, such as GARCH and its variants, have long dominated this domain because of their theoretical foundation and interpretability. However, recent advances in machine learning offer new opportunities to improve forecast accuracy by capturing complex, nonlinear patterns in large-scale data.

This paper explores the application of modern machine learning techniques, specifically the ensemble learning methods XGBoost and CatBoost, to forecast the realised volatility of the S&P 500 index. These are compared with several widely used econometric models, including GARCH(1,1), FIGARCH, EWMA, and HAR. The purpose of this study is to evaluate and compare the predictive performance of classical econometric and machine learning models for the prediction of volatility of the financial markets. To achieve this goal, we use a high-frequency data set of S&P 500 minute-level prices spanning from 2008 to 2021, a period that encompasses both major financial crises and periods of low volatility. The models are evaluated out of sample using standard accuracy measures.

The rest of the paper is structured as follows. Section 2 describes the methodology. Section 3 presents and compares the empirical results, highlighting the strengths and weaknesses of each approach. Finally, Section 4 concludes the study and discusses practical implications for financial forecasting and risk management.

2 METHODOLOGY

The data set used in this study consists of minute-level intraday price data for the S&P 500 index, spanning from March 2008 to May 2021. Data were obtained from Kaggle [5] and include open, close, and adjusted close prices in CSV format. This high-frequency dataset

captures market fluctuations across several economic cycles, including the 2008 global financial crisis and the COVID-19 pandemic; see Figure 1.

The realised volatility (RV) is computed from minute-by-minute log returns,

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right), \tag{1}$$

where $\boldsymbol{P_t}$ is the price at time *t*, and $\boldsymbol{P_{t-1}}$is the price in the previous time step. The realised volatility is then calculated as the square root of the sum of squared logarithmic returns over a given period (e.g., one day).

$$RV = \sqrt{\sum_{t=1}^{N} r_t^2}, \tag{2}$$

where *N* denotes the number of intraday time intervals. The use of logarithmic returns in computing RV helps slightly reduce heteroskedasticity.

The data set was divided into two periods: data up to December 31, 2018, which served as the in-sample set for model training, while the remaining data were used as the out-of-sample set to evaluate model performance. The daily realised volatilities for both periods are depicted in Figure 1.

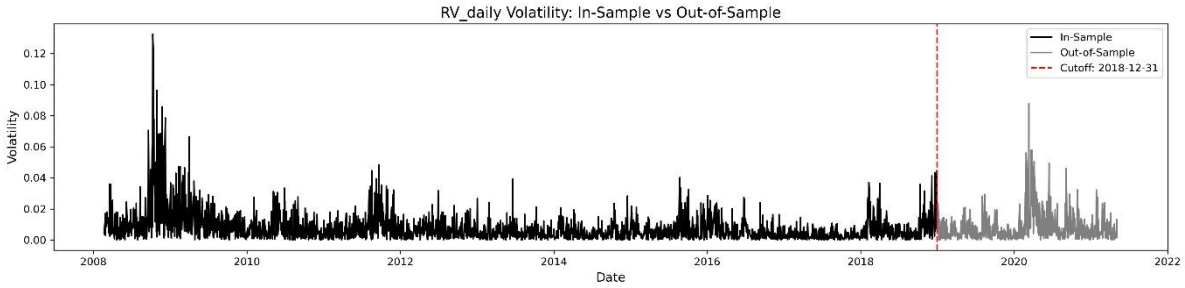


Figure 1: Daily realised volatilities.

The Generalised Autoregressive Conditional Heteroskedasticity Model (GARCH) [2] is a standard econometric tool for modelling and forecasting financial market volatility. The variant most widely used, GARCH(1,1), assumes a delay for both innovations and variances and is defined as:

$$\sigma_{t+1,t}^2 = \omega + \alpha \cdot \varepsilon_t^2 + \beta \cdot \sigma_{t,t-1}^2, \tag{3}$$

where $\boldsymbol{\omega, a, \beta \geq 0}$ and $\boldsymbol{a + \beta < 1}$ ensure stationarity. The model assumes that returns are conditionally normally distributed and that parameters are usually estimated using the maximum likelihood method.

To address the long-memory behaviour observed in volatility series, the Fractionally Integrated GARCH (FIGARCH) [1] model was developed. It generalizes the GARCH model by allowing fractional differencing with a memory parameter $\boldsymbol{d \in (0, 1)}$. This allows for a more flexible fit to empirical data with slowly decaying volatility autocorrelations.

A special case of GARCH(1,1) is the Exponentially Weighted Moving Average (EWMA) model, which simplifies the volatility updating process to a single smoothing parameter $\boldsymbol{\lambda \in (0, 1)}$:

$$\sigma_{t+1,t}^2 = (1 - \lambda) \cdot \varepsilon_t^2 + \lambda \cdot \sigma_{t,t-1}^2, \tag{4}$$

Risk Metrics [6] recommends $\boldsymbol{\lambda}$=0.94 for daily data.

The Heterogeneous Autoregressive (HAR) model, introduced in [4], is based on the heterogeneous market hypothesis, which assumes the existence of investors with varying decision horizons. This leads to volatility cascading from lower to higher frequencies and explains the long memory observed in volatility dynamics. The HAR model captures this by combining realised volatility over different time horizons, specifically daily, weekly (5 days), and monthly (22 days), into a single autoregressive framework,

$$RV_{t+1} = \beta_0 + \beta_1 RV_t + \beta_2 RV_t^w + \beta_3 RV_t^m + \varepsilon_{t+1}, \quad (5)$$

where the daily RV is computed from intraday squared returns, while the weekly and monthly RV are derived as moving averages of the daily RV over the respective periods.

CatBoost [3] and XGBoost [7] are two state-of-the-art gradient boosting frameworks widely adopted for predictive modelling in various domains, including finance. Both algorithms construct a set of decision trees sequentially, where each new tree aims to correct the errors of the previous trees combined, thus improving the overall accuracy of the prediction. XGBoost (Extreme Gradient Boosting) builds asymmetric decision trees optimised through second-order gradient information. It requires careful preprocessing of categorical features, as it natively supports only numerical input. CatBoost (Categorical Boosting), on the other hand, introduces a novel approach by growing symmetric trees, where all leaves on the same level are split using the same feature, resulting in a balanced tree structure. This design significantly speeds up training and inference. CatBoost natively handles categorical variables through an efficient ordered boosting scheme, eliminating the need for extensive preprocessing.

3 RESULTS

Table 1 summarises the forecast accuracy of the models outside the sample. The models are sorted by RMSE and MAE. The CatBoost model achieves the best performance, with the lowest MAE and RMSE, indicating a high accuracy in volatility prediction. It also has the highest logarithmic likelihood value, reflecting a strong statistical model quality. However, its main drawback is the long training time of 45 minutes, making it less suitable for applications that require fast results.

The XGBoost model ranks second, with slightly worse performance measured by MAE and RMSE, but with the smallest MAPE value. Its training time is about 1 hour, which is even less time-efficient than CatBoost. The log-likelihood score is still better than those of other models.

Table 1: Comparison of out-of-sample forecast accuracy and time complexity

	Error Measurements				***Time Complexity***
	RMSE	*MAE*	*MAPE*	*LL*	*Estimation time (in minutes)*
CatBoost	0.0052	0.0022	19.16 %	1959	45
XGBoost	0.0066	0.0024	18.81 %	1947	60
FIGARCH(1,d,1)	0.0066	0.0039	39.19 %	1878	5
GARCH(1,1)	0.0067	0.0038	38.64 %	1877	3
EWMA (λ=0)	0.0070	0.0038	34.70 %	1879	<1
HAR	0.0074	0.0035	26.19 %	1837	<1
EWMA (λ=0.94)	0.0085	0.0045	42.71 %	1806	<1

HAR provides a balance between accuracy and speed. Its MAE, RMSE, and MAPE are higher than those of CatBoost and XGBoost, indicating less accurate predictions, but its parameter estimation takes only 1 second, making it very efficient. The models GARCH (1,1) and FIGARCH (1,d,1) offer traditional econometric approaches to volatility forecasting. Both models show moderate accuracy. Their MAPE values are relatively high (around 39%),

reflecting less precise percentage predictions compared to machine learning models. The FIGARCH model slightly outperforms GARCH and takes longer to estimate parameters (5 minutes vs. 3 minutes). These models are slower than the simple HAR and EWMA models, but faster than CatBoost and XGBoost.

EWMA models with fixed lambda parameters (= 0.94) and estimated(λ=0) lambda parameters are the fastest, taking only 1 secondbut theyey lag behind in accuracy. The fixed lambda EWMA model has the highest percentage error among all methods. On the other hand, the estimated lambda EWMA achieves better results and even a higher log-likelihood than HAR. It is important to note that the EWMA with zero lambda leads to the model that forecasts the future value only from the current value (without averaging previous values).

4 CONCLUSION

This study has compared the predictive performance of traditional econometric models and modern machine learning methods in the task of forecasting financial market volatility. Using minute-level data from the S&P 500 index over a 14-year period, we evaluated models across multiple volatility frequencies and accuracy metrics.

The results demonstrate that the ensemble learning models, particularly CatBoost and XGBoost, consistently outperform classical approaches such as GARCH, FIGARCH, EWMA, and HAR in terms of predictive accuracy. CatBoost achieved the highest overall accuracy, although at the cost of increased computational time. On the contrary, simpler models such as HAR and EWMA offer fast estimation and may be more suitable in real-time or resource-constrained environments.

It is important to note that none of the models can capture current market sentiment or political risks, which often cause large volatility swings, such as during the COVID-19 pandemic, an event without precedent in recent history and therefore not reflected in the models. Future research may extend this work by incorporating sentiment analysis, macroeconomic indicators, or regime switching models to capture these dynamics more effectively.

Acknowledgement
This research was supported by the SGS project SP2025/043 at the Faculty of Economics, VSB - Technical University of Ostrava.

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THE WEIGHT OF COUNTRY RISK IN CREDIT PRICE – EVIDENCE FOR THE EUROZONE HOUSING LOANS

Ana Kundid Novokmet
University of Split, Faculty of Economics, Business and Tourism
Cvite Fiskovića 5, 21 000 Split, Croatia
E-mail: ana.kundid.novokmet@efst.hr

Bruna Čalija
Global Markets Investment Products at OTP banka d.d.
Domovinskog rata 61, 21 000 Split, Croatia
E-mail: calija.bruna@gmail.com

Abstract: This paper analyses the burden of country risk for the interest rate of housing loans. We focus on the housing loans due to their homogenous features and standardized procedures of approval, where clients' characteristics cannot significantly affect the credit price. Credit default swap spread and country risk score stand for the country risk, while the interest rate is for the euro-denominated housing loans. Our dataset consists of 14 Eurozone countries and Croatia in the time span from January 2012 to December 2021 and is analysed with multiple linear regression analysis separately for each country. The research results confirm the impact of country risk in shaping the credit price of housing loans.

Keywords: country risk, housing loans, credit default swap, interest rate, linear regression, Eurozone.

1 INTRODUCTION

Country risk is one of the systematic risks that banks and other economic agents are confronted with in their business activities. It is an outcome of a country's macroeconomic and institutional framework, which corresponds to the growth and development of a particular economy and, more broadly, to the level of democracy and progress in society. Country risk represents the economic, political and social circumstances that can consequently affect the associated business risks, financing conditions and the ability of economic agents to pay their debts. Some research also confirms its wider significance for e.g. tourism development [9] as well as for sustainability outcomes and the SDGs achievement [10]. The country risk premium reflects the benefits of living and doing business in a well-functioning institutional environment, but also the burdens in the opposite case. It shapes the price of government debt. Compared to companies and households, the state is considered a risk-free sector and therefore its debt price is usually a starting point to which various risk premiums for all other economic actors are added. So far, the impact of country risk on household loans has been neglected, as empirical studies have focussed on the corporate sector [6] or the banking sector [13].

This study focuses on the household sector's long-term loans i.e. housing loans, to show how important the circumstances under which families earn money influence their ability to repay the loans. We assume a spillover effect of country risk on the household sector. More specifically, the different levels of country risk are expected to explain the differences in home loan interest rates between the selected countries, even if they belong to a monetary union such as the Eurozone. The determinants of country risk are heterogeneous and go beyond single monetary policy. To be more precise, country risk is composed of 1) international political risks, 2) domestic political and institutional risks, 3) jurisdiction risks, 4) macroeconomic risks, 5) microeconomic risks, 6) sanitary, health, industrial and environmental risks, and finally 7) natural and climate risks [5]. Country risk is particularly important for those investing in less developed countries [2]. Nevertheless, interest rates for housing loans are not only influenced by country risk, but also by other factors. The level of competition in the banking sector, aggregate credit risk in the economy and money market interest rates are some of these [3],

while bank lending policies and the cost structure of interest rates, including, for example, operating costs [1], [12] are another factor.

2 EMPIRICAL STRATEGY AND RESULTS

In the paper, we examined the impact of country risk on the interest rate of housing loans. The interest rate on housing loans approximates long-term credit price. Household loans are approved on a standardised basis, without any or only weak impact of the client's features on the interest rate. In this way, the exposure to systematic risks such as country risk can be better analysed. Country risk is measured using the following indicators: 1) CDS spread (credit default swap spread between the observed country and the country with the AAA credit rating), 2) yield on long-term government bonds (10-year maturity), 3) country risk score and 4) sovereign risk score. Both scores have a theoretical scale of 1 to 100. All indicators imply higher country risk, for the higher value recorded.

Of the 20 Eurozone countries, we collected monthly data for 14 of them as well as for Croatia, which introduced Euro in 2023. Five countries were excluded from the analysis due to the lack of data for the entire period observed (Cyprus, Estonia, Latvia, Luxembourg and Malta). CDS spreads were sourced from the S&P Capital IQ database, while other country risk indicators were available from the Economist Intelligence Unit database. Interest rates for euro-denominated housing loans were retrieved from the websites of the relevant central banks. The dataset covers the period from January 2012 to December 2021 and we used SPSS Statistics 23 for the analysis.

Figures 1 and 2 show the mean values of country risk indicators and interest rate on housing loans and confirm that for most countries, with the exception of Greece (EL), there may be some synchronisation between interest rates and country risk measures.

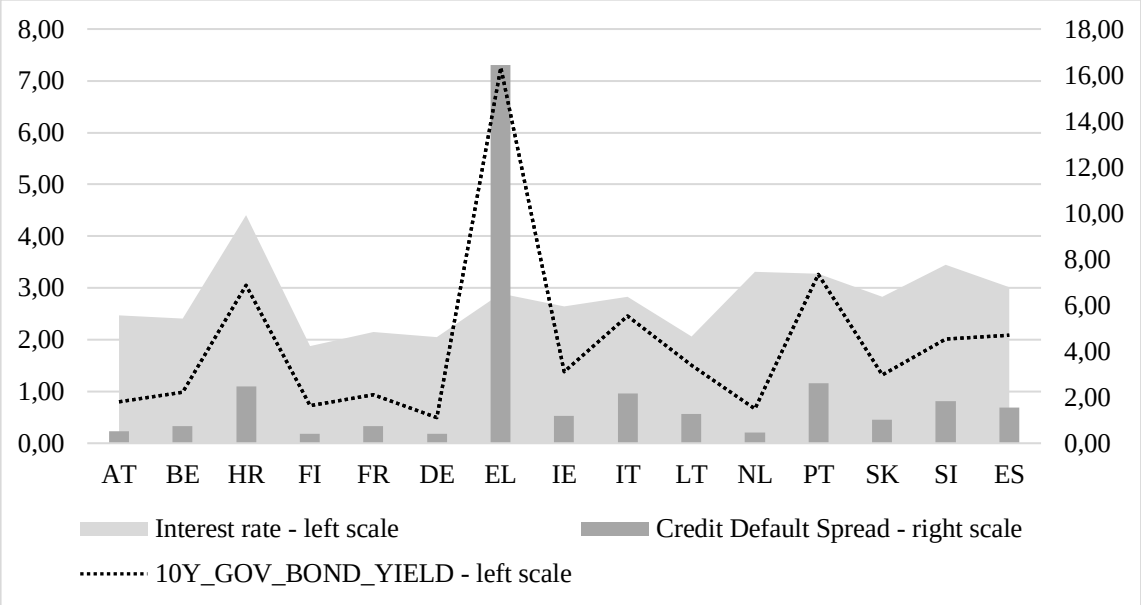


Figure 1: Mean values of interest rates, credit default spreads and yield on 10-year government bonds by countries from January 2012 to December 2021.

Note: Austria (AT), Belgium (BE), Croatia (HR), Finland (FI), France (FR), Germany (DE), Greece (EL), Ireland (IE), Italy (IT), Lithuania (LT), Netherlands (NL), Portugal (PT), Slovakia (SK), Slovenia (SI), Spain (ES).

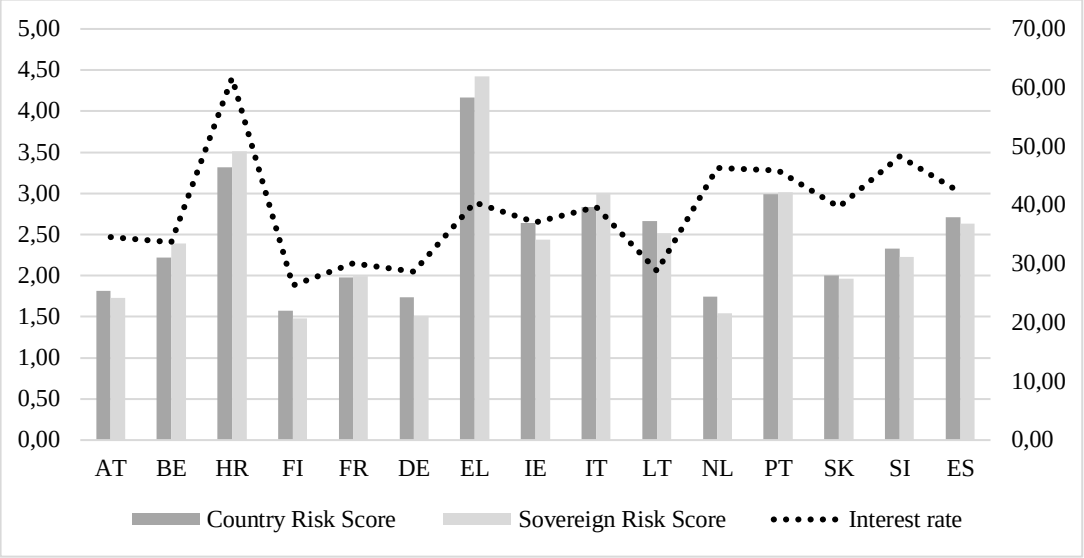


Figure 2: Mean values of interest rates, country risk scores and sovereign risk scores by countries from January 2012 to December 2021.

Note: Austria (AT), Belgium (BE), Croatia (HR), Finland (FI), France (FR), Germany (DE), Greece (EL), Ireland (IE), Italy (IT), Lithuania (LT), Netherlands (NL), Portugal (PT), Slovakia (SK), Slovenia (SI), Spain (ES).

According to the descriptive statistics (Table 1), there are huge differences in variables. For example, the interest rate on housing loans ranges from 0.16% to 9.36%, while the long-term yield on government bonds ranges from -0.550% to 29.240%. A conclusion on significant disparities between the countries and/or the observed period is tested with the analysis of variance and the Kruskal-Wallis test if the condition of homogeneity of variances was not met for ANOVA. The differences between the countries were significant for all indicators at the 1% level (sig.=0.000). When testing the differences between the observed 120-month period, the interest rates and long-term bond yields were significant at the 1% level (sig.=0.000), the CDS spreads were significant at the 5% level (sig.=0.016), while the country and sovereign risk scores were insignificant.

Table 1: Descriptive statistics.

Variable	***Mean***	***Std. Deviation***	***Minimum***	***Maximum***	***N***
Interest rate	2.7780	1.2487	0.16	9.36	1799
CDS	2.2549	9.6181	0.1627	204.0391	1800
10-Y_GOV_BOND_YIELD	1.9276	2.7450	-0.550	29.240	1798
Country Risk Score	34.28	10.221	17	67	1800
Sovereign Risk Score	33.96	11.862	15	74	1800

Due to sizeable differences between the countries, the effects of the country risk indicators on the prices of housing loans were estimated separately using multiple linear regression. Of the four country risk indicators, only the CDS spread and the country risk were used as predictors for the interest rates. The yield on long-term bonds correlated strongly with the CDS spread as well as country risk score with the sovereign risk score. To better reflect the country risk rather than the sovereign risk, we chose the CDS spread and the country risk score (CRS).

The results of the linear regression for each country are shown in Table 2. Since the tolerance was always greater than 20% and the variance inflation factor (VIF) was less than 5, we can confirm that there is no multicollinearity problem.

Table 2: Linear regression with interest rate as dependent variable.

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95.0% Confidence Interval for B		Collinearity Statistics	
		B	Std. Error	Beta			Lower Bound	Upper Bound	Tolerance	VIF
AT	Constant	**0.706**	0.449		1.573	0.118	-0.183	1.596		
	CDS	**1.729**	0.103	0.831	16.793	**0.000**	1.525	1.933	0.987	1.013
	CRS	**0.034**	0.018	0.094	1.903	0.059	-0.001	0.069	0.987	1.013
BE	Constant	**0.007**	0.467		0.016	0.988	-0.917	0.932		
	CDS	**1.159**	0.085	0.751	13.609	**0.000**	0.990	1.327	0.905	1.105
	CRS	**0.050**	0.015	0.177	3.214	**0.002**	0.019	0.080	0.905	1.105
FI	Constant	**1.260**	0.261		4.821	**0.000**	0.743	1.778		
	CDS	**1.492**	0.184	0.603	8.094	**0.000**	1.127	1.857	0.978	1.022
	CRS	**0.000**	0.012	0.002	0.020	0.984	-0.023	0.024	0.978	1.022
FR	Constant	**0.819**	0.363		2.257	**0.026**	0.100	1.537		
	CDS	**1.819**	0.085	0.910	21.292	**0.000**	1.650	1.988	0.814	1.228
	CRS	**-0.001**	0.014	-0.003	-0.074	0.941	-0.028	0.026	0.814	1.228
DE	Constant	**1.225**	0.344		3.558	**0.001**	0.543	1.908		
	CDS	**2.288**	0.152	0.836	15.099	**0.000**	1.988	2.588	0.867	1.154
	CRS	**-0.004**	0.015	-0.017	-0.303	0.762	-0.034	0.025	0.867	1.154
EL	Constant	**3.357**	0.311		10.805	**0.000**	2.742	3.973		
	CDS	**0.004**	0.001	0.467	4.719	**0.000**	0.002	0.005	0.729	1.371
	CRS	**-0.009**	0.005	-0.164	-1.654	0.101	-0.020	0.002	0.729	1.371
IE	Constant	**1.827**	0.066		27.479	**0.000**	1.695	1.959		
	CDS	**0.051**	0.008	0.369	6.489	**0.000**	0.036	0.067	0.524	1.908
	CRS	**0.021**	0.002	0.600	10.540	**0.000**	0.017	0.024	0.524	1.908
IT	Constant	**-5.110**	1.717		-2.975	**0.004**	-8.511	-1.708		
	CDS	**0.857**	0.119	0.528	7.189	**0.000**	0.621	1.092	0.936	1.069
	CRS	**0.153**	0.044	0.254	3.457	**0.001**	0.066	0.241	0.936	1.069
LT	Constant	**0.145**	0.119		1.217	0.226	-0.091	0.381		
	CDS	**0.494**	0.056	0.528	8.810	**0.000**	0.383	0.605	0.284	3.526
	CRS	**0.034**	0.005	0.448	7.467	**0.000**	0.025	0.044	0.284	3.526
NL	Constant	**1.748**	0.319		5.471	**0.000**	1.115	2.381		
	CDS	**3.156**	0.177	0.884	17.851	**0.000**	2.806	3.506	0.736	1.359
	CRS	**0.002**	0.014	0.008	0.166	0.868	-0.026	0.031	0.736	1.359
PT	Constant	**-4.303**	1.659		-2.595	**0.011**	-7.588	-1.018		
	CDS	**0.504**	0.080	0.565	6.337	**0.000**	0.347	0.662	0.325	3.077
	CRS	**0.150**	0.044	0.308	3.450	**0.001**	0.064	0.237	0.325	3.077
SK	Constant	**3.723**	1.201		3.101	**0.002**	1.345	6.101		
	CDS	**2.430**	0.212	0.700	11.448	**0.000**	2.010	2.850	0.934	1.071
	CRS	**-0.121**	0.040	-0.185	-3.025	**0.003**	-0.200	-0.042	0.934	1.071
SI	Constant	**-1.034**	0.694		-1.490	0.139	-2.410	0.341		
	CDS	**1.014**	0.061	0.812	16.719	**0.000**	0.894	1.135	0.986	1.014
	CRS	**0.081**	0.021	0.184	3.797	**0.000**	0.039	0.123	0.986	1.014
ES	Constant	**-3.708**	1.173		-3.161	**0.002**	-6.032	-1.385		
	CDS	**0.649**	0.092	0.521	7.048	**0.000**	0.467	0.832	0.630	1.586
	CRS	**0.150**	0.033	0.337	4.559	**0.000**	0.085	0.216	0.630	1.586
HR	Constant	**0.630**	0.667		0.944	0.347	-0.691	1.950		
	CDS	**0.766**	0.061	0.798	12.557	**0.000**	0.645	0.887	0.286	3.501
	CRS	**0.041**	0.017	0.152	2.399	**0.018**	0.007	0.074	0.286	3.501

Note:
(AT) R=0.847, R^2=0.717, Adjusted R^2=0.712, St. Error of the Estimate= 0.44147, F=148.269, sig.=0.000
(BE) R=0.823, R^2=0.678, Adjusted R^2=0.672, St. Error of the Estimate= 0.47980, F=122.987, sig.=0.000
(FI) R=0.603, R^2=0.364, Adjusted R^2=0.353, St. Error of the Estimate= 0.36955, F=33.507, sig.=0.000
(FR) R=0.909, R^2=0.826, Adjusted R^2=0.823, St. Error of the Estimate= 0.38683, F=277.553, sig.=0.000
(DE) R=0.830, R^2=0.689, Adjusted R^2=0.684, St. Error of the Estimate= 0.37541, F=129.628, sig.=0.000

(EL) R=0.406, R^2=0.165, Adjusted R^2=0.151, St. Error of the Estimate= 0.23858, F=11.573, sig.=0.000
(IE) R=0.895, R^2=0.802, Adjusted R^2=0.798, St. Error of the Estimate= 0.07815, F=236.218, sig.=0.000
(IT) R=0.641, R^2=0.411, Adjusted R^2=0.400, St. Error of the Estimate= 1.00982, F=40.748, sig.=0.000
(LT) R=0.938, R^2=0.881, Adjusted R^2=0.879, St. Error of the Estimate= 0.17395, F=431.536, sig.=0.000
(NL) R=0.888, R^2=0.789, Adjusted R^2=0.785, St. Error of the Estimate= 0.49897, F=218.639, sig.=0.000
(PT) R=0.837, R^2=0.700, Adjusted R^2=0.695, St. Error of the Estimate= 1.04595, F=135.389, sig.=0.000
(SK) R=0.769, R^2=0.591, Adjusted R^2=0.584, St. Error of the Estimate= 1.11516, F=84.601, sig.=0.000
(SI) R=0.853, R^2=0.728, Adjusted R^2=0.723, St. Error of the Estimate= 0.66370, F=156.519, sig.=0.000
(ES) R=0.773, R^2=0.598, Adjusted R^2=0.591, St. Error of the Estimate= 0.86131, F=86.874, sig.=0.000
(HR) R=0.930, R^2=0.865, Adjusted R^2=0.863, St. Error of the Estimate= 0.41687, F=375.236, sig.=0.000

According to the estimated coefficients, CDS spreads increase the interest rate on housing loans in all countries observed. The highest effects were found for the Netherlands (3.156), Slovakia (2.430) and Germany (2.288), and the lowest for Greece (0.004) and Ireland (0.051). The country risk score (CRS) was significant for 9 out of 15 countries. However, its impact was much smaller than that of the CDS spread and only unexpectedly negative in the case of Slovakia. In addition, CDS spreads capture changes in country risk much faster than other country risk assessments [7], [11].

The correlation between the model variables (R) and the model representativeness, i.e. the coefficient of multiple determination (R Square), are quite high in all cases (above 0.8 for R and around 70% for R Square), except in the case of Finland, Greece and Italy for both indicators and Slovakia and Spain for model representativeness (almost 60%). All estimated models are statistically significant.

These results could also be important for estimating the demand for housing loans. This is because higher interest rates are negatively associated with changes in growth and demand for housing loans [4], [8] and thus an increase in country risk level could reduce credit growth.

3 CONCLUSION

This paper analyses influence of context on financing conditions, more specifically, the impact of country risk on the interest rate of housing loans, a topic that is underrepresented in empirical estimates. In order to minimise the impact of other potential predictors such as monetary policy or client characteristics, Eurozone countries and housing loans were selected for the empirical analysis. Nevertheless, there are significant differences in the interest rates for housing loans. The results of the multiple linear regression show that interest rates react more frequently and strongly to the CDS spread than to the country risk score. This suggests that the market indicators are more important than the other country risk measures.

The positive relationship between country risk and interest rates confirms the importance of the institutional framework for decision-making and the burden of an underdeveloped environment on the credit affordability. In addition to country risk, other factors can also influence interest rates. The unemployment rate, overall credit risk, inflation, GDP growth, house prices, regulatory requirements and banking market characteristics, to name the most important. However, monthly data for these factors are limited, usually less volatile, and including them as predictors requires a change in methodology. Therefore, in this exploratory study, we focused only on country risk and employed simple multivariate statistics i.e. multiple linear regression. This could be seen as a limitation of the work, but the country risk indicators reflect the overall functionality of the country, including numerous macroeconomic variables and proxies for institutional settings. However, future research could take these variables into account, use other country risk indicators and apply a more sophisticated methodological approach.

Acknowledgement
We would like to thank Deloitte in Croatia for sharing the data from the Economist Intelligence Unit and S&P Capital IQ database during the internship of Ms Bruna Čalija.

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COMPETITIVENESS, CRONYISM AND REGULATION IN EU NEW MEMBER STATES

Velibor Mačkić
University of Zagreb Faculty of Economics and Business
J. F. Kennedy Square 6, 10 000 Zagreb
E-mail: vmackic@efzg.hr

Lucija Rogić Dumančić
University of Zagreb Faculty of Economics and Business
J. F. Kennedy Square 6, 10 000 Zagreb
E-mail: lrogic@efzg.hr

Boris Cota
University of Zagreb Faculty of Economics and Business
J. F. Kennedy Square 6, 10 000 Zagreb
E-mail: bcota@efzg.hr

Abstract: This paper investigates the role of cronyism and government regulation in the competitiveness of the eleven new EU member states from 2007-2016. The results imply that cronyism has a negative effect on competitiveness and, due to that, higher government regulation corrects for these 'crony externalities' ensuring higher competitiveness. The relevance of the results highlights that minimizing inefficiencies in the public sector and its connections with economic agents in the private sector (lowering cronyism) are crucial in achieving overall competitiveness.

Keywords: competitiveness, cronyism, regulation, new EU member states

1 INTRODUCTION AND THEORETICAL FOUNDATION

The term "competitiveness" has emerged as the paradigm for the economic well-being of a nation. Our emphasis lies on public institutions that provide 'rules of the game' – constraints and incentives – for economic agents determining a specific political economy model [3]. The goal is to empirically test the role of cronyism and government regulation on the overall competitiveness level of the eleven new EU member states. Literature distinguishes between two main approaches to competitiveness of a contemporary economy: narrow (prices and costs) and broad (living standard and welfare). While first approach centres on international comparison, the second is time centred and focuses on governance and inefficiencies in the public and non-tradable sectors of the national economies. Those sectors determine the level of competitiveness since they (i) formulate and implement competitiveness strategies and (ii) directly influence the tradable sector by providing public and intermediary goods and services. Consequently, cronyism provides explanations on the connection between public institutions, business environment, and economic outcomes in which some economic agents profit at the expense of the rest of the society (see [13]; [7]; [9]). Following [6] approach of a triad of cronyism/corruption/rent-seeking that highlights the importance of public institutions and [11] work on institutional determinants of overall competitiveness we include the impact of government regulation on economic outcomes (see [5]; [10]). Our contribution stems from using a database of public institutions that have not been used in the previous research and applying it within a context of cronyism and regulation effects on competitiveness.

2 METHODOLOGY AND DATA

We test the hypothesis that in countries plagued with cronyism, a high level of government regulation positively affects competitiveness. To test this hypothesis, we employ a linear

dynamic panel model based on the one step generalized method of moments (GMM) estimator proposed by [2]. We use a broader measure of competitiveness – GDP per employed person. This enables examination of three crucial factors of well-being: the factor incomes, level of employment, and productivity. Chosen sample of countries – Bulgaria, Croatia, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Romania, Slovakia, and Slovenia – shares a common socialist legacy, i.e. path dependency that determined the current political economic architecture. The analysis covers the period from 2007 until the end of 2016 for each country ([12] stopped collecting the data after 2016). The frequency of data is yearly. Table 1 lists variables, their description, and data sources.

Table 1: Data description and sources

Variable	Label	Measure	Source	The expected sign
GDP per person employed	GDP_PE	2011 PPP $, const.	[15]	Dependent variable
Burden of government regulation	BGR	Index, [1:7]	[12]	-
Favoritism	FAV	Index, [1:7]	[12]	+
Total population	POP	the persons, period average	[14]	-
Countries' Exports to EU15	XEU15	Euro, mn	[14]	+

Using the data described above, we estimate the following relationship:

$$GDP_{PE\,i,t} = \beta_0 + \beta_1 GDP_{PE\,i,t-1} + \beta_2 XEU15_{it} + \beta_3 POP_{it} + \beta_4 FAV_{it} + \beta_5 BGR_{it} + \varepsilon_{i,t} \quad (1)$$

We exploit the definition of cronyism by [8] and use [12] variable FAV that exactly measures to what extent government officials demonstrate favouritism to well-connected individuals and companies when deciding public policies and contracts. It the value of the variable equals' one, then government shows favouritism to a great extent, and vice versa. Positive sign (higher value of FAV variable = lower level of cronyism) reflects that resources are allocated by market mechanisms (relative prices). Variable BGR measures how burdensome it is for business to comply with governmental administrative requirements. A BGR equal to 1 indicates that it is incredibly burdensome, and vice versa. The expected negative sign of BGR reflects that higher level of government regulation (lower value of BGR) results in a higher level of competitiveness. Higher public sector performance, in countries plagued with high levels of cronyism, is ensured by following the simple rule of thumb: 'rules rather than discretion'. Two control variables in the model are population and country's export to the EU15 with expected signs.

3 RESULTS AND DISCUSSION

Table 2 reports our main econometric findings. Both of our variables of interest – FAV and BGR – are statistically significant coefficient with the expected theoretical sign. Moreover, they maintain their statistical significance and signs in all three estimated models. The results of Sargan test shows that instruments are coherent with validity. Two main findings are reported. First, the influence of public institutions (measured by the WEF's perception indicators on cronyism and government regulation) is statistically significantly related to our measure of the overall competitiveness of contemporary economies. The results show that in the new EU member states cronyism decreases overall competitiveness. It is exactly in that

environment where more government regulation positively affects competitiveness. In other words, cronyism works differently in the 'new' compared to the 'old' EU. As opposed to [4] conclusion that cronyism occurs as a side-effect of increased regulation and creates misallocation of resources, this paper finds that in already crony environment it is rules that govern the behaviour of economic agents. They provide the right incentives and minimize the discretion that results in sub-optimal results. In other words, rules rather than discretion should be the maxima for competitiveness.

Table 2: Estimation Results

	Model_1	Model_2	Model 3
L.GDP_PE	0.69	0.71	0.65
	(0.08)***	(0.08)***	(0.08)***
XEU15	0.09	0.09	0.10
	(0.02)***	(0.02)***	(0.02)***
POPULATION	-4.08	-4.65	-3.97
	(1.45)***	(1.46)***	(1.36)***
FAVOR	1544.81		2272.10
	(829.70)*		(854.59)***
BGR		-1673.58	-1761.94
		(715.03)**	(704.81)**
_cons	78512.61	62212.27	52460.90
	(15131.16)***	(15885.56)***	(14671.77)***
Sargan test	79.7	64.5	77.3
chi2	209.36	217.36	230.15
zrank	81.00	81.00	84.00

*Notes:*Significance at the 10% level; ** significance at the 5% level; ***significance at the 1% level.*
zrank – the number of instruments
Source: authors' calculation

Second, the results indicate one additional and interesting finding. One would expect that economic agents' preferences had converged, since the collapse of socialist economies, towards a market economy free of the state's interventions and influences. One would expect that especially in the light that our dataset starts from 2007 – 18 years after the fall of the Berlin wall – as [1] report for economic agents from former East Germany. Our results point to the other direction, but nonetheless economic agents, in our sample, do act rationally. The incentives and uncertainty of the economic environment led them to invest resources into relationships with political elites. This was both a short-term and a long-term strategy aimed at survival. Of course, comparatively the misallocation of resources from innovation into rent-seeking resulted in lower productivity and lower overall productivity (i.e. competitiveness).

4 CONCLUSION

This paper investigated the relationship between public institutions (cronyism and government regulation) and competitiveness – GDP per person employed – using a sample of 11 new EU member states. Dynamic panel analysis yielded following results. First, cronyism has a negative effect on competitiveness. Second, due to the crony environment, higher government regulation corrects for these 'crony externalities' ensuring higher competitiveness. Policy recommendation that results from our finding is following. Stringent regulation that minimizes discretion in favour of rules minimizes inefficiency from the general welfare point of view and increases competitiveness of contemporary economies.

Interesting avenues for further research depend on the availability of data, since this kind of political economy topics rest on expert survey questioners that are available and comparable.

Acknowledgement
This work has been fully supported by the Croatian Science Foundation under the project No. IP-09-2014-5476 and IP-2024-05-5629.

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INDUSTRY-SPECIFIC SENSITIVITY TO INVESTOR SENTIMENT: AN EMPIRICAL STUDY ON RUSSELL 1000 CONSTITUENTS

Ing. Bahate Maidiya
VSB – Technical University of Ostrava, Department of Finance
17. listopadu 2172/15, 708 00 Ostrava-Poruba, Czech Republic
E-mail: bahate.maidiya@vsb.cz

Abstract: This study examines the industry-specific sensitivity of firm-level trading behavior to investor sentiment within the Russell 1000. Incorporating a sentiment spread into extended Fama–French specifications, we estimate OLS regressions for returns, trading volume, and volatility for individual stocks across 11 industries. We find that sentiment has limited effects on returns but pronounced effects on volume and volatility. Welch's ANOVA and Games–Howell post-hoc tests show significant cross-industry differences in how sentiment influences market activity, underscoring the heterogeneous role of sentiment across industries.

Keywords: investment sentiment, Russell 1000, behavioral finance, GICS industries

1 INTRODUCTION AND METHODOLOGY

Prior research establishes that investor sentiment significantly affects asset prices and trading patterns (Baker et al., 2006; Baker et al., 2007; Kostopoulos et al., 2018; Lei et al., 2012). A key finding is that the strength of these effects varies across industries, as some sectors are more susceptible to shifts in investor psychology than others (Huang et al., 2014; Niu et al., 2023; Xie, 2024). However, the empirical evidence remains mixed; for instance, while sentiment derived from traditional surveys is impactful, measures extracted from internet postings may lack predictive power (Kim et al., 2014). This heterogeneity underscores the need for a robust comparative framework. We revisit this issue using the broad Russell 1000 universe, which offers enhanced statistical power for cross-industry comparisons relative to narrower large-cap benchmarks.

This study investigates whether firms exhibit differential sensitivity to investor sentiment across industries, with a focus on volatility and trading volume channels. Our contributions are threefold. First, we extend standard Fama-French models by incorporating a direct sentiment proxy to capture behavioral effects. Second, we analyze not only returns but also trading volume and conditional volatility to provide a holistic view of sentiment's impact. Third, we implement a rigorous multi-stage testing procedure—including diagnostic checks, Welch's ANOVA, and Games-Howell post-hoc tests—to ensure robust comparisons across industries with heterogeneous variances.

We obtain daily stock returns and pricing data for all constituents of the Russell 1000 Index sourced from Yahoo Finance, covering the period from 2000-01-01 to 2024-12-31. To ensure data quality and mitigate bias, we exclude firms with fewer than 2,000 daily observations (approximately 8 years), resulting in a final sample of 853 stocks. This filter enhances cross-industry comparability by removing firms with limited trading history or delisting events. We employ the Global Industry Classification Standard (GICS) for sector-level categorization. Sentiment data are from the weekly AAII Investor Sentiment Survey, which reports the percentages of bullish, neutral, and bearish respondents (summing to 100%). Our sentiment proxy is the bull-bear spread, defined as the difference between bullish and bearish percentages. We employ the Fama–French five-factor model as our baseline asset pricing framework. For each constituent stock i of the Russell 1000 Index, we estimate the following time-series regression models via Ordinary Least Squares (OLS):

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_{1,i} \cdot MKT_t + \beta_{2,i} \cdot SMB_t + \beta_{3,i} \cdot HML_t + \gamma_i \cdot SENT_t \,, \quad (1)$$

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_{1,i} \cdot MKT_t + \beta_{2,i} \cdot SMB_t + \beta_{3,i} \cdot HML_t + \beta_{4,i} \cdot RMW_t + \beta_{5,i} \cdot CMA_t + \gamma_i \cdot SENT_t \quad (2)$$

where $r_{i,t} - r_{f,t}$ is the realized excess return of stock i on time t, $R_{f,t}$ is the risk-free return at time t, α_i is the intercept term, and all $\beta_{k,i}$ (k=1,...,5) parameters quantify the extent to which each factor affects stock returns. MKT_t refers to the market excess return at time t, SMB_t is the size premium (Small Minus Big) at time t. HML_t is the value premium (High Minus Low) at time t. RMW_t is the profitability premium (Robust Minus Weak) at time t. CMA_t is the investment style premium (Conservative Minus Aggressive) at time t. γ_i is the parameters for sentiment, and $SENT_t$ is the sentiment index at time t. To address autocorrelation and heteroskedasticity in the residuals, we report Newey-West robust standard errors for all regressions. We model the time-varying conditional volatility of each stock using a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model, which captures the well-documented phenomenon of volatility clustering in financial returns (Bollerslev, 1986). Specifically, we estimate a GARCH(1,1) process for each stock i:

$$r_{i,t} = \mu_i + \sigma_{i,t} \cdot \epsilon_{i,t} \,, \tag{3}$$

$$\sigma_{i,t}^2 = \omega + \alpha \cdot \sigma_{i,t-1}^2 + \beta \cdot \epsilon_{t-j}^2 \,, \tag{4}$$

where $r_{i,t}$ is the return on asset *i* at time *t*, μ_i is the average return on asset *i*, which is usually assumed to be a constant, $\sigma_{i,t}$ is the volatility of asset *i* at time *t*, and $\epsilon_{i,t}$ is the random disturbance term of the standard normal distribution (i.e., $\epsilon_{i,t} \sim$ N(0,1)). $\sigma_{i,t}^2$ is the square of the volatility of asset *i* at time *t*. ω, α, and β are model parameters that need to be estimated from historical data. ω is the mean of the long-run volatility, α is the effect of the previous period's volatility on the current volatility, and β is the effect of innovations (i.e., stochastic perturbations) in the previous j periods on the current volatility. The estimated conditional volatility series is then utilized as a dependent variable in a separate cross-sectional regression against the sentiment index to explicitly test for a direct link between investor sentiment and stock return volatility.

To assess cross-industry heterogeneity, Welch ANOVA and Games-Howell post-hoc tests are employed, accommodating unequal variances and sample sizes.

2 RESULT

First, we performed four regression calculations on all 853 stocks according to 11 industry classifications, namely FF3F, FF5F, trading volume, and volatility. Table 1 presents the proportion of firms within each industry exhibiting significant sensitivity ($p < 0.05$) to investor sentiment in the extended regression models.

Table 1: Industry-level Proportion of Firms with Significant Sentiment Sensitivity ($p < 0.05$)

Industry	***FF3_Ratio***	***FF5_Ratio***	***Volume_Ratio***	***Volatility_Ratio***
Health Care	0.1075	0.1075	0.6129	0.6882
Industrials	0.0625	0.0625	0.8194	0.8403
Information Technology	0.0490	0.0490	0.5294	0.7157
Consumer Discretionary	0.0943	0.1038	0.7358	0.8868
Utilities	0.0789	0.0526	0.9474	0.8421
Financials	0.0730	0.0730	0.8832	0.9270
Real Estate	0.0612	0.0612	0.7755	0.8571
Materials	0.0345	0.0345	1.0000	0.9655
Consumer Staples	0.0476	0.0476	0.6667	0.9524
Communication Services	0.1351	0.1081	0.8378	0.6757
Energy	0.0455	0.0909	0.9545	1.0000

Limited Impact on Expected Returns: Sentiment shows a remarkably weak influence on stock returns after controlling for common risk factors. The proportion of firms with significant

sentiment coefficients in both the FF3F and FF5F models is strikingly low across all industries (mostly below 10%, with a maximum of 13.5% in Communication Services for FF3). This suggests that investor sentiment, as measured by the AAII spread, is not a pervasive determinant of cross-sectional expected returns. In stark contrast, sentiment exerts a powerful and widespread influence on trading activity and risk. For volume and volatility, the proportions are high, frequently exceeding 80% of firms within an industry. Notably, the Energy and Materials sectors show near-universal sensitivity, with 100% of Materials firms showing a significant sentiment-volatility link. This indicates that sentiment primarily drives investor trading behavior, rather than directly affecting priced returns. While the high prevalence in volume and volatility is a universal theme, the degree varies by industry. Utilities and Financials show exceptionally high sensitivity in trading volume (>88%), whereas Health Care and Information Technology appear relatively less affected, though still with a majority of firms being significant.

Prior to assessing differences in sentiment sensitivity across industries, we conducted comprehensive diagnostic tests to validate the assumptions of standard parametric tests. Context with diagnostics. Shapiro–Wilk and Levene tests reject normality and homoscedasticity at the industry level for the four metrics; hence, the use of Welch's ANOVA and Games–Howell—which do not assume equal variances or balanced group sizes—is warranted. The Games-Howell tests, which control for multiple comparisons, reveal the structure of the heterogeneity identified by the ANOVA. Return & Volume Models: The number of statistically significant pairwise differences was limited (13/55 for FF5F, 18/55 for FF3F, and 11/55 for Volume). For volume, a clear pattern emerged: the Energy sector was significantly different from most others (e.g., Health Care, Industrials, IT, Utilities, Real Estate, Materials, Consumer Staples), with a much higher mean sentiment coefficient.



Figure 1: Cross-Industry Differences in Volatility Response to Sentiment (p-values heatmap)

Figure 1 matrix reports FDR-adjusted p-values (Benjamini–Hochberg) Games-Howell pairwise test results for all 55 industry pairs; light cells indicate significant differences in industry means of the firm-level sentiment coefficients. The heterogeneity is pervasive: 40/55 pairs are significant. A clear separation emerges. Industries such as Information Technology and Health Care exhibit higher average sentiment coefficients than several capital-intensive or rate/commodity-exposed sectors — e.g., Financials, Real Estate, and Energy. Illustrative contrasts include IT vs Energy ($p \approx 0.0000$; mean diff ≈ 0.0151), Health Care vs Energy ($p \approx 0.0000$; ≈ 0.0148), IT vs Financials ($p \approx 0.0000$; ≈ 0.0119), and Health Care vs Real Estate ($p \approx 0.0000$; ≈ 0.0117). Together with Welch's ANOVA ($F=23.359$, $p<10^{-6}$, $\eta^2=0.218$—large), the evidence confirms strong cross-industry dispersion in how sentiment maps into conditional volatility.

3 DISCUSSION

Our results delineate a clear and nuanced narrative: investor sentiment is not a priced risk factor but a potent behavioral driver of market activity (trading volume) and volatility, with effects that are critically contingent on industry context. First, the near absence of significant sentiment coefficients in the Fama-French models challenges the view that sentiment consistently predicts cross-sectional expected returns. This suggests that the AAII sentiment measure, while capturing prevailing investor mood, may be largely arbitraged away in equity pricing or subsumed by the established five factors for the average stock. This finding aligns with models of limited arbitrage where mispricing exists but is not easily captured by linear factor exposures. The profound cross-industry heterogeneity, particularly in volatility sensitivity ($\eta^2 = 0.218$), is our central finding. It reveals that sentiment's impact is not uniform but is instead filtered through sector-specific characteristics. Cyclical, opaque, and commodity-driven sectors (e.g., Energy, Financials) are highly susceptible, while defensive and growth sectors (e.g., Health Care, IT) are more insulated.

This has critical implications. For asset pricing theorists, our results argue for behavioral models that explicitly incorporate industry heterogeneity. For practitioners, it provides a refined map of risk, highlighting that portfolios concentrated in cyclical sectors are not only exposed to fundamental economic cycles but are also disproportionately vulnerable to waves of investor sentiment. Therefore, this confirms that models of asset pricing and market behavior must account for industry-specific effects to accurately capture the role of investor sentiment.

Several limitations warrant acknowledgment. Our analysis is confined to the U.S. market and a specific time period, and despite our robust models, the potential for omitted variables remains. This naturally suggests fruitful avenues for future research. Extending this analysis to other global markets, incorporating alternative sentiment proxies from news or social media, and examining how sensitivity varies across industry cycles would greatly strengthen the external validity and nuance of these findings. Ultimately, accurately capturing the role of investor sentiment requires accounting for its industry-specific effects.

Acknowledgement

The research was financially supported by the SGS project no. SP2025/043 of VSB – Technical University of Ostrava.

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CONFIRMATORY FACTOR ANALYSIS – TESTING OF A MODEL FOR MEASURING SOCIAL MARKETING ORIENTATION OF END USERS IN AN EDUCATIONAL SYSTEM

Zoran Mihanović
University of Split, Faculty of Economics, Business, and Tourism
Cvite Fiskovića 5, 21000 Split, Croatia
zmihanov@efst.hr

Abstract: Within the framework of this research, the instrument for measuring social marketing orientation was defined and tested by confirmatory factor analysis in the context of final user (students) in primary and secondary education institutions in Croatia. From the research, it was determined that the constructs are highly correlated, have a positive direction and a strong intensity. The significance of the model as a whole has been confirmed. When the measurement model is validated and satisfactory, it is possible to measure the degree of social marketing orientation towards end users and link it to the performance of institutions.

Keywords: confirmatory factor analysis; social marketing orientation, education system; end users

1 THEORETICAL CONSIDERATIONS

The social marketing orientation states that the main task of the institution is in determining the needs, desires and interests of its consumers and in its adaptation to enable satisfactions that preserve or increase the well-being and long-term interests of the users and society as a whole [15]. When it comes to the for-profit sector, the term "marketing orientation" is appropriate. The term "social marketing orientation" has been accepted as a proposal for a term defining the implementation of marketing orientation in the non-profit and public sectors (education system). In the 20th century, social marketing orientation was not significantly applied in public sector institutions for various reasons. Namely, political philosophy was not aligned with the marketing concept, and it was also believed that the lack of competition for predominantly state-owned enterprises did not justify the use of marketing tools and a more significant marketing focus. In addition, financial resources were significantly limited and managers did not consider it expedient to invest in public sector marketing [1]. It was also believed that satisfying consumer needs and expectations was the primary task of the for-profit sector, while the purpose of the public sector should primarily be to enforce laws and respect procedures within its domain of activity. Thus, it became common knowledge that increasing end-user satisfaction should not be a priority for the public sector [10]. However, this work relies on the doctrine of new public management with its origins in managerial approaches that advocate a higher degree of marketing orientation than before. Likewise, these approaches believe that networking of different stakeholders in the public sector is an important way of increasing the efficiency of the public sector. In addition, they encourage decentralization and a lower level of hierarchy [6], [21]. The use of marketing tools in the practice of public sector institutions and the adoption of a customized concept of marketing through a social marketing orientation enables better execution of services that meet the needs of users and are better directed towards the neediest [3]. In this paper, emphasis is placed on the perspective of the end users of the services of public sector institutions, i.e. educational system. Looking at the broader perspective of social marketing orientation research resulted in looking at the approach of multiple stakeholder market orientation. It is this stakeholder approach that has shown the positive impact of such social marketing orientation on the performance of public sector institutions and is often preferred over the existing paradigm of market orientation [4], [5], [7], [17]. Although it seems realistic to expect that social marketing orientation has a positive

impact on the performance of institutions providing public services, some research has found a negative impact of social marketing orientation on performance or service quality [8], [22]. Therefore, it is necessary to test the research model and measurement instrument for specific institutions and conduct research to determine the degree, direction and intensity of the connection. This research tested an adapted and newly created questionnaire for measuring social marketing orientation by end users, students of primary and secondary education institutions. Based on the study of the theoretical framework, it seems that the research area from the perspective of end users in general and in the context of social marketing orientation has been very little researched, and in the context of primary and secondary education students it has not been researched at all. The questionnaire that is tested consists of questions that describe the social marketing orientation towards the end users of primary and secondary education institutions. The adapted measure MARKOR (Measure of Market Orientation) was used as a measure for measuring social marketing orientation. The questionnaire was originally taken and adapted for the purposes of this research from the research of Kohli, Jaworski and Kumar [13], [14], and its reliability and validity were investigated in the educational sector mainly through the higher education sector [9], [11], [18], [20].

2 SUITABILITY OF THE MEASUREMENT MODEL OF SOCIAL MARKETING ORIENTATION AND RESULTS – MEASURED BY END USERS (STUDENTS)

In this study, confirmatory factor analysis was applied to confirm whether the theoretically established measurement model for measuring the degree of social marketing orientation by end users, in this case students, is appropriate. In this way, the validity of the measurement instrument is also checked. Confirmatory factor analysis is applied for each construct individually. Constructs are unobservable variables that become measurable by applying the measurement model proposed here. The levels of social marketing orientation are exogenous constructs (observed as causes), while the levels of performance are endogenous constructs (observed as consequences) since the influence of social marketing orientation on performance is assumed. Of course, the exogenous construct of the degree of social marketing orientation becomes endogenous when it is decomposed into its components. There are three components to the social marketing orientation questionnaire: (1) intelligence generation; (2) intelligence dissemination and (3) reaction.

The model that combines measured models with a structural model is known in the literature as Structural Equation Modeling (SEM). Ultimately, SEM combines a set of regression equations with confirmatory factor analysis, and its main advantage is that it can simultaneously analyze a set of multiple cause-and-effect relationships between unobserved variables or variables that are not directly measurable [2]. In structural equation modelling, the standard method for estimating unknown parameters is the Maximum Likelihood (ML), which requires a large number of observations given the large number of dimensions of constructs that are formed according to inherent manifest variables. Since each manifest variable is one question from the questionnaire, and there are many questions, consequently the entire model would be oversized and a large number of parameters would have to be estimated, which is not possible with the ML method on such a large sample of schools and students. To avoid this, confirmatory factor analysis is used, and then regression analysis individually, after the constructs become measurable (either endogenous or exogenous) [12]. Before confirming whether the measurement model is appropriate in the context of conducting inferential statistics based on the sample and appropriate tests, descriptive statistics of the students is provided. The research collected a total of 1,476 valid, or correctly completed, questionnaires from end users, students, 16% (235) from primary schools and 84% (1,241) respondents from secondary schools. Therefore, confirmatory factor analysis was conducted to identify and confirm the

three dimensions of social marketing orientation by users (students) and to verify the validity of the social marketing orientation measurement model itself. The measurement model of the confirmatory factor analysis is shown in Figure 1. Each of the three dimensions of social marketing orientation (intelligence generation, intelligence dissemination and reaction) represents one construct that is measured by the corresponding manifest variables, or individual questions from the questionnaire. There are nine manifest variables related to the construct "intelligence generation", six related to the construct "intelligence dissemination", and nine to "reaction". The Figure 1 shows the standardized factor loadings between each manifest variable and the corresponding construct. From Figure 1, it is important to note that the manifest variables "IG5" and "IG9" do not contribute to the construct "intelligence generation", just as manifest variable "R21" does not contribute to the construct "reaction". For the above reason, the following questions are omitted from the questionnaire: "IG5- The school is slow to detect changes in the wishes of our students for certain school and extracurricular programs (e.g., the desire for new elective subjects or some new forms of extracurricular activities, sports activities, etc.); IG9- Our school is slow to notice new technologies (smart boards, newer and better computers), laws (e.g., the New Education Act) or competitors (schools that offer additional content unlike our school); R21- The services offered by our school are more related to what suits the school staff than to the needs of the students.

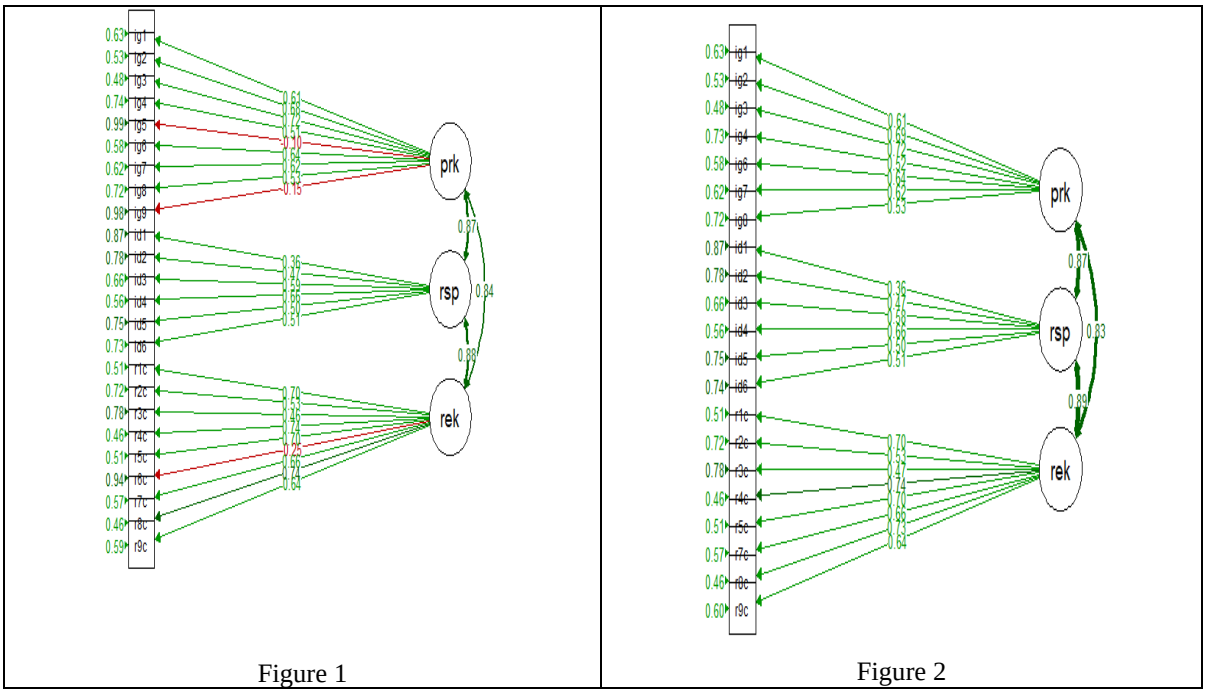


Figure 1 Figure 2

Figure 1 and 2: Graphical representation of the measurement model of confirmatory factor analysis with all manifest variables (Figure1) and with reduced manifest variables (Figure2)
Source: author constructions using R Studio within lavaan package

Therefore, given that the aforementioned manifest variables do not contribute to individual constructs, a factor analysis was conducted with reduced manifest variables and is shown in Figure 2. Figure 2 shows how all manifest variables contribute to individual constructs, and by comparing standardized factor loadings, it can be concluded that the manifest variable IG3- In our school, staff often ask us about our needs, desires, and satisfaction with school is the most important in creating the data collection construct. The manifest variable ID13- Information about our students' satisfaction or dissatisfaction reaches interested staff in our school through

conversations, on the school website, through the school magazine, or in a similar way is the most important in creating the data distribution construct. The manifest variable R23- When the school discovers that we students are dissatisfied with the quality of services at school, the school immediately reacts to provide better services is the most important in creating the reaction construct.

Table 1: Parameter estimates, parameter significance tests, covariances and variances

Number of free parameters			45	
Number of observations			1476	
Test statistic			1679.231	
Degrees of freedom			186	
P-value (Chi-square)			0.000	
Comparative Fit Index (CFI)			0.866	
Tucker-Lewis Index (TLI)			0.848	
RMSEA			0.074	
SRMR			0.013	
Latent Variables:				
Estimate	Std.Err	z-value	P(>|z|)	
ig =~				
ig1students	1.000			
ig2studnets	1.128	0.054	21.080	0.000
ig3studnets	1.161	0.053	21.747	0.000
ig4studnets	1.152	0.068	16.846	0.000
ig6studnets	0.980	0.049	20.109	0.000
ig7students	1.004	0.051	19.527	0.000
ig8students	1.165	0.068	17.121	0.000
id =~				
id1students	1.000			
id2students	1.766	0.159	11.116	0.000
id3students	1.566	0.130	12.068	0.000
id4students	1.617	0.129	12.538	0.000
id5students	1.222	0.107	11.418	0.000
id6students	1.407	0.122	11.509	0.000
reac =~				
r1studnets	1.000			
r2students	1.344	0.071	18.813	0.000
r3students	1.217	0.073	16.726	0.000
r4students	1.098	0.042	25.949	0.000
r5students	1.148	0.046	24.716	0.000
r7students	0.992	0.042	23.395	0.000
r8students	1.134	0.044	25.868	0.000
r9students	1.125	0.050	22.600	0.000
Covariances:				
Estimate	Std.Err	z-value	P(>|z|)	
ig ~~				
id	0.230	0.021	10.946	0.000
reac	0.347	0.022	15.961	0.000
id ~~				
reac	0.222	0.020	11.364	0.000

Source: author constructions using R Studio within lavaan package

The results are presented graphically and in tables. The tables include: measures of the measurement model's suitability; parameter estimates, i.e. factor loadings; tests of parameter

significance, covariance and variance. The parameter estimates in the table are unstandardized (with the first manifest variable assumed as the reference), while Figure 2 shows standardized values of estimates, which are, however, comparable to each other. Table 1 shows that a total of 45 parameters were estimated in the factor analysis model based on 1476 observations. The significance of all parameters in the model was tested using the chi-square test (explained below). The test value is 1679.231, and the associated empirical significance level is approximately 0, confirming the significance of the model as a whole.

The assessment of the overall suitability of the model is analyzed using the following suitability indicators: (1) absolute; (2) incremental; (3) parsimonious. Most measures of model fit are based on the observed difference between the estimated and actual covariance matrices. The goal is to reject the null hypothesis and accept the alternative hypothesis that the difference between the covariance matrices is not at the level of statistical significance. This was confirmed in the study by the chi-square test as a statistical test of the stated difference. To further determine the suitability of the model, it is proposed to analyze several suitability indicators. Since different softwares may produce fit indices in slightly different ways, it is important to note that all results in this study were obtained using the lavaan package (Latent Variable Analysis) in R Studio and were appropriately interpreted. Furthermore, the sample size and the number of manifest variables affect the target values of the suitability indicators. For example, the CFI and TLI indicators should be greater than 0.80 if there are more than 12 manifest variables. In this case, this condition is met. The Root Mean Square Error of Approximation (RMSEA) indicator should be less than 0.09. This condition is also met because the RMSEA in this case is 0.074.

A significant chi-square can be misleading when assessing model fit because the p-value is often underestimated in large samples. On the other hand, the test may fail to detect poor fit when the sample size is small. Therefore, the significance of the chi-square should be taken with caution, while greater emphasis should is placed to goodness-of-fit indices as well as the ratio between chi-square and degrees of freedom [12]. Considering all of the above, the measurement model of social marketing orientation for students appears to be satisfactory. After creating the constructs and confirming the validity of the measurement model, the relationships between them can be examined using regression analysis. That part concerns the structural parameters, that is, the structural model presented by a set of regression equations, which allow endogenous constructs to be regressed on exogenous constructs.

2.1 Correlation between dimensions of the social marketing orientation measurement model – measured by students

In the measurement model of factor analysis, a significant correlation is also observed between the constructs themselves, that is, the dimensions of social marketing orientation, which is stated in the following table. Table 2 shows that all constructs are highly correlated, with a positive direction and strong intensity, and the most correlated are reaction and intelligence dissemination (correlation coefficient 0.89). All correlations are statistically significant at the 1% significance level. By comparing the mentioned correlations in pairs, the importance of individual dimensions can be determined. The importance of “intelligence generation" is 33%, the importance of "intelligence dissemination" is 34%, while the importance of "reaction" is 33%. The mentioned importance is not particularly different, so it is concluded that they are equally important to students. When the measurement model is confirmed and satisfactory, the degree of social marketing orientation of end users can be accurately measured. In doing so, the values of manifest variables, to which an ordinal or sequential measurement scale belongs, are transformed into constructs of a numerical continuous measurement scale in the range from 0 to 1. Therefore, by combining manifest variables, constructs are obtained in a way that they

are normalized. By normalizing manifest variables, the value of the construct is achieved in the range from 0 to 1, which is interpreted as from 0% to 100%, which allows the degree of social marketing orientation to be accurately assessed and interpreted more simply.

Table 2: Correlation between different dimensions of social marketing orientation

		IG	ID	R
Inteligence generation (IG)	Pearson Correlation	1	,87**	,83**
	Sig. (2-tailed)		,000	,000
	N	1476	1476	1476
Dissemination (ID)	Pearson Correlation	,87**	1	,89**
	Sig. (2-tailed)	,000		,000
	N	1476	1476	1476
Reaction (R)	Pearson Correlation	,83**	,89**	1
	Sig. (2-tailed)	,000	,000	
	N	1476	1476	1476

**. Correlation is significant at the 0.01 level (2-tailed).

Source: author constructions using R Studio within lavaan package

3 CONCLUSION

In previous research on social marketing orientation, it was not found that anyone tested and define the measure for the social marketing orientation of minor end-users in the public sector, in this case in primary and secondary education institutions. This research tested an adapted and newly created questionnaire for measuring social marketing orientation among end users, students. From the research, it was determined that the constructs are highly correlated, have a positive direction and a strong intensity. Summing up, the measurement model of social marketing orientation for students proves to be satisfactory considering everything stated in the research. Finally, it can be concluded that the importance of a valid measuring instrument lies in the fact that the main assumptions of the theoretical part of the research is that the performance of the public sector can be increased if the implementation of the marketing concept is carried out well and if social marketing orientation is accepted.

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CRYPTO OWNERSHIP AND FINANCIAL LITERACY IN THE EUROPEAN UNION: A SPATIAL ECONOMETRIC ANALYSIS

Ivana Mravak
University of Split, Faculty of Economics, Business and Tourism
Cvite Fiskovića 5, Split, Croatia
E-mail: ivana.mravak@efst.hr

Abstract: This paper examines how financial literacy influences cryptocurrency ownership across European Union countries using spatial econometric analysis. The EU's common regulatory framework provides a unique setting to isolate this effect. Prior studies report mixed findings: some link financial literacy to greater crypto adoption, others to increased risk aversion. Results show financial literacy is the strongest positive driver of crypto ownership, while higher interest rates reduce it. GDP, inflation, and unemployment are not significant. Spatial spillovers across countries suggest unobserved factors shape ownership patterns. These findings highlight the value of enhancing financial education to support more informed crypto participation.

Keywords: cryptocurrency ownership, retail investors, financial literacy, European Union, spatial regression analysis

1 INTRODUCTION AND LITERATURE REVIEW

Over the past decade, cryptocurrencies have evolved from a small, emerging trend into an important element of financial markets. Although cryptocurrency markets are very volatile, this does not discourage people from investing in them and their popularity is still growing, especially among retail investors [2]. In Europe, interest in crypto assets remains high, as recent Eurobarometer surveys show that many people in European Union (EU) own or have owned cryptocurrencies [9]. A rising number of studies aim to identify the factors influencing crypto adoption and trading. Observed determinants include factors such as GDP, unemployment, institutional quality, VIX index, etc. [11,15,20,22]. Furthermore, recent research on cryptocurrencies highlights financial literacy as an important factor [13,17,18,21].

Financial literacy involves a comprehensive understanding of financial terminology, as well as the ability to apply this knowledge to make good financial decisions [19]. The growing use of cryptocurrencies, with their decentralized structure and complex technology, presents both opportunities and challenges for individuals possibly demanding a higher level of financial literacy to make informed decisions. This is in line with some studies that have shown positive relationship between financial literacy and crypto market participation [12,23]. However, some research suggest that higher financial literacy is associated with lower cryptocurrency ownership because of a greater awareness of its risks [13,17,18]. Such research suggest that individuals with a deeper understanding of financial principles may approach cryptocurrencies with caution and recognize the potential for volatility and losses. Overall, the relationship between financial literacy and cryptocurrencies deserves closer study.

In 2022, crypto trading in the EU was regulated under shared rules. At that time, all EU countries followed common anti-money laundering (AML) rules, and crypto platforms were required to register and verify their customers' identities. These rules were introduced through amendments to the EU's 5th Anti-Money Laundering Directive, which extended AML requirements to crypto asset service providers, including Know Your Customer (KYC) checks and the reporting of potentially suspicious transactions [24]. In 2023, EU adopted the Markets in Crypto-Assets Regulation (MiCA), a new legal framework that introduced licensing, consumer protection, and transparency requirements for crypto service providers. Although MiCA only started to apply gradually from July, 2024, with most rules in force by December 2024, its adoption had signaled the EU's commitment to consistent and predictable crypto

regulation. Compared to other countries with different or unclear rules and regulations regarding crypto trading, EU offered a more unified legal setting during this time [14], which helps isolate the effect of financial literacy in the analysis.

While investigating cryptocurrencies, most papers focus on price, trading or crypto adoption while this paper shifts focus on owning crypto. There are several reasons for that. Firstly, while the term cryptocurrency adoption encompasses a broad range of activities including trading volumes, payment use, business acceptance, etc., in practice, many of these dimensions are difficult to measure consistently across countries. This especially applies to trading volumes since people usually trade on international exchanges and there is limited availability on these data by countries. Ownership data, such as those from the Eurobarometer or similar surveys, provides more comparable cross-sectional data, making it better suited for cross-country analysis of influence of financial literacy. Secondly, focus of this paper is on retail investors. Other variables such as trading and prices are more influenced by big investors. Moreover, a significant amount of retail investors keep cryptos as a long-term investment and do not actively trade with it. Therefore, such people would not appear in short-term trading volumes [7]. A survey in Germany showed that only 27% of the individuals are classified as short-term investors that trade frequently, 41% have predominantly long-term investment interests and hold their cryptos for longer periods, while the remaining 32% are categorized as 'indifferent' [25]. Furthermore, ownership is less volatile than trading volume or prices, which are heavily affected by short-term market trends, investor sentiment, or speculative dynamics [6,25]. Using ownership as an indicator allows for a more stable analysis of crypto engagement across countries.

Key findings from previous literature on cryptocurrencies indicate a significant relationship between crypto activities and traditional market interest rates [3,16]. Most socioeconomic variables, such as GDP and inflation, although significant for crypto trading volumes [8,11], are shown not to be significant for crypto adoption [5]. From macroeconomic indicators, only unemployment has been shown to have positive impact on some crypto adoption aspects [4,26].

This paper contributes to the literature in several ways. First, it focuses on EU countries, where unified regulation enables a clearer analysis of financial literacy's role in cryptocurrency ownership. It addresses an unsettled question in existing research regarding the relationship between financial literacy and crypto ownership, using recent cross-country data. Unlike most prior studies that focus on trading or adoption, this paper shifts the focus to ownership as a more reliable indicator. Finally, it captures previously overlooked spatial spillover effects in crypto ownership across EU countries by applying spatial analysis.

2 DATA AND METHODOLOGY

Data regarding cryptocurrency ownership are taken from European Commision's Eurobarometer survey [9]. This survey, conducted in June 2022, included 27,862 respondents across all 27 EU countries. Therefore, the dataset provides cross-sectional data well suited for examining spatial variation in cryptocurrency engagement across Europe. Among other questions, respondents were asked whether they currently own or have previously owned cryptocurrencies which serves as the dependent variable (*CRYPTO*) in this research. Socioeconomic indicators used in the analysis include GDP, unemployment rates, interest rates, and the Harmonised Index of Consumer Prices (HICP) as a measure of inflation which is comparable across countries. These data were obtained from the World Bank database. Financial literacy data are also taken from a Eurobarometer survey conducted by European Commission [10]. In this paper, as an indicator of financial literacy, percentage of respondents who showed middle to high financial literacy is used (*FIN_LITERACY).* Although the fieldwork for the financial literacy data was conducted in March and April 2023, while other

data are from 2022, financial literacy is generally considered a stable attribute over short and medium time horizons, up to six years [1]. Therefore, the one-year difference is unlikely to affect the structural relationships explored in this study. Moreover, this survey is conducted on adults, in contrast to student-based surveys such as the OECD's PISA financial literacy assessment. Given that adults are already active participants in financial markets, this type of data is more suitable for the purposes of this research.

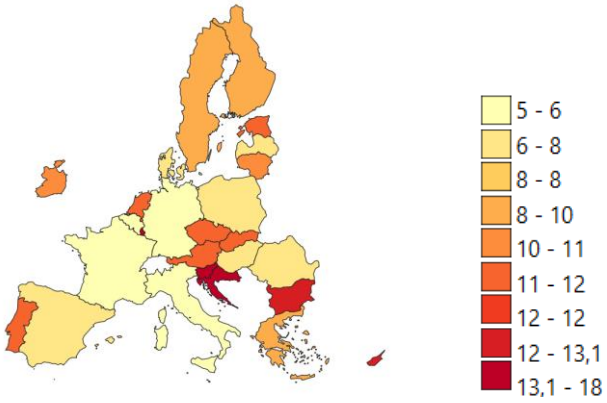


Figure 1: Percentage of respondents who own or have owned crypto assets.

Figure 1. shows percentage of survey respondents who own or have own crypto assets across EU countries. Figure suggests that crypto ownership data cluster together which motivates us to test if spatial dependencies exist. Although the reason for using spatial regression is not the spatial dependence solely in the dependent variable, but in residuals of regression, these values motivated us to conduct the spatial analysis.

A simple regression model is first estimated using Ordinary Least Squares (OLS)

$$Y = X\beta + \varepsilon \tag{1}$$

and the model is then tested for spatial autocorrelation in the residuals. This is done using Moran's I statistics. If spatial autocorrelation is present, then a more suitable spatial regression model needs to be chosen. To choose between different spatial regression models, Robust Lagrange Multiplier test is used which tests if spatial error, spatial lag term or both should be included in the model. Spatial error model described as:

$$\begin{aligned} Y &= X\beta + u \\ u &= \lambda W u + \varepsilon \end{aligned} \tag{2}$$

expands (1) with an error term u that consists of two parts. One part includes spatial error term Wu which incorporates the error terms from neighboring locations multiplied by the autoregressive coefficient λ and the other, ε, are the error terms specific to each location. W is the weight matrix:

$$w_{ij} = \begin{cases} 1, \text{ if } i \text{ and } j \text{ are neighbors} \\ 0, \text{ otherwise} \end{cases} \tag{3}$$

that is then normalized. On the other hand, spatial lag model (Spatial Autoregressive, SAR) written as

$$Y = \rho WY + X\beta + \varepsilon \tag{4}$$

expands the model (1) with the spatial lag WY multiplied by the spatial lag coefficient ρ.

3 RESULTS AND DISCUSSION

In the first regression, previously shown determinants of crypto adoption, namely unemployment and interest rates are used together with the financial literacy to assess crypto ownership. Results of running OLS on regression:

$$CRYPTO = \beta_0 + \beta_1 UNEMPLOY + \beta_2 INTEREST_RT + \beta_3 FIN_LITERACY + \varepsilon \quad (5)$$

are shown in Table 1.

Table 1: Results of OLS estimation of regression (5).

INTERCEPT	-14.431 (0.371)
UNEMPLOY	-0.1521 (0.597)
INTEREST_RT	-0.658 (0.117)
FIN_LITERACY	0.319* (0.096)
Moran's I	0.215** (0.027)
Adj. LM lag	2.525 (0.112)
Adj. LM error	3.666* (0.056)
Adj. LM sarma	3.941 (0.139)

Notes: * p < 0.1, ** p < 0.05, *** p < 0.01.

Results in Table 1 show spatial autcorrelation in residuals with observed Moran's I statistic value of 0.215 and p-value 0.027 meaning that significant spatial autocorrelation is present and it should be tested how to improve the model. *p*-value of adjusted lagrange multiplier test for error model is 0.056, for lag model 0.112, and for SARMA model which includes both spatial error and spatial lag term 0.139. Therefore spatial error model is used as described in (2).
The maximum likelihood (ML) estimator is now applied to the spatial error model (SEM):

$$CRYPTO = \beta_0 + \beta_1 UNEMPLOY + \beta_2 INTEREST_RT + \beta_3 FIN_LITERACY + u, u = \lambda W u + \varepsilon \quad (6)$$

Robustness checks are then made by replacing *UNEMPLOY* with other macroeconomic variables which were significant in some literature for crypto trading volumes: GDP and HICP.

Table 2: Results of ML estimation of spatial error model regression (6).

INTERCEPT	-11.987 (0.325)	-18.126 (0.120)	-21.189* (0.084)	-19.094 (0.113)
UNEMPLOY	-0.338 (0.141)			
GDP		0.151 (0.526)		
HICP			0.143 (0.383)	
INTEREST_RT	-1.035*** (0.003)	-0.934** (0.013)	-0.943** (0.012)	-0.905** (0.013)
FIN_LITERACY	0.312** (0.028)	0.354** (0.014)	0.377** (0.010)	0.371** (0.011)
λ	0.357** (0.046)	0.466** (0.046)	0.191** (0.033)	0.241**(0.011)

Notes: * p < 0.1, ** p < 0.05, *** p < 0.01.

Results in Table 2 reveal three key patterns. First, financial literacy shows a positive and statistically significant coefficient across all specifications of the model indicating that higher financial-literacy scores are associated with higher levels of owning or having owned cryptocurrencies. Second, the interest rate (*INTEREST_RT*) coefficient is negative and significant in every regression, suggesting that higher interest rates correspond to lower levels of crypto ownership. Third, the spatial-error coefficient (λ) is positive and significant in all four models confirming that unobserved shocks in one EU country tend to spill over into

neighboring countries, and that OLS would have been biased had spatial dependence been ignored. GDP, HICP and unemployment are insignificant when included, implying that macroeconomic variables other than interest rates do not drive cross-country differences in crypto ownership within the EU's regulated environment.

The positive association between financial literacy and crypto ownership aligns with prior studies that argue that financially more literate investors feel more comfortable navigating decentralized, technologically complex assets [12,23]. Those authors claim that when individuals understand financial concepts they are likelier to trust and adopt cryptocurrencies. On the other hand, our findings diverge from research suggesting that greater financial literacy breeds skepticism thus leading to lower crypto holdings due to heightened awareness of volatility and regulator uncertainty [13,17]. In the EU context, where uniform anti-money-laundering rules have reduced regulatory ambiguity, financially literate individuals may perceive cryptocurrencies more as legitimate investment opportunities rather than speculative pitfalls.

The negative influence of interest rates corroborates studies highlighting that when traditional assets yield higher returns retail investors shift away from speculative assets such as crypto. Basher and Sadorsky [3] also found that rising interest rates depress Bitcoin price growth, and Bezirgan [4] noted that regions with higher real-yield opportunities exhibit lower crypto search intensity. Within our EU sample, higher interest rates likely signal more attractive returns on liquid, low-risk instruments (e.g. savings accounts, government bonds), reducing the appeal of volatile crypto holdings. Meanwhile, the insignificance of GDP and HICP suggests that, once interest rates and financial literacy are accounted for, overall economic size or inflation differences across EU members do not meaningfully explain cross-country variation in crypto ownership.

Finally, the significance of λ underscores the importance of spatial econometric techniques when modeling crypto ownership. This means that some unobserved factors such as cultural diffusion or cross-border media are correlated among neighboring countries. Collectively, these results imply that, within the EU's comparatively homogeneous regulatory regime, individual-level attributes (financial literacy) and interest rates, along with spatial spillovers, jointly shape national rates of crypto ownership.

4 CONCLUSION

In summary, findings in this paper underscore financial literacy as the most important factor driving cryptocurrency ownership in EU countries. This implies that individuals who understand digital assets are more likely to hold crypto. Lower interest rates also play a role. When traditional savings and bonds look less attractive, people turn toward more speculative assets like cryptocurrencies. The positive spatial-error effect suggests that unobserved regional influences, whether shared policies, subtle regulatory differences, or cultural and media spillovers further shape adoption patterns across neighboring countries. Taken together, these results imply that strengthening financial education should be the top priority for policymakers aiming to foster informed crypto engagement. Complementing this with sensible interest-rate monitoring and more harmonized crypto rules across borders will help guide adoption toward responsible use. On the other hand, traditional macroeconomic indicators like GDP, inflation and unemployment are insignificant.

This paper uses cross-sectional data and therefore reflects relationships at a single point in time. In future research, it would be useful to include longitudinal data, which was not possible here due to data limitations. Future studies could also examine the role of behavioral factors and cultural differences between countries, which may further explain variations in cryptocurrency ownership.

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EURO ADOPTION IN CROATIA: ASSESSING THE IMPACT ON INFLATION

Josip Visković
Faculty of Economics, Business and Tourism, Department of Finance
Cvite Fisković 5, Croatia
jviskovi@efst.hr

Slavko Šodan
Faculty of Economics, Business and Tourism, Department of Accounting and Auditing
Cvite Fisković 5, Croatia
ssodan@efst.hr

Paško Burnać
Faculty of Economics, Business and Tourism, Department of Finance
Cvite Fisković 5, Croatia
pburnac@efst.hr

Abstract: This study examines the impact of the introduction of the euro in Croatia on 1 January 2023 on inflation. It uses a difference-in-differences method with panel data from the EU member states. The results suggest that there is no statistically significant impact on headline inflation, which is consistent with previous empirical results that estimate only marginal price effects following the introduction of the euro. However, there is a pronounced inflation reaction in the restaurants and hotels sector, where prices have risen by 1.38 percentage points compared to the control group. This sectoral heterogeneity is in line with theoretical expectations and previous cross-country findings and reflects the influence of market concentration, lower price transparency and rounding practises. Although measured inflation remained moderate, consumer perceptions indicate a heightened sense of price increases, characterised by frequent experience of rising service prices and reinforced by media discourse. The results emphasise the need for a detailed, sector-specific analysis and call for targeted policy interventions to ensure price stability during the currency changeover, particularly in vulnerable service sectors.

Keywords: euro adoption, Croatia, inflation, difference-in-differences

1 INTRODUCTION

On 12 July 2022, the European Council formally approved Croatia's accession to the eurozone and set the conversion rate of the kuna at 7.53450 per euro. On 1 January 2023, Croatia officially adopted the euro and became the 20th member of the eurozone (Falagiarda & Gartner, 2023). In contrast to previous enlargements of the euro area, the transition in Croatia took place against a backdrop of unprecedented inflationary pressures. Moreover, the transition coincided with one of the most significant monetary tightening cycles in history, which made the transition even more complex. Geopolitical uncertainties also exacerbated the economic and political challenges associated with the introduction of the euro (Vujčić, 2024) and increased concerns about possible price increases, as observed in other euro area countries.

Empirical evidence suggests that the introduction of the euro may lead to modest inflationary effects, particularly due to price rounding practises, conversion costs and adjustments to companies' pricing strategies. These factors primarily affect frequently purchased goods and services and contribute to short-term price increases. While the inflationary impact of the introduction of the euro was generally limited and ranged between 0.05 and 0.4 percentage points, there are differences between countries (Pufnik, 2018). For example, Finland recorded the estimated inflationary effect with an increase of 0.27 percentage points, while Italy recorded a minimal increase of only 0.004 percentage points (Sturm et al., 2009). However, consumers' perception of inflation often exceeds the actual inflation rate,

which is largely influenced by media reports and psychological distortions. Many EU citizens have perceived a significant price increase after the introduction of the euro, although empirical results indicate a relatively moderate impact. This discrepancy arises because price changes are unevenly distributed across product categories and frequently purchased items are disproportionately affected, which in turn influences consumers' inflation expectations (Dziuda and Mastrobuoni, 2009).

A similar pattern was observed in Estonia, where the actual impact of the introduction of the euro on inflation was relatively small, but public perception suggested a significant increase in consumer prices. Studies have shown a marked divergence between perceived and measured inflation, with perceived inflation rising sharply even though actual price changes were only marginal. This phenomenon is partly attributed to the more pronounced inflationary impact on food prices, a category to which consumers are particularly sensitive due to their frequent purchasing cycle (Rõõm and Urke, 2014). Such findings highlight the limitations of aggregate inflation metrics, which can mask sector-specific price increases that have a disproportionate impact on consumer sentiment.

According to Pufnik (2018), the following sectors are most affected by price increases following the introduction of the euro in various eurozone countries include: Cleaning, repair and rental of clothing (eurozone, France, Germany, Estonia); restaurants and cafés (eurozone, Finland, France, Germany, Slovenia, Slovakia, Latvia); hair salons and beauty centers (eurozone, France, Germany, Spain, Estonia, Latvia, Lithuania); recreational and sports services (Finland, Ireland, Slovenia); and services related to the maintenance and repair of residential buildings (Finland, Slovakia, Estonia, Latvia). These results suggest that the short-term inflationary effects in the services sector are more pronounced during the euro changeover.

This study aims to analyse whether Croatia’s experience with the introduction of the euro follows a similar pattern to that observed in other euro area countries. In particular, it analyses whether short-term price increases have been observed, both in terms of overall inflation and sector-specific price dynamics. The rest of the paper is organised as follows: Section 2 reviews the relevant literature. Section 3 describes the data and methodology. Section 4 presents the empirical results and discussion, while section 5 contains concluding remarks.

2 LITERATURE REVIEW

Inflationary pressures after the introduction of the euro were particularly pronounced in Spain and were fuelled by asymmetric demand shocks, lower interest rates and reduced monetary policy autonomy. Lower interest rates stimulated demand beyond supply capacity and exacerbated inflationary tendencies. In addition, inflation expectations were self-fulfilling, pushing up price levels further, and political discourse reinforced inflation concerns as policymakers emphasised significant price increases in eurozone countries (Nowaczyk & Hernik, 2020).

Empirical analyses provide mixed results regarding the inflationary impact of the euro. Angeloni, Aucremanne and Ciccarelli (2006) found no significant inflationary impact when the euro became an electronic unit in 1999. The introduction of the physical currency in 2002 led to a temporary increase in the frequency of price adjustments, but this soon normalised again. However, the decreasing persistence of inflation since the mid-1990s suggests that it was not Economic and Monetary Union (EMU) but more general macroeconomic trends that were responsible.

Dziuda and Mastrobuoni (2009) used a treatment and a matched control approach to assess inflation after the introduction of the euro (further details on matched-sample research methodology can be found in Šodan (2025)). Their analysis of the Harmonized Indices of

Consumer Prices (HICP) showed that inflation was more pronounced for low-priced goods. This effect was particularly strong in countries where consumers had difficulty adapting to the new currency, leading to a perception of inflation that went beyond actual price movements. Similarly, Rõõm and Urke (2014) used a difference-in-differences method to analyse the introduction of the euro in Estonia. In this method, the treated group (Estonia) was compared with a control group consisting of other EU member states. The analysis used quarterly time series of the HICP compiled from monthly data from Eurostat. They found a modest impact on inflation, particularly in the services sector, with an increase in inflation rates of only 0.35 percentage points. However, the inflation perceived by consumers was significantly higher than the measured rates, due to psychological biases related to food price changes. The authors showed that the euro changeover led to distortionary price effects, particularly affecting cheaper goods, which contributed to consumers' perceptions of higher inflation even though average inflation in the eurozone remained relatively low. This discrepancy shows how analysing overall inflation can mask specific price increases for commonly purchased items.

Retailers played an important role in shaping the perception of inflation. Dziuda and Mastrobuoni (2009) argue that lower price transparency during the transition allowed retailers to raise prices, especially for cheaper goods. Consumers found it difficult to compare prices, which weakened the competitive pressure on retailers. As a result, price increases on frequently purchased items contributed to perceived rather than actual inflation. Market concentration played a moderating role, as larger retailers increased price transparency and thus mitigated the inflationary effects. Similarly, Rõõm and Urke (2014) confirmed that a negative correlation between market concentration and euro-related inflation suggests that larger retailers may have adopted practises that increased price transparency. Market dynamics also influenced inflation trends during the currency changeover. Companies often rounded up prices to make the euro amounts psychologically more attractive, especially for cheaper items. Consumer expectations also influenced the perception of inflation, as the expected price increases prompted companies to adjust their pricing strategies accordingly. In highly competitive markets, however, companies were under greater pressure to keep their prices stable. The reduction in transaction costs and exchange rate risks associated with the introduction of the euro may also have contributed to price stabilisation, especially for consumer durables (Pufnik, 2018).

Kotlinski (2020) concluded that the introduction of the euro in Central and Eastern European countries has led to a decline in overall inflation in the medium term. However, in certain sectors such as the restaurant and hotel industry as well as alcohol, tobacco and narcotics, there was a faster rise in prices, which may reflect sector-specific price behaviour and influence public perception. There is often a gap between perceived and actual inflation, which is partly due to sector-specific price changes and the influence of the media.

The service sector recorded the most significant price increases after the currency changeover. Personal services such as hairdressing, restaurants and dry cleaning saw significant price increases, which increased the perception of inflation (Rõõm & Urke, 2014). Hobijn, Ravenna and Tambalotti (2006) also found that restaurant prices rose sharply after the introduction of the euro. Eurostat assessments suggest that country-specific factors have influenced inflationary outcomes. Differences in economic conditions, market structures and consumer behaviour contributed to different inflation trajectories. Rising prices in the services sector had only a limited impact on overall inflation due to their small share of the Consumer Price Index (CPI) basket (Pufnik, 2018). Studies confirm that the hospitality sector, particularly restaurants and cafés, experienced the highest price adjustments. Conversely, the prices of goods remained stable, except for frequently purchased items, where rounding effects contributed to significant price increases.

Recent research on Croatia's euro adoption further elucidates these trends. Falagiarda et al. (2023) employed a multi-method approach, benchmarking monthly price data against historical

trends to isolate anomalies. Their analysis estimated that headline inflation would have been 0.4 percentage points lower without the euro transition. Despite stable prices for 65% of products, services, especially in bars, restaurants, and personal services, experienced higher month-on-month inflation. These findings align with earlier studies highlighting the disproportionate inflationary effects on services.

Sorić (2024) applied the Synthetic Control Method (SCM) to construct a counterfactual Croatia without the euro transition, comparing inflation trends with EU economies. Using the Augmented Synthetic Control Method (ASCM) to correct for bias, the study identified Spain, Slovakia, Malta, Portugal, and Slovenia as key comparable economies. The findings indicate a modest inflationary impact, with the most affected sectors being food, clothing, and restaurants. Notably, clothing prices accounted for 61.97% of category-specific inflation in July 2023, driven by rounding effects, retailer pricing strategies, retailers' desire to generate extra profit and market concentration. Similarly, the reduced price transparency following the changeover meant that consumers had a harder time comparing prices and detecting price hikes, leading retailers to take advantage of this by raising prices on cheaper goods disproportionately.

3 DATA DESCRIPTION AND METHOD

This study uses the aggregated indices of Eurostat's Harmonised Indices of Consumer Prices (HICP), which are aligned with the COICOP (Classification of Individual Consumption by Purpose) framework. While consumer price data is reported by Eurostat at monthly intervals, macroeconomic variables such as GDP growth and the unemployment rate are only available at quarterly intervals. In order to harmonise the temporal resolution, the HICP data are converted into quarterly inflation rates. The dataset comprises 29 European Union (EU) Member States, including the United Kingdom (Member State until 31 January 2020), and covers the period from the third quarter of 2013 to the fourth quarter of 2024.

To assess the causal impact of euro adoption on inflation, a difference-in-differences (DiD) method is applied. The treatment group comprises countries that adopted the euro during the period under study — Croatia, Latvia and Lithuania — while the control group consists of other EU Member States. The DiD approach compares the differences in inflation rates between the treatment period (defined as a four-quarter window around the euro adoption that includes two quarters before and after the adoption) and the non-treatment periods for the countries that adopted the euro compared to the corresponding differences in the control group.

The validity of the DiD estimator depends on the assumption of parallel trends: without the introduction of the euro, the inflation trajectories in the treatment and control countries would have developed similarly over time. Following the model specification proposed by Rõõm and Urke (2014), the cyclical economic dynamics are taken into account by including GDP growth and the unemployment rate as covariates. This inclusion strengthens the plausibility of the assumption of parallel trends over longer horizons. To account for demand-side dynamics and price rigidities, lagged GDP growth rates (with a lag of two and three quarters) are included in the regression framework. In addition, a lagged dependent variable (with a lag of one quarter) is included to attenuate the autocorrelation arising from the inertia of inflation.

The empirical specification, adapted from Rõõm and Urke (2014), is formalized as:

$$HICP_{it} = \beta_0 + \beta_1 HICP_{i,t-1} + \beta_2 GDP_{i,t-2} + \beta_3 GDP_{i,t-3} + \beta_4 U_{i,t-1} + \beta_5 (C_i * Euro_t) + \Sigma\beta Q_t + \Sigma\beta C_t + \varepsilon_{it} \quad (1)$$

where *i* stands for a sub-index for countries and *t* for time periods (quarters), *HICP* (Harmonised Indices of Consumer Prices) for the inflation rate, *GDP* for the growth rate of gross domestic product, *U* for the unemployment rate, *C* for a dummy for countries, *Euro* for ae dummy for the period of the euro introduction (four-quarter window around the introduction

of the euro, comprising two quarters before and after the introduction), *Q* for a dummy for quarters. Before estimation, the stationarity of the inflation series is evaluated through panel unit root tests, a prerequisite for the validity of the specified model.

4 RESULTS AND DISCUSSION

The difference-in-differences (DiD) methodological approach used in this analysis requires the fulfilment of stationarity assumptions within the time series data. To rigorously assess this requirement, a Fisher-type unit root test — a composite procedure that integrates the statistics of the augmented Dickey-Fuller test (ADF) across the cross-sectional units — is applied. The test results decisively reject the null hypothesis of non-stationarity (i.e. the presence of a unit root) in all specifications at the usual significance thresholds. This empirical evidence confirms the stationarity of the series, thus underpinning the statistical validity of the DiD framework and mitigating concerns regarding spurious regression bias.

The difference-in-differences (DiD) analysis shows different inflation dynamics at the aggregate and sectoral level after the introduction of the euro in Croatia. Table 1 shows the results for the Harmonised Index of Consumer Prices (HICP) across all items, which show no statistically significant euro-induced inflation at the aggregate level. The coefficient for the Croatia × euro interaction term is 0.006 (p = 0.990), suggesting that the introduction of the euro has not systematically changed overall inflation trends compared to EU countries that have not adopted the euro. This is in line with the more general findings of Pufnik (2018) and Falagiarda et al. (2023), who estimate the contribution of the euro to annual inflation at 0.05–0.40 percentage points, suggesting that macroeconomic factors dominated price movements during the transition period.

Table 1: Results of the model with HICP (all items)

$HICP_t$	*Coef.*	*St.Err.*	*t-value*	*p-value*
$HICP_{t-1}$	0.858***	0.015	56.80	0.000
GDP_{t-2}	0.010	0.014	0.73	0.464
GDP_{t-3}	0.022	0.014	1.52	0.129
U_{t-1}	-0.008	0.018	-0.47	0.639
*Croatia*Euro*	0.006	0.476	0.01	0.990
*Latvia*Euro*	0.207	0.912	0.23	0.820
*Lithuania*Euro*	-0.361	0.474	-0.76	0.447
Constant	0.126	0.282	0.45	0.656
R-squared				0.949
F-test				281.061
Prob > F				0.000
Number of obs				1247
Quarter fixed effect				Yes
Country fixed effect				Yes

Source: author's calculation

Figure 1 illustrates the trajectory of the aggregate HICP, with the red line marking the date of the introduction of the euro. While the index shows increased inflation at the end of 2022 (consistent with general trends in the eurozone), the post-euro adoption slope remains congruent with pre-existing patterns, supporting the regression-based conclusion of limited aggregate effects. However, Croatia's transition took place under special circumstances - it was the first euro changeover in a period of high inflation in the eurozone. This context may have

amplified certain transmission channels while masking others, as the underlying inflationary pressures may have allowed firms to make larger price adjustments than would have been possible under stable price conditions. This divergence emphasises the importance of sectoral analysis in capturing price dynamics in the Eurozone.

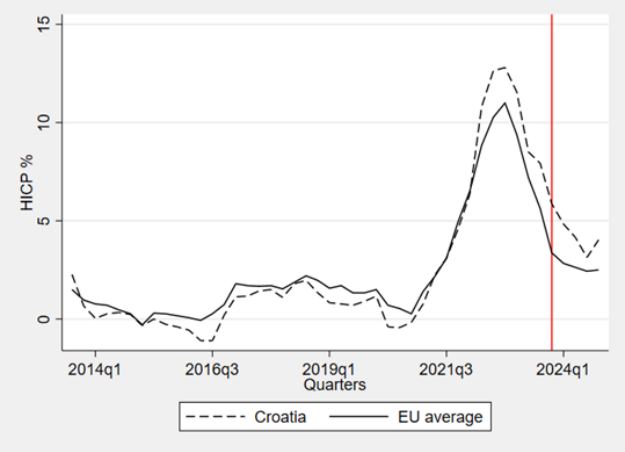


Figure 1: Harmonised Indices of Consumer Prices (HICP) for all items
Note: Red line represents Euro adoption in Croatia

Table 2 shows considerable sectoral heterogeneity, particularly in the restaurants and hotels category. The Croatia × euro coefficient of 1.382 (p = 0.035) indicates a pronounced inflation effect, with prices in this sector increasing by 1.38 percentage points more than in the control group during the four-quarter treatment period. This result confirms historical patterns observed in previous euro adoptions, where the services sectors showed increased sensitivity to currency movements due to lower price transparency and rounding practices (Sorić, 2024). Figure 2 visually underlines this result and shows a clear upward deviation of the HICP for restaurants and hotels immediately after the January 2023 introduction date, which deviates from the trend before the changeover.

Table 2: Results of the model with HICPH (restaurants and hotels)

$HICPH_t$	*Coef.*	*St.Err.*	*t-value*	*p-value*
$HICPH_{t-1}$	0.823***	0.017	49.52	0.000
GDP_{t-2}	-0.005	0.025	-0.18	0.853
GDP_{t-3}	-0.007	0.025	-0.27	0.785
U_{t-1}	-0.029	0.024	-1.17	0.241
*Croatia*Euro*	1.382**	0.656	2.11	0.035
*Latvia*Euro*	1.519	1.250	1.21	0.225
*Lithuania*Euro*	0.484	0.650	0.75	0.456
Constant	0.653*	0.386	1.69	0.091
R-squared				0.920
F-test				174.516
Prob > F				0.000
Number of obs				1247
Quarter fixed effect				Yes
Country fixed effect				Yes

Source: author's calculation

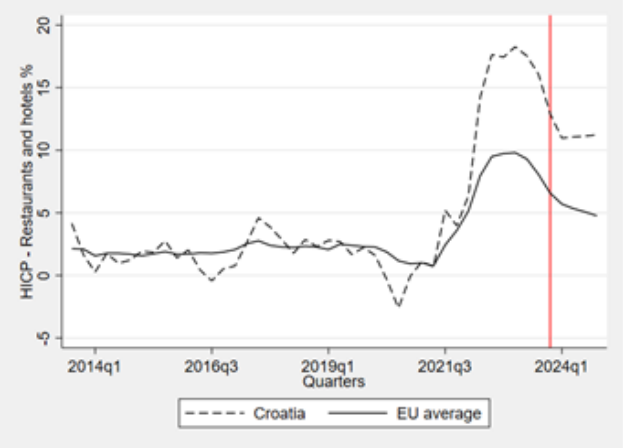


Figure 2: Harmonised Indices of Consumer Prices (HICP) for restaurants and hotels
Note : Red line represents Euro adoption in Croatia

The pronounced impact on restaurant and hotel prices deserve particular attention, as they are in line with theoretical predictions and empirical findings from other countries. The vulnerability of the restaurant and hotel sector stems from structural features typical of personal services: localised market power, infrequent consumer price comparisons and labour-intensive cost structures. These factors create conditions under which firms can impose price increases with limited competitive pressure, especially for services that consumers rarely buy and where price comparisons are difficult.

Despite modest measured effects, survey data show that most Croatian consumers perceived significant euro-induced price increases (Vujčić, 2024). This perception gap mirrors patterns observed in other countries and reflects the psychological and behavioural factors that influence how consumers process and interpret price changes during the currency changeover. The concentration of price increases on frequently purchased services amplifies these perceptual effects, as consumers see higher prices for items they buy regularly, while they may overlook price stability in other categories.

The role of media coverage and public discourse in shaping these perceptions should not be underestimated. Previous research has shown how media coverage and political rhetoric can influence inflation expectations and consumer behaviour during currency changes (Pufnik, 2018). In the case of Croatia, the timing of euro adoption at a time of already elevated inflation may have heightened the public's sensitivity to price changes, making it difficult for consumers to distinguish between euro-induced effects and broader macroeconomic trends. This context emphasises the importance of clear communication strategies and monitoring mechanisms during the currency changeover.

5 CONCLUSION

This study provides a nuanced assessment of the inflationary consequences of euro adoption by using a difference-in-differences methodology to evaluate Croatia’s recent experience. While no significant inflationary impact was observed at the aggregate level, the analysis at the sectoral level, particularly in the restaurant and hotel sector, revealed a measurable price increase of 1.38 percentage points, which is consistent with similar patterns observed in other countries. These results highlight the importance of disaggregated analysis when assessing macroeconomic transitions, as aggregate stability can mask sector-specific vulnerabilities.

The Croatian experience reconfirms the role of the euro as an anchor of stability for small and open economies, with the elimination of exchange rate risk outweighing temporary

inflation costs. However, the presence of concentrated inflationary effects in the services sectors underlines the need for structural precautions, especially in markets with limited competition, where market power and reduced price transparency can favour excessive price increases. From a policy perspective, the proactive measures taken by the Croatian authorities, such as dual pricing and enhanced market surveillance, appear to be effective in containing overall inflationary pressures. These tools, together with robust price monitoring, enforcement of competition policy and transparent public communication, should be an integral part of any future currency changeover strategy.

Finally, while the applied methodological framework is well suited to identify causal effects, some limitations remain. The short observation period after the introduction and the aggregation of different country experiences may dilute the specific national dynamics. Future research should aim to extend the time frame of the analysis in order to better understand the medium to long-term effects of euro adoption.

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SENSITIVITY OF PROFITABILITY DETERMINANTS TO THE CHOICE OF INDICATORS: A PANEL STUDY OF THE CROATIAN FOOD INDUSTRY

Josipa Višić
University of Split, Faculty of Economics, Business and Tourism
Cvite Fiskovića 5, 21000 Split, Croatia
E-mail: josipa.visic@efst.hr

Petra Penga
University of Split, Faculty of Economics, Business and Tourism
Cvite Fiskovića 5, 21000 Split, Croatia
E-mail: ppenga00@live.efst.hr

Abstract: This paper examines the impact of firm determinants on the profitability of companies in the food industry in Croatia in the period from 2014 to 2023. The main objective was to assess how sensitive these determinants are to the choice of profitability and size variables, using ROA, ROE and profit margin as dependent variables and two different size measures. The results show that the effects of indicator choice vary significantly depending on the profitability and size indicator used, especially in relation to firm size and age. These results clearly highlight the significance of indicator selection and underscore the inherent complexity of profitability analysis.

Keywords: profitability determinants, panel data analysis, firm size, firm age, food industry, Croatia

1 INTRODUCTION

The food industry is an important branch of manufacturing in the Republic of Croatia and makes a significant contribution to the national economy. In 2021, the food industry accounted for 1.9% of Croatia's gross domestic product (GDP), and in 2023, this sector accounted for around 3% of total employment in Croatia [9]. Given the essential nature of food products and their role in daily life, the food industry is of strategic importance for every country. Ensuring the nutritional needs of the population through domestic production is an important national goal, as it reduces dependence on imports and increases food security. Despite Croatia's considerable natural resources and its potential to increase domestic food production, the country is still highly dependent on food imports. The ratio of exports to imports of food, beverages, and tobacco products stood at 57.1 percent in 2023, compared to 63.8 percent in 2010 [9]. This persistent trade imbalance underlines the need for a deeper understanding of the sector's dynamics and performance.

Given the economic and social importance of the food industry, it is of utmost importance to recognize and understand the factors that influence the profitability of companies in this sector. Achieving positive financial results is the fundamental goal of any business, but profitability is influenced by a variety of factors that are often categorized into global, national, industry and company-specific factors, with company-specific determinants such as company size and age, liquidity, debt, operational efficiency and sales growth being the most frequently emphasized in the literature.

While there is an extensive body of research on the determinants of profitability in various industries, studies focusing specifically on the food industry are rarer (e.g. [3], [5], [19], [3]) and those examining the Croatian context are rather scarce. Notable contributions in Croatia include the studies by [10], which examined the impact of firm-level determinants on the profitability of the food and beverage industry, [12] which focused exclusively on the food industry, while [6] focused on the impact of subsidies for agriculture. However, there are more

studies on the determinants of profitability in the manufacturing and other sectors in Croatia (e.g. [15], [11], [16], etc.)

It is important to note that the effects of firm size and age on profitability remain the subject of ongoing academic debate, while the effects of other financial variables (such as liquidity, debt and growth) are generally more consistent across studies. Research shows that the relationship between company size and profitability can be both positive (due to economies of scale and market power) and negative (due to bureaucratic inertia). Similarly, the impact of firm age ranges from positive (due to more stable resources and experience) to negative (due to technological obsolescence and less flexibility). In contrast, variables such as debt consistently show a negative impact on profitability due to increased financial risk, while sales growth generally has a positive effect. This inconsistency in terms of size and age highlights the need for contextual adaptation of analytical models, while financial indicators are usually considered reliable predictors of general trends.

Despite the extensive literature on firm profitability, there is still much room for further research in this area. In particular, it is important to investigate how sensitive the results of profitability analysis are to the choice of specific profitability and size variables, as different indicators may lead to different conclusions regarding the influence of individual determinants. This topic is of practical importance for corporate management, investors, policy makers and researchers, as the choice of profitability and size indicators can significantly influence strategic decisions and the interpretation of financial results. Accordingly, this paper makes a notable contribution to the literature by not only applying alternative measures of profitability as a robustness check, but also employing two different approximations of firm size to examine the sensitivity of the results. Therefore, this approach provides a more comprehensive understanding of the determinants of profitability.

The following chapter presents the data description and methodology. Chapter three discusses the results, while the concluding remarks can be found in chapter four.

2 DATA DESCRIPTION AND METHODOLOGY

The sample includes companies operating in the food industry in the Republic of Croatia. According to the NACE Rev. 2 classification, these companies belong to the manufacturing industry (C), more precisely to Division 10 – Manufacture of food products. The data on the companies were obtained from the Orbis Europe database, and for each company a period of 10 years is observed, covering the years from 2014 to 2023. The unbalanced panel dataset consists of 1356 companies when ROA is used as the dependent variable, 1196 companies when ROE is used as the dependent variable, and 1366 companies when profit margin is used as the dependent variable. The data were processed using Stata 18.

As mentioned earlier, three dependent variables were used, while the following five variables were used as independent variables: firm size, firm age, liquidity, debt, and firm growth. In addition, two measures of firm size were used – size is calculated using total assets and operating revenues. The definitions of the variables and the codes used are listed in Table 1. The static panel data analysis was carried out using the POLS (Pooled Ordinary Least Squares) approach, which is represented by the following model:

$$y_{it} = \beta_0 + \beta_1 x_{it1} + \beta_2 x_{it2} + \cdots + \beta_K x_{itK} + u_{it};\ \ i = 1, \ldots n,\ t = 1, \ldots T \qquad (1)$$

In this equation, *i* denotes the company (ranging from 1 to n), *t* represents the year (from 1 to T), and y_{it} stands for profitability in company *i* during year *t*. The total number of observational units is given by *n*. Here, β_0 is the intercept, while the remaining β coefficients correspond to

the five explanatory variables included in the analysis. The error term u_{it} is assumed to be independently and identically distributed across both countries and time periods.

In addition to the POLS model, both fixed-effects (FE) and random-effects (RE) models were specified, taking into account the properties of their parameter estimates. To determine which of the three models best fit the data, a series of statistical tests were performed: the overall F-test (to compare POLS and FE), the Breusch-Pagan test (to compare POLS and RE), and the Hausman test (to distinguish between FE and RE).

Table 1: Description of variables

Variables	Definition	Formula/Description	Source
Independent Variables			
ROA	Return on assets using profit (loss) before tax	(Profit before tax / Total assets) * 100	Orbis Europe
ROE	Return on equity using profit (loss) before tax	(Profit before tax / Equity) * 100	Orbis Europe
ProfitM	Profit margin	(Profit before tax / Operating revenue) * 100	Orbis Europe
Dependent Variables			
Size_TA	Firm size	Natural logarithm of total assets	Authors' calculation based on Orbis Europe data
Size_OR	Firm size	Natural logarithm of operating revenue	Authors' calculation based on Orbis Europe data
Age	Firm age	Year - year of company establishment	Authors' calculation based on Orbis Europe data
Liquidity	Liquidity ratio	(Current assets - Inventories) / Current liabilities	Orbis Europe
Debt	Debt ratio	(Non-current liabilities + Current liabilities) / Total assets	Orbis Europe
Growth	Company growth	(Operating revenue in the current period - Operating revenue in the previous period) / Operating revenue in the previous period	Authors' calculation based on Orbis Europe data

Source: Authors

There is no potential problem of multicollinearity as the VIF (Variance Inflation Factor) values for each of the independent variables were below 5. Detailed calculations of the VIF values and the correlation matrix are available on request.

3 RESULTS AND DISCUSSION

Since the aim of the analysis is to test the sensitivity of the results to the choice of two size and three profitability indicators, 18 different models were formed. The first group of models (Table 2) uses total assets as an indicator of firm size, and the second group of models uses operating revenue as an indicator of firm size (Table 3). However, for clarity of results, only fixed-effects models are presented, as the results of the overall F-test, Breusch-Pagan test and Hausman test indicate that they are appropriate for all three profitability measures and two firm size indicators.

The results obtained confirm the sensitivity of the profitability factors to profitability and the company size indicator. However, not all profitability determinants show the same degree of sensitivity. Indeed, the results related to company size and age show contradictory results, which means that their impact vary from statistically positive to statistically negative.

Table 2: FE estimates with – company's size measured by total assets

	Dependent variable		
Dependent variables	***ROA FE model***	***ROE FE model***	***Profit margin FE model***
Size_TA	-1.119762 *** (0.4342233)	4.610579** (1.960967)	2.436866*** (0.3247162)
Age	0.148471* (0.0857546)	-3.530877*** (0.3777867)	-0.1268837* (0.0668541)
Liquidity	0.0891193 (0.060099)	-0.3685761 (0.2377111)	0.2121873*** (0.0484881)
Debt	-6.62877*** (0.3699803)	-29.45928*** (6.629785)	-0.0771774*** (0.0216416)
Growth	0.0023874*** (0.0006187)	0.0032313 (0.0023451)	0.0003282 (0.000488)
Constant	22.53032*** (5.145085)	31.10438 (22.78254)	-26.724*** (3.784721)
Observations	8853	7411	8958
POLS vs FE (F-test and Prob > F)	3.06 (0.000)	2.90 (0.0000)	3.38 (0.0000)
POLS vs RE (chibar2 and Prob > chibar2 – BP test)	363.58 (0.000)	67.87 (0.0000)	948.68 (0.0000)
Hausman test (chibar2 and Prob > chibar2)	61.99 (0.000)	92.02 (0.0000)	83.58 (0.0000)

Source: Authors

Note: ***, **, * denote significance at the 1%, 5% and 10% significance levels. Robust standard errors are in parenthesis.

Table 3: FE estimates with – company's size measured by operating revenue

	Dependent variable		
Dependent variables	***ROA FE model***	***ROE FE model***	***Profit margin FE model***
Size_OR	5.421049*** (0.3578618)	14.48014*** (1.573973)	6.247221*** (0.3141701)
Age	-0.3074992*** (0.0812058)	-4.105304*** (0.3502672)	-0.3636713*** (0.064341)
Liquidity	0.1858979*** (0.0599656)	-0.1644426 (0.2391367)	0.2733349*** (0.0475584)
Debt	-5.97308*** (0.3679863)	-30.19504*** (6.473172)	-0.1063296*** (0.0206648)
Growth	0.0024627*** (0.0006088)	0.0031976 (0.0023304)	0.0002855 (0.0004774)
Constant	-55.22872*** (0.0006088)	-90.51174*** (19.9945)	-73.24953*** (3.821202)
Observations	8824	7398	8958
POLS vs FE (F-test and Prob > F)	3.24 (0.0000)	3.04 (0.0000)	3.70 (0.0000)
POLS vs RE (chibar2 and Prob > chibar2 – BP test)	367.36 (0.0000)	71.36 (0.0000)	1000.11 (0.0000)
Hausman test (chibar2 and Prob > chibar2)	178.81 (0.0000)	138.13 (0.0000)	267.26 (0.0000)

Source: Authors

Note: ***, **, * denote significance at the 1%, 5% and 10% significance levels. Robust standard errors are in parenthesis.

According to the results of the FE model using ROA as the dependent variable and total assets as the size indicator, the impact of size on firm profitability is negative and statistically

significant, as in [5]. This is in contrast to the findings of [19], where a positive impact of company size on its ROA was confirmed. On the other hand, the results of the FE model using ROE and profit margin as dependent variables indicate a positive impact on the company's profitability, which is confirmed by the research results of numerous authors (e.g. [1], [13], [4], etc.) whose research results show that larger firms achieve higher profitability. However, when company size is based on operating income, size has a statistically positive impact regardless of the profitability measure.

The use of total assets as a measure of size also seems to lead to opposite results regarding the effect of company age on profitability. According to the results of the FE model with ROE and profit margin as dependent variables, company age has a negative impact on the profitability of company [14]. On the other hand, the results of the FE model with ROA as the dependent variable have a positive effect on company profitability, as confirmed in the studies of [7] and [12]. As with the previous variable, the alternative size measure leads to consistent results indicating negative effects on profitability for each of the three indicators used.

There are also differences between models in the impact of liquidity and growth on profitability, depending on the size indicator and profitability measure used, but these differences are not so drastic. The effect of liquidity varies from statistically significant positive to non-significant as in [12]. When size is measured by total assets, higher liquidity increases profitability only when measured by profit margin. On the other hand, the positive impact of liquidity is confirmed in models that measure profitability with ROA and profit margin in models where size is represented by operating revenue. Firm growth appears to increase profitability only when it is measured by ROA regardless of the size measure, as in [5]. In other cases, it does not seem to influence profitability.

Of the five independent variables selected, the results related to debt are the most robust and indicate a negative impact on company profitability, regardless of the company size and profitability indicator used as in [18] and [5]. In other words, an increase in corporate debt decreases the profitability of the company.

4 CONCLUSION

The results of this study significantly enhance the understanding of factors influencing profitability in the Croatian food industry, a sector of great strategic importance. The analysis reveals that profitability factors are sensitive to the choice of profitability and company size indicators. The results relating to company size and age are particularly variable and often contradictory - their impact can be either positive or negative depending on the measures used. For example, company size measured by total assets has a negative effect on profitability when ROA is used, but a positive effect when ROE and profit margin are used as measures of profitability, while there are no differences between the models when size is measured by operating revenue. Similarly, the effect of company age on profitability varies according to the chosen size and profitability indicators. Liquidity and company growth also show some differences between models, though these are less pronounced. The most robust and consistent result concerns the variable related to debt, which shows a negative impact on profitability in all models. In summary, while liquidity, company growth and debt are reliable predictors of profitability, the role of size and age needs to be further explored.

The present study is limited by the fact that the sample includes only companies operating in Croatia and is based solely on firm-level variables. Future research could address these limitations by incorporating a wider range of variables, including industry and macroeconomic factors, and by analysing a larger and more diverse sample of companies from different countries or regions. Such an approach would provide a more comprehensive understanding of the determinants of profitability in the food industry.

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COMPARISON OF PER VALUE ADDED CARBON INTENSITY IN CROATIA AND SLOVENIA ACROSS SELECTED ISIC SECTIONS

Berislav Žmuk
University of Zagreb, Faculty of Economics and Business, Department of Statistics
Trg J. F. Kennedyja 6, HR-10000 Zagreb, Croatia
E-mail: bzmuk@efzg.hr

Abstract: This paper analyses and compares the carbon efficiency of Croatia and Slovenia across three key economic sectors: agriculture, forestry, and fishing; mining and quarrying; and construction. Using per value added carbon intensity as a metric, the study finds that Slovenia consistently exhibits lower carbon intensity and greater reductions over time compared to Croatia. The cluster analysis, in which the Partitioning Around Medoids clustering method was applied, places Slovenia in a group of countries with low carbon intensity, while Croatia is grouped with those exhibiting medium to high levels. The findings highlight Slovenia's stronger alignment with sustainability goals, emphasizing the need for further improvements in Croatia's carbon efficiency.

Keywords: cluster analysis, Croatia, International Standard Industrial Classification (ISIC), Slovenia.

1 INTRODUCTION

Carbon efficiency has become a critical indicator for evaluating the sustainability of economic activities, particularly in developing countries [6]. To address the growing demand for sustainable practices, the per value added carbon intensity variable has emerged as a key metric. Evaluating this metric across various economic sectors can provide valuable insights into the relative sustainability of national economies and their developmental trajectories. However, attention should be given to the different definitions of carbon efficiency, and further research should be conducted in this area as well [9].

The focus of this paper is to measure and compare the carbon efficiency levels of two neighbouring countries, Croatia and Slovenia. Both were members of the Socialist Federal Republic of Yugoslavia from 1945 to 1991 and are now members of the European Union (Croatia since 2013 and Slovenia since 2004). Therefore, the objectives of this paper are to: (1) analyse the temporal trends of per value added carbon intensity in Croatia and Slovenia, and (2) identify their relative positions among other countries using cluster analysis. By addressing these objectives, this study contributes to the understanding of sustainability metrics in key economic sectors and offers a methodological framework for cross-country comparisons.

The paper is organized as follows. After a brief introduction, the second chapter presents and describes the data and methods used in the analysis, explaining the steps involved. The third chapter presents and discusses the main results of the conducted analyses. The fourth chapter concludes the paper.

2 DATA AND METHODS

The main variables under study are the per value added carbon intensity variables for the three observed International Standard Industrial Classification of All Economic Activities (ISIC) sections: Agriculture, forestry, and fishing; mining and quarrying; and construction [7]. These three ISIC sections are considered fundamental economic activities that support developing countries. Additionally, each country should have at least some activities in these sections, which could be used to compare the success of economic activities across different countries. The per value added carbon intensity variable measures the amount of carbon dioxide

emissions per unit of value added in a given ISIC section, expressed in kilograms of CO_2 per USD, adjusted for purchasing power parity (PPP) of 2015. Therefore, the per value added carbon intensity variables were calculated as follows:

$$\text{Per value added carbon intensity} = \frac{\text{Total } CO_2 \text{ emissions (kg)}}{\text{Value added in USD (PPP 2015)}} \quad (1)$$

This indicator evaluates the carbon efficiency of a section. Lower values signify greater sustainability, as they indicate lower emissions for the same level of economic activity.

The data source is the Energy and emissions per value added database, which includes value added data and energy efficiency indicators for countries worldwide [2]. The analysis will focus on comparing the carbon efficiency in the three observed sections between Croatia and Slovenia. In the first step, the per value added carbon intensity levels in the three observed sections from 2000 to 2020 will be examined and compared between Croatia and Slovenia. Descriptive statistical methods will be used to calculate basic descriptive indicators for each of the three observed ISIC sections in both Croatia and Slovenia. Additionally, to better understand the development of the per value added carbon intensity from 2000 to 2020, Croatia and Slovenia will be compared using fixed base indices, with the values from the first observed year in 2000 serving as the base for the fixed base indices.

In the next step, to determine the position of Croatia and Slovenia regarding per value added carbon intensity in the three observed ISIC sections, cluster analysis will be conducted. The preliminary data analysis has shown that most countries are covered by data in 2019. Data for per value added carbon intensity in all three observed ISIC sections are available for 52 countries, including Croatia and Slovenia. The preliminary analysis has also shown that outliers can be detected in each of the three observed variables. Consequently, it has been decided to apply the Partitioning Around Medoids (PAM) clustering method with Euclidean distances as a distance measure. The PAM clustering method was introduced by [4] as an algorithm for mapping a distance matrix into a predetermined number of clusters. A major advantage of the PAM clustering method is that the clustering process is less sensitive to the presence of outliers [8]. In the PAM clustering method procedure, the goal is to minimize the average dissimilarity of objects to their nearest selected object by identifying a sequence of centrally located objects (e.g., medoids) within clusters [1]. The low sensitivity of the PAM clustering method to outliers arises from the fact that the medoids are not chosen randomly. The first medoid is selected as the data point with the smallest distance to all other points, placing it at the center. The subsequent medoid is then chosen in a way that minimizes the total distance of each data point to its nearest medoid [5]. As suggested by [4], the number of clusters will be determined by maximizing the average silhouette width. Silhouette width measures how similar an object is to its own cluster compared to other clusters, and it is calculated as follows:

$$S_{x(i)} = \frac{b_{x(i)} - a_{x(i)}}{max\left(a_{x(i)}, b_{x(i)}\right)} \quad (2)$$

where $x(i)$ is a data point in the cluster, $a_{x(i)}$ is average distance between $x(i)$ and every data point in the same cluster, $b_{x(i)}$ is minimum average distance between $x(i)$ and every data point in other clusters [3]. The value of the silhouette width ranges from -1 to +1, where positive values indicate that the object is well clustered.

3 ANALYSIS AND DISCUSSION

The main descriptive statistics for per value-added carbon intensity in the three observed ISIC sections (Agriculture, forestry and fishing; mining and quarrying; construction) are presented in Table 1. The descriptive statistical indicators are calculated for the period from 2000 to 2020.

Table 1: Descriptive statistics of per value added carbon intensity for the three observed ISIC sections in Croatia and Slovenia during the period from 2000 to 2020, with values expressed in kg CO_2/USD PPP 2015

Statistics	Agriculture, forestry and fishing		Mining and quarrying		Construction	
	Croatia	Slovenia	Croatia	Slovenia	Croatia	Slovenia
Mean	0.21	0.18	0.46	0.29	0.08	0.04
Median	0.22	0.18	0.47	0.27	0.08	0.03
Stand. dev.	0.02	0.02	0.18	0.04	0.01	0.02
Minimum	0.17	0.14	0.24	0.22	0.07	0.02
Maximum	0.25	0.24	0.92	0.36	0.10	0.10

The results in Table 1 reveal that, on average, Croatia exhibits higher levels of per value added carbon intensity than Slovenia across all three observed ISIC sections during the studied period. The smallest difference in per value added carbon intensity is observed in the agriculture, forestry, and fishing section. In the mining and quarrying section, the descriptive statistical indicators (primarily the mean and median) highlight that Croatia has significantly higher levels of per value added carbon intensity compared to Slovenia. In the construction section, both countries show relatively low levels of per value added carbon intensity, making even small absolute differences appear substantial in relative terms.

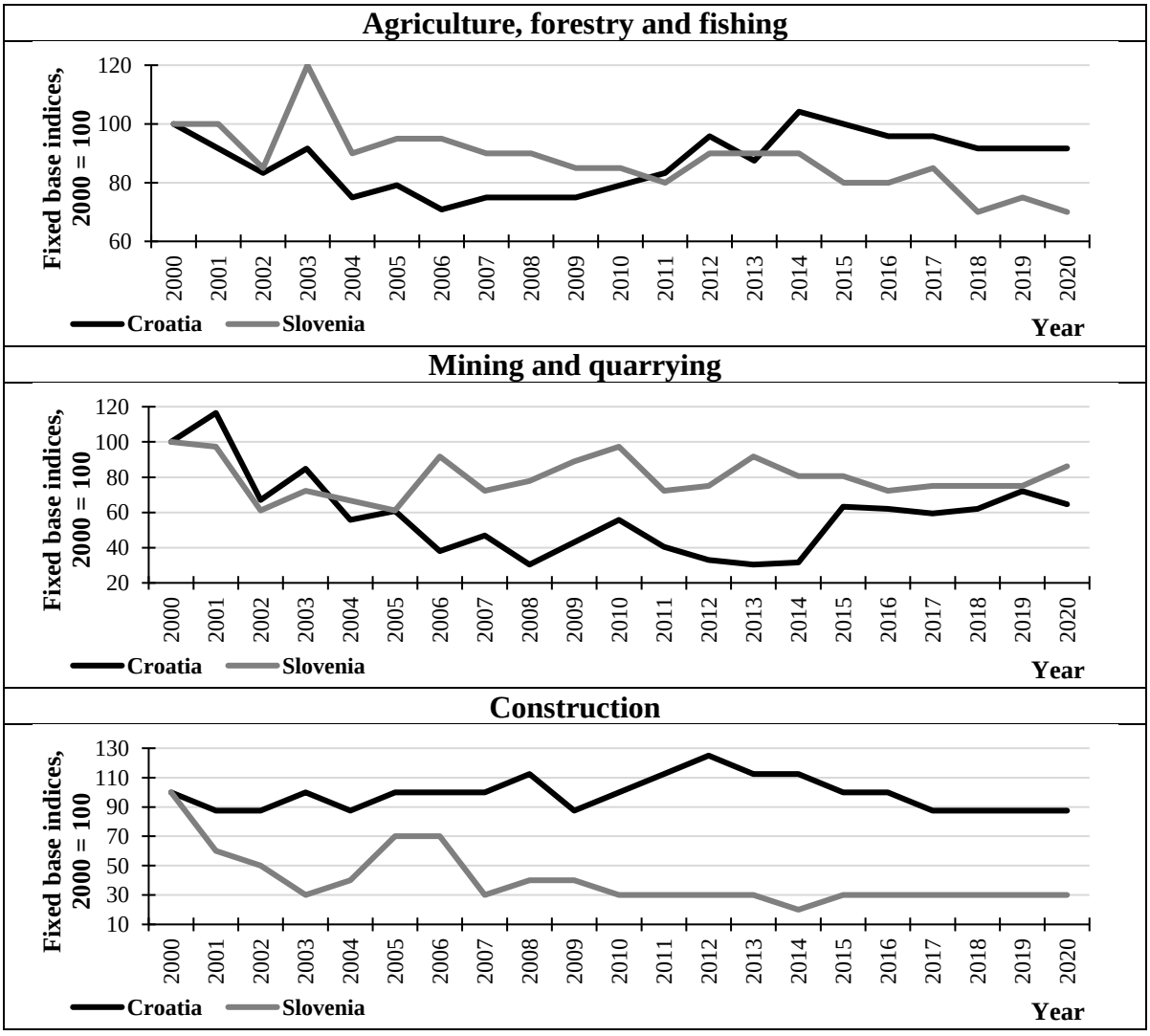


Figure 1: Development of fixed base indices of per value added carbon intensity for the observed ISIC sections in Croatia and Slovenia during the period from 2000 to 2020, with 2000 = 100

Figure 1 illustrates the development of per value added carbon intensity in the three observed ISIC sections in Croatia and Slovenia from 2000 to 2020, presented as fixed base indices. This approach was chosen to provide better insight into how Croatia and Slovenia have addressed sustainability in carbon intensity within the observed ISIC sections.

In the agriculture, forestry, and fishing section, Croatia managed to reduce per value added carbon intensity during the first half of the observed period; however, it subsequently returned to 2000 levels. Conversely, Slovenia initially maintained the same per value added carbon intensity as in 2000 but soon began reducing it, achieving a 30% decrease by 2020 compared to 2000.

In the mining and quarrying section, Slovenia managed to lower its per value added carbon intensity by 13.9% by 2020 compared to 2000, though the reduction was not substantial. Croatia was more successful in this section, reducing per value added carbon intensity by 35.4% over the same period. However, since 2015, Croatia's per value added carbon intensity has started to rise.

In the construction section, Croatia did not achieve significant reductions in per value added carbon intensity, maintaining levels similar to those in 2000 throughout the observed period. As a result, Croatia's per value added carbon intensity in this section was only 12.5% lower in 2020 compared to 2000. On the other hand, Slovenia achieved a substantial reduction in this section, lowering per value added carbon intensity by 70% by 2020 compared to 2000 levels.

Since most data for the observed per value added carbon intensity variables are available for the most countries (52) in 2019, further analyses will focus on this year. The conducted outlier analysis revealed that some outliers exist for each per value added carbon intensity variable. Outliers are defined as values deviating by more than two standard deviations from the variable mean. For the per value added carbon intensity variable related to agriculture, forestry, and fishing, the outliers are the Netherlands ($\boldsymbol{z = 2.56}$), Belgium ($\boldsymbol{z = 2.41}$), and Suriname ($\boldsymbol{z = 2.15}$). In mining and quarrying, Canada ($\boldsymbol{z = 4.03}$) and Cyprus ($\boldsymbol{z = 2.29}$) are identified as outliers. In construction, the outliers are Namibia ($\boldsymbol{z = 3.58}$) and Russia ($\boldsymbol{z = 2.63}$). In all cases, the identified values are significantly higher than the others.

The correlation analysis revealed that all per value added carbon intensity variables are positively correlated. A statistically significant correlation at the 5% level was found between the agriculture, forestry, and fishing section and the mining and quarrying section ($\boldsymbol{r = 0.3312}$). Additionally, a statistically significant correlation at the 10% level was observed between the agriculture, forestry, and fishing section and the construction section ($\boldsymbol{r = 0.2342}$). However, the correlation between the mining and quarrying section and the construction section was not statistically significant at any commonly used level of significance ($\boldsymbol{r = 0.0360}$).

To apply the PAM clustering method, the optimal number of clusters must first be determined. The silhouette width analysis indicated that the highest average silhouette width (0.34) was achieved with a three-cluster solution. Within this solution, cluster 2 had the highest average silhouette width (0.46), while cluster 3 had the lowest (0.15). The average silhouette width for cluster 1 was 0.21. The results of the PAM clustering method are visually presented in Figure 2, and the countries assigned to each cluster are listed in Table 2.

According to the PAM clustering membership results shown in Table 2, Croatia and Slovenia are placed in different clusters. Croatia is in Cluster 1, along with 13 other countries. This cluster includes Croatia's eastern neighbours, Hungary and Serbia, as well as highly developed countries like Germany and the United States. Slovenia is placed in Cluster 2, the largest cluster, consisting of 29 countries. Unlike Croatia, Slovenia is grouped with its western neighbours, Austria and Italy. Cluster 2 also includes other highly developed countries such as Japan and the United Kingdom.

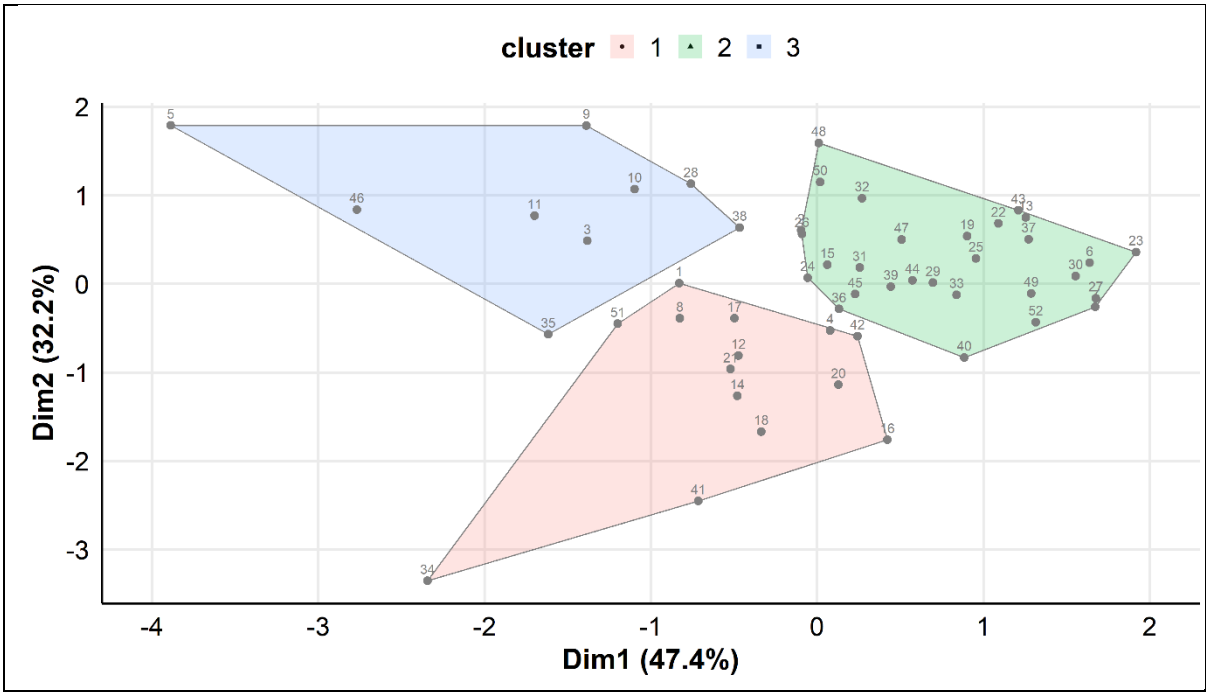


Figure 2: Partitioning Around Medoids (PAM) clustering method results visualization, including per value added carbon intensity variables in the year 2019 across 52 countries
Note: Croatia is represented by number 8, and Slovenia is represented by number 44 in the figure.

Table 2: Cluster membership, determined using the Partitioning Around Medoids (PAM) clustering method, for the year 2019 across 52 countries

Cluster	Countries
Cluster 1 (n=14)	Australia, Bulgaria, **Croatia**, Estonia, Finland, Georgia, Germany, Greece, Hungary, Iceland, Namibia, Russia, Serbia, United States of America
Cluster 2 (n=29)	Austria, Colombia, Costa Rica, Eswatini, France, Guyana, India, Indonesia, Ireland, Italy, Japan, Kenya, Latvia, Lithuania, Luxembourg, Mexico, Mozambique, Norway, Philippines, Portugal, Romania, Slovakia, **Slovenia**, Spain, Sweden, Thailand, Turkey, United Kingdom, Zambia
Cluster 3 (n=9)	Belgium, Canada, Cyprus, Czechia, Denmark, South Korea, Netherlands, Poland, Suriname

Table 3: Main characteristics of clusters based on mean per value added carbon intensity levels, determined using the Partitioning Around Medoids (PAM) clustering method, for the year 2019 across 52 countries

Statistics	Agriculture, forestry and fishing	Mining and quarrying	Construction
Cluster 1	Medium (0.27)	Medium (0.29)	High (0.08)
Cluster 2	Low (0.14)	Low (0.25)	Low (0.02)
Cluster 3	High (0.53)	High (0.71)	Medium (0.03)

Note: mean per value added carbon intensity levels are given in brackets.

Table 3 presents the main characteristics of the clusters based on the mean per value added carbon intensity levels. According to the results, Croatia was grouped into a cluster of countries that, on average, exhibit medium levels of per value-added carbon intensity in the agriculture, forestry, and fishing, and mining and quarrying sections, but high levels in the construction section. Conversely, Slovenia was placed in a cluster of countries characterized by low per value added carbon intensity levels across all three observed ISIC sections.

Therefore, it can be concluded that Slovenia demonstrates a stronger alignment with sustainability goals compared to Croatia, particularly in reducing carbon intensity across the observed ISIC sections. This highlights the need for targeted efforts in Croatia to address its high carbon intensity in the construction section and further improve its performance in other sections.

4 CONCLUSION

The research demonstrates that Slovenia not only has lower per value added carbon intensity levels in the three observed ISIC sections compared to Croatia but has also achieved significantly greater reductions over the period from 2000 to 2020. As a result, the PAM clustering method's findings were unsurprising, placing Slovenia in a cluster of countries with low per value added carbon intensity levels across all three ISIC sections. In contrast, Croatia was grouped with countries characterized by medium to high per value added carbon intensity levels in the same sections. These findings suggest that Slovenia has achieved a higher level of sustainability than Croatia, with the gap between the two countries widening over time.

However, the research is subject to several limitations. The cluster analysis focuses solely on a single year (2019) and includes only 52 countries, which may restrict the generalizability of the findings. Additionally, the study is limited to three ISIC sections (agriculture, forestry, and fishing; mining and quarrying; and construction) leaving out other important economic activity sections.

Future research should address these limitations by conducting a panel analysis over an extended period to capture temporal dynamics more comprehensively. Expanding the dataset to include more countries and incorporating all economic activity sectors would provide a more holistic view of per value added carbon intensity trends. Such an approach could yield deeper insights into global sustainability patterns and identify best practices for reducing carbon intensity across various sectors.

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The 18th International Symposium on
Operational Research in Slovenia

SOR '25

Bled, SLOVENIA
September 24 - 26, 2025

Session 2:

Human Resources

MEASURING THE LEARNING ANALYTICS ADOPTION: HOW IMPORTANT ARE DIFFERENT CAPABILITIES?

Nina Begičević Ređep, Tena Jagačić, Nikola Kadoić, Jelena Gusić Munđar
University of Zagreb Faculty of Organization and Informatics
Pavlinska 2, 42 000 Varaždin, Croatia
E-mail: nina.begicevic@foi.unizg.hr, tjagacic@foi.unizg.hr, nikola.kadoic@foi.unizg.hr, jelena.gusic@foi.unizg.hr

Abstract: Nowadays, many higher education institutions (HEIs) use the benefits of learning analytics (LA) to achieve their strategic goals related to low dropout and high competitiveness of students in the labour market. Different institutions can adopt and apply LA differently, and there is a need to measure LA adoption and application. To measure LA adoption and application, knowing what to measure (which HEI's capabilities) and how is mandatory. While the HEIs' LA-related capabilities are identified in previous research, this paper deals with measuring their importance because some capabilities contribute to LA maturity more than others by identifying the importance of each capability in the LA maturity model. There are 28 capabilities grouped into eight categories. The multicriterial methods analytic hierarchy process (AHP) and social network analysis process (SNAP) were applied to measure their importance. The research involved experts in LA who are/were at responsible university positions related to the teaching process (deans and vice deans responsible for teaching). Experts identify *People Competences* as the most important LA category related to LA maturity and *Training programs on LA for teachers* as the most important LA capability under that category. Although, expert from the same institutions doesn't often overlap in assessment of the importance of LA categories, they have similar opinion about the level of LA maturity of their institution.

Keywords: learning analytics, maturity, capabilities, higher education, LA adoption, analytic hierarchy process

1 INTRODUCTION

The nature of LA is multidisciplinary, as it lies at the intersection of data science, educational technology, and the learning sciences [14]. It is a key area within educational technology that, through the application of data analysis, provides tangible benefits for both teachers and learners [11]. LA is focused on improving student success and can be seen as a tool through which stakeholders—such as students, educators, and administrators—gain access to valuable insights and information on factors that contribute to the enhancement of teaching and learning processes. It also catalyzes the development and implementation of educational reforms [16]. Furthermore, LA offers an opportunity to integrate data-driven decision-making processes into existing educational practices [12]. One of the significant challenges in adopting learning analytics is the lack of specifically developed policies or guidelines, including both legal regulations and non-legislative principles, to guide its implementation while taking into account the cultural, economic, political, and technical context of each institution [19]. The successful implementation of LA requires systematic attention to seven key factors—pedagogy, complexity, ethics, power, regulation, validity, and affect—alongside strong and thoughtful engagement from all involved stakeholders [1, 8]. The authors recommend integrating participatory design into LA-related projects to increase stakeholder engagement and ensure the relevance of the systems [3]. LA is a valuable tool for course design and pedagogical approaches that educators can apply to foster student engagement, and it is closely linked to machine learning techniques used to detect patterns and predict outcomes [7, 10].

The development and application of learning analytics (LA) have led to the emergence of numerous models and frameworks that provide institutions with guidance for the successful

integration of LA into educational practices and policies and some of them will be mentioned bellow.

Supporting Higher Education to Integrate Learning Analytics (SHEILA) is a policy and strategy framework for LA, developed based on the experiences of 51 European higher education institutions and interviews with 78 senior managers, adapted from the RAPID Outcome Mapping Approach (ROMA) [21]. The framework is illustrated through four case studies. It is intended for iterative use in strategic planning and policy development, particularly for large-scale institutional implementation of LA in higher education [20]. The Learning Analytics Readiness Instrument (LARI) was designed to help institutions prepare for the successful implementation of LA. It consists of 90 items grouped into five key factors: ability, data, culture and process, governance and infrastructure, and overall readiness perception [2]. The LA Maturity Model outlines four levels, ranging from initial ad hoc initiatives to full institutionalization, and enables institutions to assess their current status and progressively enhance the use of LA through organizational, technical, and pedagogical alignment. It includes five key categories: data management, administration and training, pedagogical support, data analysis, legislation, privacy, and ethics. Although LARI, SHEILA, and the proposed Maturity Model share the same focus, each operates at different stages of the LA adoption process and can be used complementarily. LARI serves as an initial diagnostic tool, the Maturity Model offers guidelines for gradual development, and SHEILA supports the creation of institutional policies to reach higher levels of maturity [9].

Drachsler and Greller developed the DELICATE checklist, consisting of eight key elements (Determination, Explain, Legitimate, Involve, Consent, Anonymise, Technical, and External), which serves as a guide for researchers, policymakers, and institutional leaders to ensure the reliable and ethical implementation of LA [4]. Aiming to strengthen trust in data-driven technologies, Ramaswami et al. propose a Learning Analytics (LA) system that combines predictive and prescriptive analytics based on machine learning, with a focus on transparency and user-centeredness [16]. Effective communication among stakeholders is crucial for the coordinated and sustainable implementation of LA, supported by the Orchestrating Learning Analytics (OrLA) tool, which facilitates collaboration and complements existing strategic frameworks, such as ROMA and SHEILA [15]. To promote awareness and understanding of LA and to strengthen teamwork through shared understanding and terminology, the Fellowship of Learning Activities and Analytics (FoLA2) method was developed. This method supports the integration of LA into curriculum and learning activity design across various subjects [17].

The research questions this paper answers are:

1. What is the importance of LA categories related to LA maturity?
2. What is the importance of LA capabilities under categories related to LA maturity?
3. What is the level of agreement on LA categories weights of different experts?
4. What is the level of LA maturity of higher education institutions in Croatia?

To answer those questions, the following methodological framework was applied and detailly explained in methodology section:

- The selection of a multi-criteria decision-making method for the determination of the importance of LA categories in LA maturity,
- The selection of a multi-criteria decision-making method for the determination of the importance of LA capabilities under categories in LA maturity,
- The identification of the experts that will give judgements to determine the importance of LA capabilities and LA categories,
- The creation of data collection forms,
- The analysis of the collected data, and
- Reporting.

2 METHODOLOGY

The development of Capability Maturity Model for adopting Learning Analytics in Higher Education (HELA-CMM) was based on the design science research paradigm and a procedural model for maturity model design. The proposed maturity model comprises 28 capabilities grouped into eight categories: (1) Teaching / Learning / Assessment, (2) Data Management and Analytics, (3) Ethics, Privacy, and Legal Issues, (4) People Competences, (5) Management / Leadership, (6) Culture, (7) Infrastructure, and (8) Quality Assurance [18].

Regarding research participants, the research covered 12 higher education experts from Croatian universities. All of them have a PhD level of education, and more importantly, they are or were responsible persons on management boards of their higher education institutions (dean or vice dean positions). The data were collected in special sessions organised in the scope of the HELA project from September 2024 to January 2025. For data collection purposes, we created forms in Microsoft Excel.

When deciding on a selection of multi-criteria decision-making methods, we consider the existence of influences and dependencies among the capabilities and categories. The dependencies can be easily identified, which means that the multi-criteria decision-making method employed for weighting purposes has to model dependencies and influences. The number of such methods is low. Additionally, very often, due to the complexity of such processes, even though the dependencies/influences are presented in the decision-making model, the decision-making models were analysed with methods that do not support dependencies/influences modelling [13]. Consequently, we decided to apply the social network analysis process (SNAP) methodology at the level of LA categories and the analytic hierarchy process (AHP) at the level of capabilities. Employing SNAP for both purposes (identification of weights of capabilities and categories) would be more beneficial since SNAP enables modelling dependencies/influences. However, since this would result in a high number of inputs to be collected by experts, for the level of capabilities, the AHP was applied (which does not model dependencies/influences among the elements in the problem structure). There is evidence of using a combined approach in history [5, 6].

Further, for the determination of the level of agreement on LA categories and LA capabilities' weights of different experts, the correlation analysis has been implemented.

Also, hierarchical cluster analysis and dendrogram for visual representation were used to compare level of agreement with LA maturity of their higher education institution between the experts from the same institution.

3 RESULTS

The weights of LA categories and LA capabilities are presented in Tab 1. *People Competences* and *Management / Leadership* were identified as the most important LA categories related to LA maturity, while *Data Management & Analytics* and *Teaching / Learning /Assessment* were identified as the less important LA categories. Under each category the most important LA capabilities were identified (category (1): *Implementation of teaching*, category (2): *Data reporting*, category (3): *Training programs on LA for teachers*, category (4): *Transparent information and communication*, category (5): Operationalization of the strategy plan for LA, category (6): *The development of a blended teaching and learning environment*, category(7): *Supporting, recognizing and rewarding excellence in teaching*, and category(8): *Implementation of the quality assurance cycle of LA*).

Table 1: The weights of LA categories and LA capabilities

Category	weight	Capability	weight
Teaching / Learning /Assessment (1)	0.101	Learning design	0.178
		Curriculum design	0.179
		Implementation of teaching	**0.361**
		Implementation of assessment	0.282
Data Management & Analytics (2)	0.087	Data reporting	**0.330**
		Data analysis	0.277
		Data acquisition	0.236
		Data storage	0.158
People Competences (3)	**0.180**	Training programs on LA for teachers	**0.337**
		Training programs on LA for students	0.158
		Communities of practice	0.244
		Career development	0.262
Ethics, Privacy, Legal Issues (4)	0.116	Designing and implementing guidelines for personal data protection, and rights of data subjects	0.259
		Transparent information and communication	**0.456**
		Designing and implementing Code of conduct for LA	0.286
Management / Leadership (5)	0.157	Development and implementation of a strategy plan for LA	0.269
		Operationalization of the strategy plan for LA	**0.288**
		Vision and goals development for LA	0.268
		Resilience plan	0.175
Infrastructure (6)	0.114	IT support services (for staff and learners)	0.238
		IT services management	0.225
		IT equipment management	0.217
		The development of a blended teaching and learning environment	**0.321**
Culture (7)	0.136	Integrating digital elements and data driven approaches in the learning process	0.186
		Fostering learning centered approach	0.249
		Supporting, recognizing and rewarding excellence in teaching	**0.334**
		Fostering sharing and openness	0.230
Quality Assurance (8)	0.109	Integration of LA in quality assurance cycle	0.384
		Implementation of the quality assurance cycle of LA	**0.616**

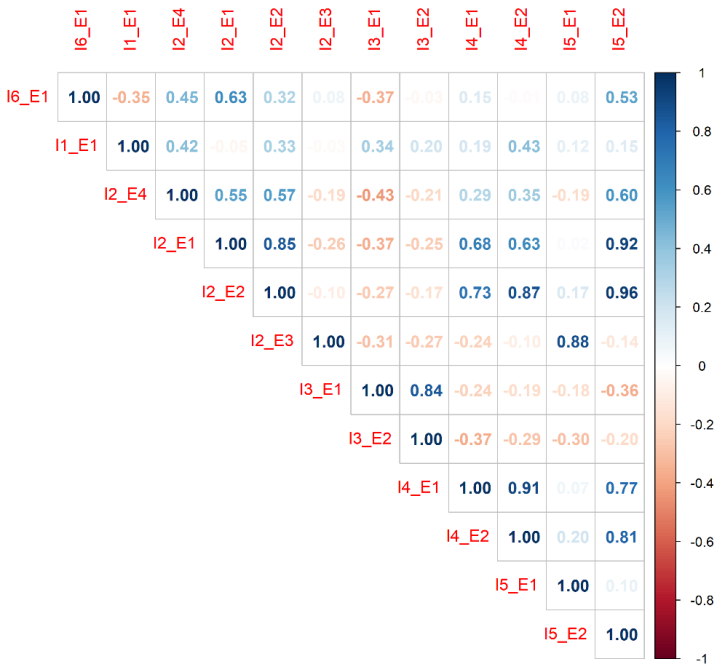


Figure 1: Correlation matrix for the level of agreement on LA categories weights of different experts.

Because we decided to apply the SNAP methodology for calculating weights of LA categories, correlation matrix presents the level of agreement on LA categories SNAP weights of different experts. (Fig 1.).

In most cases expert from the same institutions doesn't overlap in their assessment of the importance of the LA categories. Expert E1, E2, and E4 from the institution 2 together with experts E1 and E2 from institution 4, E2 from institution 5, and maybe E1 from institution 6 have a high level of agreement on importance of LA categories. On the other hand, expert from the institution 3 have similar opinion but their opinion differs from the other experts. Similar conclusion can be made from the experts E1 from institution 5 and E3 from the institution 3. (Fig. 1)

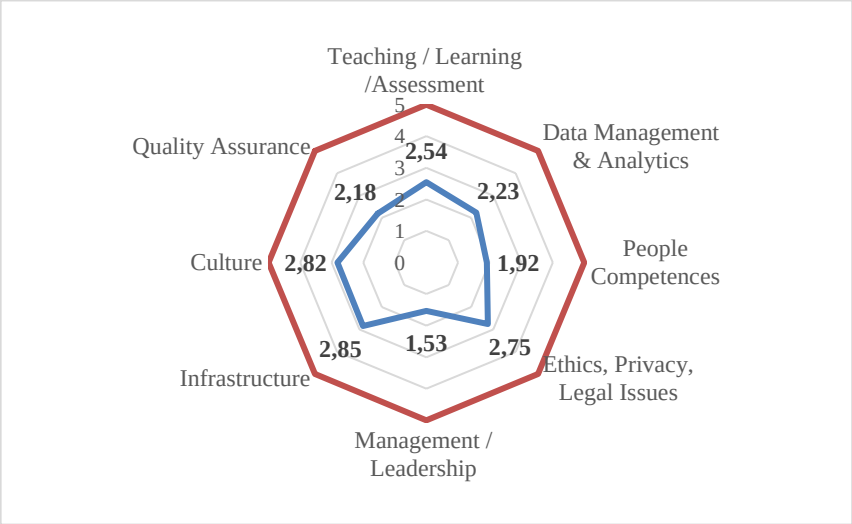


Figure 2: Average maturities of experts' higher education institutions

Fig. 2 presents the average maturities of experts' higher education institutions for 10 out of 12 experts. They evaluated the *Management / Leadership* category as the less mature and the *Infrastructure* as the most mature LA category. Detailed analysis of the similarities in rating maturity of institution for the expert from the same institution can be seen on Fig. 3.

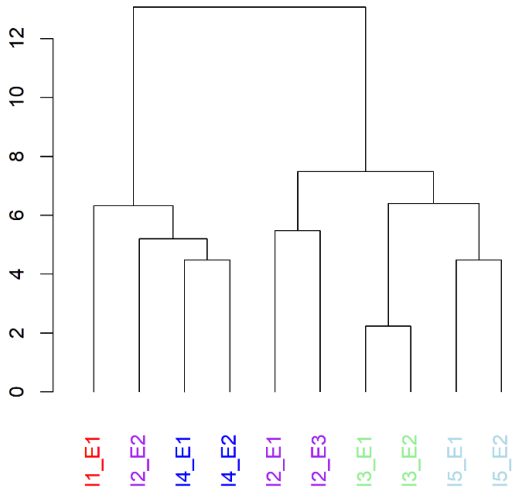


Figure 3: Dendrogram for visualization of similarities how experts rated the LA maturity of their institution.

In most cases experts from the same institution evaluated most similar maturity of their institution. The only exception was expert E2 from institution 2, who evaluated the maturity of his institution more similar to experts from institution 4 than to experts from his institution. Reason for that can be that expert E2 from institution 2 wasn't on dean or vice dean positions in institution in the same period as experts E1 and E3.

4 CONCLUSION

Based on the analysis of LA categories, it is evident that *People Competences* and *Management / Leadership* are the most critical factors influencing LA maturity, whereas *Data Management & Analytics* and *Teaching/Learning/Assessment* are comparatively less important. According to the level of agreement how certain LA categories are important, experts can be divided into three groups. In the future work, we plan to calculate weights of LA categories and LA capabilities for each group and compare results to see which LA categories and capabilities are considered the most important in each group. The maturity assessment of the institutions indicates that Management/Leadership tends to be less developed, while Infrastructure is more advanced across institutions.

Notably, experts from the same institution typically assess their institution's maturity similarly, with occasional deviations likely influenced by time when they had specific roles within the institution. Overall, these findings underscore the importance of targeted improvements in leadership and management to advance LA maturity of the higher education institutions in Croatia, while also recognizing existing strengths in infrastructure and culture.

Acknowledgement
This work has been fully funded by the Croatian Science Foundation under the grant IP-2020-02-5071.

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QUANTIFYING THE BURDEN: UNPAID WORK AND LABOUR MARKET DECISION TRADE-OFFS

Ema Kelin
University of Rijeka, Faculty of Economics and Business
Ivana Filipovića 4, 51000 Rijeka, Croatia
E-mail: ema.kelin@efri.uniri.hr

Abstract: Women in the EU are now participating more in the labour force, but still do most of the unpaid work, which means that they are overburdened. This paper shows the market value of unpaid work and its opportunity cost for two countries: Hungary and France. The results show that highly educated women have the highest opportunity cost of unpaid work. There is a need for policy measures that at least partially relieve women of the burden of unpaid work when they decide to what extent they want to participate in the labour market.

Keywords: unpaid work, value of unpaid work, highly educated, women, decision, labour force participation

1 INTRODUCTION

For years, the creation of economic value in an economy was attributed only to the work performed in the labour market. Historically, this paid work performed through the labour market was predominantly done by men. At the same time, women have specialised in housework (preparing meals, cleaning, etc.) and childcare, i.e. unpaid work at home [8]. Unpaid work includes all services that, if they were not performed by someone in the household, would have to be purchased on the market [13]. So, although it is not formally recognised as an economic value producing activity in the System of National of Accounts, unpaid work still creates economic value for the household members and recipients of these services.

The gender-specific division of paid and unpaid work has changed as a result of economic development and the expansion of education. In the EU, the rise in educational attainment has also led to an increase in labour force participation, particularly among women [2]. By 2053, the EU labour force is expected to have a higher share of men than women, but a higher share of women than men will have a tertiary education [10]. Already in 2024, 39.1% of EU female population had tertiary education, compared to 33% of male population. In addition, 84.3% of highly educated women aged 20 to 64 were employed, compared to 69% of women with medium level of education and 47% of women with low level of education [5] .

Therefore, higher level of education goes hand in hand with higher labour force participation, especially among women. Spending more time in paid work affects the amount of time spent in unpaid work, as well as the type of unpaid work performed. Research by Kelin, Istenič and Sambt [9] has shown that while educational level does not affect the amount of housework done by men, it does affect the amount done by women. Women with a tertiary education spend the least time doing housework other than childcare, compared to women with low and medium education.

However, this does not apply to all activities of unpaid work. Childcare is specific and therefore often considered separately, as it is concentrated at a particular age in the life cycle and substantially increases the total amount of unpaid work at that age. Although women of all educational levels are the main producers of childcare spending about two and a half times as much time on it than men, both men and women with tertiary education spend the most time on childcare compared to other educational levels [9] . However, the total workload (paid and

unpaid work combined) is still higher on average for women. In Italy, for example, it is almost 30% higher [1].

As they are the main producers of unpaid work and nowadays participate more and more in paid work, women of all educational levels often must balance the time they spend on paid and unpaid work. This paper aims to show the market value of unpaid work produced and its opportunity cost depending on the educational level of the person doing it. The results of this study can therefore shed light on the economic value of unpaid work and help the person doing it to make an economic decision on how much time should be spent on it and how much should be delegated to another person. For this reason, this paper analyses the time spent on paid and unpaid work activities and their market value over the economic life cycle by gender and educational level for two countries: Hungary as a representative of Eastern Europe and France as a representative of Western Europe.

2 METHODOLOGY

The main data source for analysing time devoted to unpaid work is the Multinational Time Use Study (MTUS) for Hungary and France for the year 2009 as these are the most recent surveys available [6]. The surveys cover all daily activities of the respondents as well as their socio-economic characteristics. Respondents record the time spent on a particular activity in a diary that they keep for one or more days.

To analyse the production of unpaid work, which is measured in minutes per day, this paper uses the National Time Transfer Accounts (NTTA) methodology [4]. The aim of this methodology is to construct the age profiles, i.e. the age-, education- and gender-specific averages of time spend on unpaid (and paid) work each day. For this purpose, all daily activities are divided into five groups: (1) paid work (2) unpaid work (3) education (4) leisure and (5) personal care. The activities belonging to the unpaid work group are determined by the "third party criterion", which includes all activities for which a third person could be employed while the respondent benefits from it, such as cooking, cleaning, childcare, etc. [11]

To determine the monetary value of unpaid work, two methods are used. The specialist replacement (SR) method is commonly used to determine the value of unpaid work as it uses the wages of occupations that would be hired in the market for a particular activity of unpaid work. The second method, the opportunity cost (OC) method, uses the average wage determined by the educational level of the person performing the unpaid work. Although as such it overestimates the value of unpaid work, it gives a good insight into the opportunity cost of unpaid work when comparing the time spent on unpaid work across different levels of education [7][13] .

3 RESULTS

For most of their economic life cycle, men of all educational levels do more paid than unpaid work. In both Hungary and France, the time spent on unpaid work exceeds the time spent on paid work only between ages of 60 and 65, i.e. after retirement (Figure 1).

Figure 1 further shows that the situation is quite different for women. Over their entire economic life cycle, women of all educational levels do more unpaid than paid work. The only exception is highly educated women between the ages of 45 and 55 in both Hungary and France (in France also at the beginning of their careers), where the time they spend on paid work briefly exceeds the time they spend on unpaid work. Highly educated women also devote more time to paid work than women with low and medium levels of education. This can be explained by the higher wages they receive for their paid work which leads them to prioritise paid work more [12].

However, the difference between paid and unpaid work is much smaller for both highly educated men and women. Although highly educated men continue to prioritise paid work, the amount of time spent on unpaid work is higher than for men with low and medium level of education. This is particularly true between the ages of 35 and 40, when the higher proportion of unpaid work is spent on childcare. In addition, highly educated men and women tend to devote similar amounts of time to paid work after childbearing age. Although the difference between the time highly educated men and highly educated women spend on unpaid work is smaller, highly educated women continue to do more unpaid work for the rest of the life cycle. However, this shows that highly educated men and women share paid and unpaid work more equally.

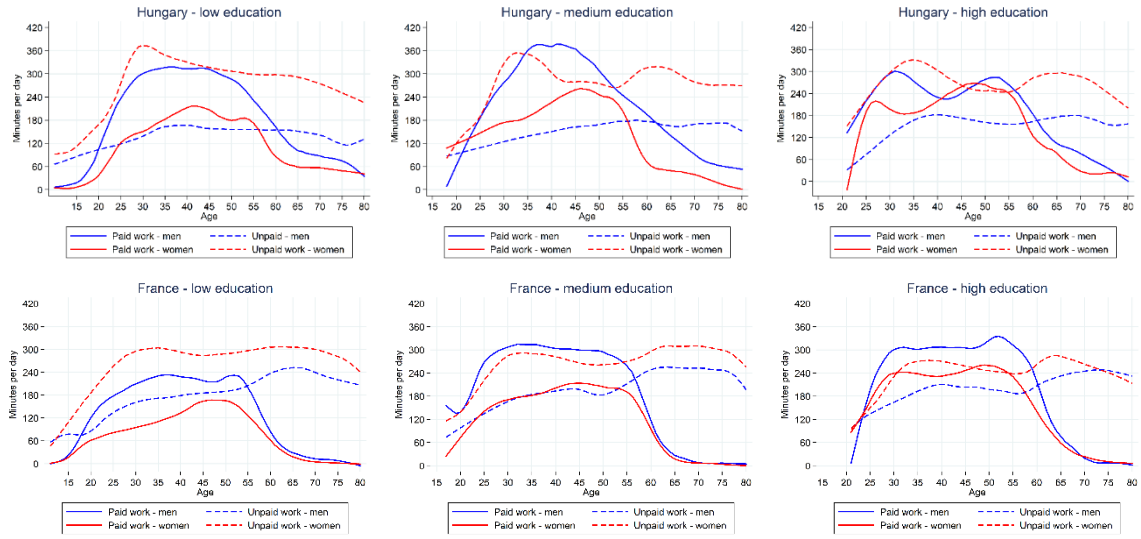


Figure 1: Time spent on paid and unpaid work in minutes per day – Hungary and France, 2009

This is not surprising because the better educated are on average also better paid on the labour market [12]. However, this means that highly educated women spend a similar amount of time doing unpaid work during their working years alongside paid work, which can lead to overworking [14]. These results are especially important at a time when women tend to participate more in the labour force and when the share of highly educated women in the EU is increasing. It is therefore beneficial to look at the market value unpaid work and its opportunity cost to see how much economic is lost as a result of unpaid work.

Figure 2 shows that the value of unpaid work is almost the same for both men and women with a low level of education according to the SR and OC methods. This is due to the fact that most activities of unpaid work such as cooking, cleaning, gardening or maintaining are the activities that would receive a wage of a person with low level of education in the labour market. Therefore, for those with low level of education, the opportunity cost of substituting the unpaid work with paid work is very low in terms of the monetary value created. They, could, however, benefit from other advantages of paid work such as health insurance, pensions etc., but this analysis is beyond the scope of this paper. The opportunity cost of substituting unpaid with paid work is slightly higher for those with a medium level of education than for those with low level of education.

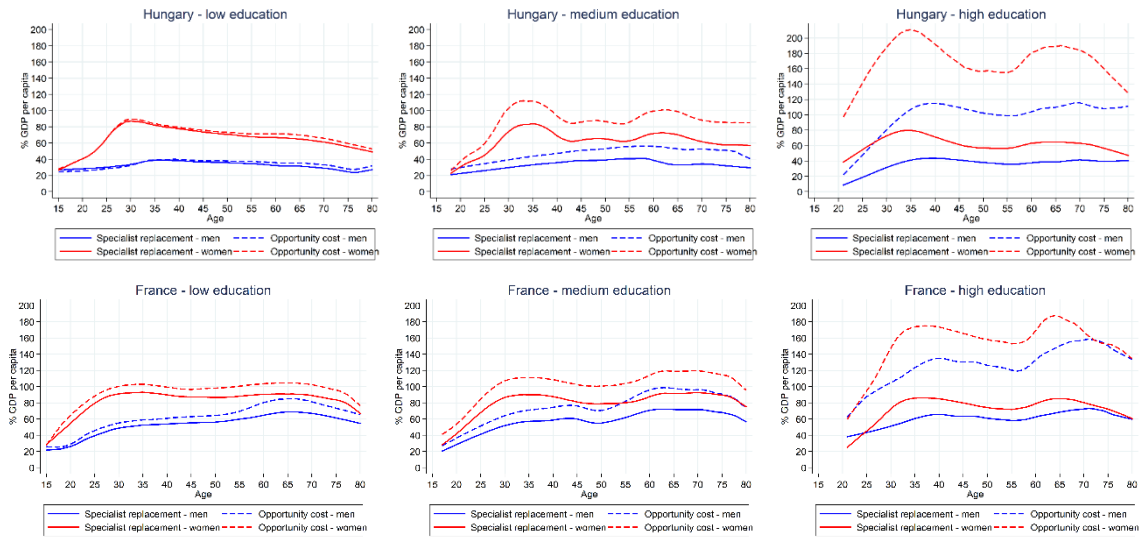


Figure 2: Unpaid work in Hungary and France, valued by the specialist replacement and opportunity cost methods, expressed as % of national GDP per capita for 2010.

The opportunity cost of unpaid work is the highest for both men and women with tertiary education as their wages are on average much higher than the wages that would be paid for unpaid work. This is particularly true for the highly educated women, whose opportunity cost of unpaid work is more than twice as high as the market value of unpaid work throughout their economic life cycle. The difference between the market value of unpaid work and its opportunity cost is the highest at the childbearing age and around retirement age for highly educated women. In both cases, these are times when women are caring for either their own children or their grandchildren.

4 DISCUSSION AND CONCLUSION

Men of all levels of education do more paid than unpaid work for most of their economic life cycle, while women still do much more unpaid than paid work. Only highly educated women perform similar amounts of paid and unpaid work. For women of all educational levels, unpaid work is highest when they are caring for small children in addition to doing housework. Previous research has shown that this "rush hour of life", where the total workload (paid and unpaid) exceeds leisure time, can be much more intense and prolonged for women than for men in some countries [14]. This can lead to women postponing the decision to have children or even giving up work altogether to care for children. As previous studies have shown, highly educated women already have their first child later in life than women with a low or medium level of education. They are also more likely to become mothers, but only if they are fully integrated into the labour market and have a secure job [3].

Therefore, these results are also extremely important for policy makers and companies. The analysis of the opportunity cost of unpaid work clearly shows that the provision of unpaid work services is necessary. This is particularly true for highly educated women whose opportunity costs of unpaid work are the highest. By promoting policies and services that could reduce the amount of unpaid work, such as more available, convenient, flexible and high quality childcare facilities, the availability of healthy and cooked food, the opportunity costs of unpaid work could also be reduced. This would make enable women decide for themselves on how much time they want to spend on paid work, housework and childcare without being overworked.

By not including unpaid work in the System of National Accounts, a large part of economic value created in this way remains invisible, but burdens women who are its main producers. At the same time, the share of highly educated women in the labour force is increasing in the EU. Giving them the opportunity to be more present in the labour market could be one of the solutions to keep the EU economy sustainable.

Acknowledgement
This work has been fully supported by the University of Rijeka project [uniri-mladi-drustv-23-43].

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GENERATIONAL DIFFERENCES IN WELL-BEING PERCEPTION: A MIXED-METHODS ANALYSIS

Terezie Krestová
VSB – Technical University of Ostrava, Faculty of Economics, Department of Management
17. listopadu 2172/15, 708 00 Ostrava
E-mail: terezie.krestova@vsb.cz

Pavlína Forstová Kuráňová
VSB – Technical University of Ostrava, Faculty of Economics, Department of Mathematical Methods in Economics
17. listopadu 2172/15, 708 00 Ostrava
E-mail: pavlina.forstova.kuranova@vsb.cz

Nicola Kubacká
VSB – Technical University of Ostrava, Faculty of Economics, Department of Management
17. listopadu 2172/15, 708 00 Ostrava
E-mail: n.kubacka.n@gmail.com

Abstract: Generational differences in the perception of well-being represent a key challenge in human resource management (HRM). This paper aims to explore generational differences in perception of well-being and identify key factors influencing well-being within Generations X, Y and Z. A mixed-methods approach with a quantitative focus was used. The results show that Generation X reported the highest and most stable levels, while Generation Z consistently showed the lowest, especially in terms of financial, spiritual, and mental well-being. The findings support inclusive HR strategies that address generational needs, improve satisfaction, engagement, and motivation, while reducing burnout in an age-diverse workforce.

Keywords: well-being, generational differences, human resource management

1 INTRODUCTION

The concept of well-being has become increasingly relevant in the context of modern work and life. The traditional work-life balance model shifts towards holistic fulfilment spanning physical, mental, social, and emotional dimensions. Well-being, a key indicator of life quality and resilience [9], is gaining prominence both in public policy and in HRM, underpinning strategies that address diverse employee needs. Research show that perceptions of well-being differ significantly across generations. Generations X, Y, and Z demonstrate different patterns in engagement, values, and workplace expectations [8], [3].

The aim of the paper is to examine how Generations X, Y, and Z perceive well-being, focusing on work-life balance, technology use, and organisational support. Understanding these generational differences is essential for developing HR strategies that support diverse employee needs in today's multigenerational workforce.

2 THEORETICAL BACKGROUND

Well-being is a dynamic state in which both individuals and societies achieve a balance between the physical, mental, and social dimensions of life. Within the scope of sustainable development, well-being is increasingly viewed as the intersection of health, social justice, and environmental sustainability, forming a foundation for new strategies in policy making and organisational governance [9]. It is becoming evident that achieving this balance is not a universal process but rather an individual journey influenced by a range of internal (biological and mental predispositions) and external factors (social conditions, available resources, and the extent to which organisations or societies foster a supportive environment) [4].

Organisations capable of recognising and accommodating these differences have the potential to attract a broader spectrum of talent. This expands the human resource base and enhances the quality and diversity of team skills, generating value beyond conventional expectations [2]. The following well-being pillars within social work are classified [5]:

- Building meaningful relationships with partners, family, friends, colleagues, and the wider community is vital, particularly in times of stress.
- Being mentally and physically active and having a sense of purpose, through work, caregiving, or personal interests, provides life with direction and fulfilment.
- Continual learning and the commitment to human values help individuals, including social workers, find meaning and improve their practice.
- Giving to others through compassion, empathy, and support, in the form of time, care, or material resources, strengthens individuals and contributes to the well-being of society.
- Practising mindfulness and presence supports the development of authentic relationships, effective communication, and reflective listening, all of which are foundational to the values and principles of social work.

Another multidimensional approach is represented by the PERMA model (Positive emotion, Engagement, positive Relationships, Meaning, and Accomplishment) [7]. Drawing from further literature [3], [6] and insights from experts at the Gallup Institute, well-being can also be understood through five core pillars that reflect key domains of human life: meaning, relationships, health, finances, and community. Based on data collected from individuals in more than 150 countries, it has been concluded that these five factors are decisive in determining whether a person is truly thriving or merely surviving. This concept formed the basis for the survey conducted in this study.

3 METHODOLOGY

This research used a quantitative survey to investigate generational differences in the perception of well-being and to identify key factors influencing this. The main objective was to evaluate subjective well-being in Generations X, Y and Z, and to explore its relationship with employment status, lifestyle, and broader socioeconomic variables.

The survey instrument was based on the Gallup well-being framework, adapted by Brixová [1]. It consisted of two sections: the first collected demographic and classification data; the second included 22 closed-ended items divided into five domains of well-being: physical, mental, social, spiritual and financial. Responses were measured on a five-point Likert scale (1 = strongly disagree, 5 = strongly agree), allowing quantification and statistical analysis.

Data collection was carried out in January 2025 through an online questionnaire. The survey was primarily disseminated through social media and personal networks. Two research questions were set: i) Are there statistically significant differences in the perception of well-being across generations? ii) What factors most significantly influence well-being within each generational group? To test for significant generational differences in well-being, the Kruskal-Wallis test was applied. This method was selected for its suitability to compare median values across multiple groups. Separate tests were performed for each of the five dimensions to identify where significant intergenerational differences existed.

4 RESULTS

Of the total sample of 260 respondents, 7 responses were excluded, as they did not meet the criteria of the selected research sample. In the paper, only summary results for the entire questionnaire, across all five well-being dimensions, are presented. Each dimension of

subjective well-being was quantified as the arithmetic mean of responses to a set of relevant items, all measured using a five-point Likert scale. The average score for each dimension was calculated as the mean of all subitems within the given category. These results are in Table 1.

Table 1: Mean Scores for Well-Being Dimensions by Generation (own elaboration)

Dimension / Generation	***Physical***	***Mental***	***Social***	***Spiritual***	***Financial***
Z (2000–2013)	3.41	3.25	3.63	3.63	3.26
Y (1981-1999)	3.67	3.44	3.79	4.11	3.52
X (1965-1980)	3.43	3.82	3.78	4.24	3.45

Principal Component Analysis was conducted to verify the unidimensionality of the individual constructs. Factor analysis was applied separately to each of the five dimensions of well-being. The Kaiser-Meyer-Olkin measure ranged from 0.729 to 0.829 across all cases, indicating an adequate level of sampling adequacy. The Bartlett sphericity test was statistically significant in all cases ($p < 0.001$), confirming the suitability of factor analysis. In each dimension, a single factor was extracted, confirming the unidimensional structure of all five dimensions.

The Kruskal-Wallis test was then performed. The independent variable was generational affiliation, categorized into three groups: Generation X, Generation Y, and Generation Z. For each dimension of subjective well-being, the analysis tested whether median scores differed significantly across these generational groups. The results indicated statistically significant differences between generations in three dimensions: mental ($p = 0.003$), spiritual ($p < 0.001$), and financial ($p = 0.041$). No significant differences were found for physical and social dimension ($p > 0.05$). The $\epsilon 2$ indicated small to medium effects, with the highest observed in spiritual dimension ($\epsilon^2 = 0.062$).

The level of statistical significance was set at $\alpha = 0.05$. For the dimensions in which the Kruskal-Wallis test was significant, post-hoc pairwise comparisons were conducted using the Dunn-Bonferroni test. The results are shown in Table 2.

Table 2: The results of the Kruskal-Wallis test (own elaboration)

Dimension of Well-being	**Test Statistics**	***p*-value**	**ϵ^2**	**Effect Size Interpretation**
Physical	4.29	0.117	0.009	Small to negligible effect
Mental	11.85	0.003	0.039	Small effect
Social	0.06	0.973	0.000	Negligible
Spiritual	17.47	< 0.001	0.062	Medium effect
Financial	6.41	0.041	0.018	Small effect

The Kruskal-Wallis test identified statistically significant generational differences in mental, spiritual, and financial dimensions, with medium effect sizes indicating a meaningful influence of generational affiliation. Post-hoc comparisons revealed that Generation Z reported significantly lower mental well-being than Generation X ($p = 0.003$). In terms of spiritual well-being, Generation Z scored significantly lower than both Generation X ($p < 0.001$) and Generation Y ($p = 0.005$). Similarly, Generation Z ($p = 0.042$) reported significantly lower financial well-being compared to both older cohorts.

These findings address the first research question by confirming that subjective well-being differs significantly across Generations X, Y, and Z. Generation Z consistently scored lowest in mental, spiritual, and financial dimension.

Regarding the second research question, distinct generational well-being determinants were identified. For Generation Z, the key factors include financial insecurity, stress management difficulties, and weak value anchoring, often compounded by instability in education and

employment. Generation Y's well-being is shaped by work-life balance, self-fulfilment, and career development opportunities. Generation X benefits primarily from workplace stability, health security, and a sustained sense of purpose.

5 DISCUSSION AND CONCLUSION

The study confirmed significant generational differences in subjective well-being, particularly in the mental, spiritual, and financial domains. Generation X reported the highest and most stable levels of well-being, while Generation Z consistently showed the lowest, especially in terms of financial, spiritual, and mental well-being. Generation Y showed more varied, mid-range results.

These findings are consistent with international research that highlights lower organisational engagement and increased stress among younger cohorts. Although the survey focused on Czech employees, the patterns reflect broader global trends and underscore the importance of age-sensitive HR strategies that address the specific needs and values of each generation.

Further research will focus on analysing a larger population sample to validate the findings. Additional tests, such as cluster analysis, may also be applied, or the focus may shift to other categories (e.g., gender, education, etc.).

Recognising the diversity in well-being perceptions across generations can help organisations promote engagement, reduce burnout, and improve employee satisfaction in an increasingly age-diverse workforce.

Acknowledgement
This article was supported by the Student Grant Competition of the Faculty of Economics, VSB-Technical University of Ostrava, project registration number: SP2025/023.

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RETHINKING LEAN THROUGH A SOCIAL SUSTAINABILITY LENS

Klara Ljubi
University of Ljubljana, School of Economics and Business, Kardeljeva ploščad 17, 1000 Ljubljana, Slovenia
E-mail: klara.ljubi@ef.uni-lj.si

Marko Jakšič
University of Ljubljana, School of Economics and Business, Kardeljeva ploščad 17, 1000 Ljubljana, Slovenia
E-mail: marko.jaksic@ef.uni-lj.si

Abstract: This paper looks at lean through a social lens – social lean (SL) – and examines its overlap with social sustainability (SS). The distinguishing and overlapping features of SL and SS are identified and an SL-SS Interaction Typology is proposed that categorises the interactions as synergistic, conflicting and contingent. A focused review of 10 peer-reviewed studies revealed that the majority of studies to date report synergistic SL-SS interactions and unidirectional (SL→SS) effects. A worthwhile direction for the future is the opposite relationship (SS→SL) and the conflicting and contingent interactions.

Keywords: lean management, social lean, social sustainability, interaction typology, sustainable operations

1 INTRODUCTION

When the lean philosophy was originally established, it was more productivity-driven than socially-driven and was often criticised as exploitative and dehumanising [12]. An open question is how the social aspect of lean is anchored in lean from the very beginning. We will refer to this aspect with the term **social lean** (SL). With the increasing emphasis on **social sustainability** (SS) that has come to the forefront of business in the last decade, research followed by exploring the relationship between lean management and SS, which has raised the question of the necessary adaptations of lean to support SS. However, most of this current research has focused on understanding whether the influence of lean on SS is positive or negative, while less emphasis has been placed on the inverse relationship and other interactions.

Our study begins by clarifying the definition of SL and its elements, which is not established in the literature, and outline the overlap between lean and its social component SL and SS. We then explore the interactions between SL and SS and propose a simplified SL-SS Interaction Typology, which is the main objective of this study. Due to the exploratory nature of the study, we have limited our review of the literature to a limited number of peer-reviewed academic studies that specifically examine the relationship between SL and SS, which is in line with our research aim, rather than a larger but less focussed sample. On this basis, our aim is not to provide empirical confirmation, but to map the emerging landscape and identify opportunities for future research.

2 DISTINGUISHING FEATURES AND OVERLAP OF SL AND SS

Lean's main goal is to establish a culture of continuous improvement by identifying activities that create added value for customers and employees and eliminating inefficiencies. Lean can be defined on three levels: philosophy, principles, and tools and techniques [21]. Even though lean is mostly considered from a technical point of view (tools and techniques), the social elements and practices concern people and their relationships with the following relevant aspects: training/knowledge, continuous improvement/kaizen, empowerment, team-based problem solving, human resource management, management commitment, leadership [19, 20].

In addition, the authors have established a concept of the social pillar of lean that deals specifically with employees and requires a supportive HR strategy of skills enhancement, motivation and empowerment, as well as lean culture with a clear lean strategy that empowers teams, is based on exemplary leadership behaviour and enables for continuous learning [16]. For the purpose of this study, SL is not a standalone concept, but we understand SL as a people-centred dimension of lean that focuses on employee empowerment, continuous learning, team-based problem solving and supportive management practices to sustain a culture of continuous improvement.

The success of lean implementation depends on the culture – either organisation or employee oriented [3]. Without a lean culture, the implementation of lean practices cannot be successful either. This shows that employees play a major role in lean. The way in which people are respected and how their suggestions are incorporated into company practices will therefore determine the extent to which employees will internalise the culture and work according to it. Motivated, involved and **empowered employees/teams** support the continuous improvement processes by finding new/better ways to **problem solving** [7]. Involvement in problem solving can reduce stress, which contributes to a more positive working environment [10]. Management at different levels serve as role models in lean, and **management support** at all organisational/hierarchical levels determines whether the benefits will be predominantly positive or negative [2, 10]. Managers are coaches who ensure efficient communication and support employees [16]. Another SL element is **continuous learning**. Lean organisations should spend a lot of time training their employees and communicate openly with them. Training and skills development enable continuous learning and change, and are necessary for employees to align the culture with lean [3].

SL is still primarily productivity-focused, but we have just shown that lean has several social elements as its core. SS, on the other hand, is primarily about people. Dimensions of SS are employee well-being, communication, management support, reward and control system, learning and growth, education and training, job security, fair labour practices, and health and safety [1, 2]. In SS, employees learn from management [15], and therefore require frequent, clear and sufficiently broad **communication** [2]. When **management support**, **learning** and **growth**, and **job security** are prioritised, employees feel more valued, respected and safe, which increases their overall **well-being**. Well-being (either physical or emotional well-being, or work-life balance), in conjunction with **equity practices**, can promote and improve employee **health and safety** as well as **human rights** [2, 22]. In addition, [2] identified **training** and **reward systems** as SS elements. Training should prioritise learning by doing, followed by in-class training and work assignments, while a fair and consistent rewarding system can be motivating for employees. SS also directly addresses **fairness of employment** (e.g. wages), **equal opportunities** and diversity in various aspects (e.g. gender, age and disability inclusion) [1, 15].

The distinguishing features of SL and SS presented above can be seen from each extreme side of the diagram in Figure 1, and SL and SS also have many overlapping features shown in the intersection of the diagram. Management support at SL relates primarily to establishing and spreading the lean culture, sharing the vision of continuous improvement, actively encouraging ideas to solve problems, and working as a coach for employees/teams. This cannot be achieved without clear and open communication. The same applies to SS, where the role of management is to implement policies for social equity, inclusion, diversity and fairness, which need to be communicated clearly and openly. In both SS and SL, well-being can be physical or emotional. Employee well-being in SL is a prerequisite for engaged and motivated employees who contribute to problem solving. On the other hand, SS adds work-life balance to well-being, which is not emphasised in SL. If well-being is employee-related, health and safety refers to the physical environment. In SL, a positive influence of various lean tools on health, safety and

ergonomics as aspects of the physical environment has been confirmed [7]. Training and skills development in SL are an enabler for problem solving and continuous improvement, while in SS more emphasis is placed on equitable access to education and training that support the core dimensions of SS.



Figure 1: Venn diagram showing the distinguishing features of SL and SS and their overlap

3 SL-SS INTERACTION TYPOLOGY AND LITERATURE-BASED INSIGHTS

In the previous chapter, we pointed out parallels and distinguishing features of SL and SS, while we have not yet addressed their interaction. To better understand the latter, we propose a simplified typology consisting of three interaction types: synergistic, conflicting, and contingent on implementation context, which is in line with previous research (e.g. [8], [9], [18]). However, the authors did not focus on SL/SS specifically, but either on lean or sustainability in general or on their unidirectional relationship. Table 1 categorises the reviewed studies into the proposed SL-SS Interaction Typology. The 10 categorised studies are either empirical, based on questionnaires or case studies.

78 % of the reviewed studies fall into the **synergistic** interaction type, meaning that either SL improves SS or vice versa. Despite lean being initially more productivity-focused, redefining lean in its social terms outlines its high focus on social aspects, suggesting that lean could be a SS tool as already proposed by [8] for employee SS. The implementation of SL can have a synergistic effect on SS. In contrast, there is not much research confirming the potential positive impact of SS on SL. [18] have confirmed that the initial implementation of lean (or sustainability) helps organisations to implement sustainability (or lean) next, but they have not investigated the social aspects. [6] have emphasised that lean may have evolved over time, balancing its technical and social components, which suggests that more synergistic impacts can be expected. However, they have mainly addressed the SL→SS relationship, while the reverse relationship is barely addressed and requires further attention.

11 % of the reviewed studies fall into the **conflicting** interaction type, meaning that either SL worsens SS or vice versa. Lean has been accused of being exploitative and dehumanising [12]. This suggests that a conflictual environment can arise when the goals of lean and sustainability are not aligned. A faster pace of work can lead to more stress and pressure on workers [10]. Potential conflicts and barriers between SL and SS are often mentioned in the literature but rarely empirically documented, which represents a research gap.

The **contingent** interaction type is also under-researched. 22 % of the reviewed studies are categorised as neutral, meaning that there is no inherent positive or negative effect, with the impact depending on the context. These studies were mostly more complex and divided SL and SS into several dimensions, thus reaching contingent conclusions. For example, [10] examined 21 relationships between different work practices and job stress levels. The direction of these relationships ranged from positive to negative or statistical insignificance. They also

confirmed a time-dependent stress level with stress increasing at the beginning of lean implementation and decreasing after an inflection point. This means that the influences between SL and SS depend on the implemented dimensions, the stage of implementation and other organisational factors (e.g. management style, industry).

Table 1: Interaction between SL and SS in reviewed studies

Ref.	***Interaction SL-SS***	**SL→ SS**	***SS→SL***	***Important findings***
[4]	Conflicting	X		* Lean negatively impacts on ergonomics, safety and environment * SS in conflict with economic factors * SS factors can degrade each other
[5]	Synergistic	X		- Internal lean practices positively impact SS - SS (mediator) further positively impacts economic performance and operational performance
[6]	Synergistic	X		* Internal lean practices positively impact SS * Lean has evolved into a more balanced socio-technical system * Digital technology advancement moderates relationships
[10]	Contingent	X		- Worker job stress depends on the phage of lean implementation - Lean (different shop floor practices) can increase/decrease workers' stress or have no influence on it
[11]	Contingent	X	X	* The lean impact on SS can be direct/indirect and positive/negative
[13]	Synergistic	X		- Customer lean processes positively influence SS
[14]	Synergistic	X	X	* Researched corporate social responsibility (CSR), not SS * Positive effects of lean on CSR
[17]	Synergistic	X		- Lean positively impacts SS orientation and sustainability (including SS) - SS orientation positively impacts implementation of SS practices
[23]	Synergistic	X		* Lean positively impacts SS, which can be intensified by the lean-sustainability interaction
[24]	Synergistic	X		- Lean practices and social practices are positively linked to SS - Synergies can be achieved when more than one practice (lean, green, social) is implemented

4 CONCLUSIONS

Our study conducted a focused review to categorise previous SL-SS studies into an SL-SS Interaction Typology. First, we defined SL through a social lens and outlined the overlapping dimensions between SL and SS. We also proposed three interaction types: synergistic, conflicting and contingent. The majority of studies fall into the synergistic type, indicating the first gap in the existing literature. Future research should empirically test conflicting and contingent SL-SS relationships. In addition, a lack of studies investigating the relationship from SS to SL was identified, which requires further attention. The positive impact of lean on SS was confirmed and the lean philosophy was also suggested as a tool to support SS [8], while a reverse relationship did not receive as much attention. A limitation of the study lies in the small sample of reviewed studies.

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INTERTEMPORAL CONSUMPTION BEHAVIOUR IN POST-COMMUNIST EU COUNTRIES

Martin Lukáčik
Bratislava University of Economics and Business, Department of Operations Research and Econometrics
Dolnozemská cesta 1, 852 35, Bratislava, Slovak Republic
E-mail: martin.lukacik@euba.sk

Adriana Lukáčiková
Bratislava University of Economics and Business, Department of Operations Research and Econometrics
Dolnozemská cesta 1, 852 35, Bratislava, Slovak Republic
E-mail: adriana.lukacikova@euba.sk

Karol Szomolányi
Bratislava University of Economics and Business, Department of Operations Research and Econometrics
Dolnozemská cesta 1, 852 35, Bratislava, Slovak Republic
E-mail: karol.szomolanyi@euba.sk

Abstract: This paper investigates intertemporal consumption behaviour in eleven post-communist countries that have become members of the European Union. Using annual data from 1995 to 2019, we estimate the Euler equation for nondurable consumption based on a standard consumption-based asset pricing model. The Generalised Method of Moments (GMM) is applied to account for potential endogeneity and measurement errors. The internal rate of return, constructed from Penn World Tables as the real return on capital, is the proxy for the intertemporal substitution rate. Our findings provide evidence of consumption smoothing consistent with the theoretical model, with significant variation across countries.

Keywords: discount factor, constant relative risk aversion, Euler equation, Generalised Method of Moments

1 INTRODUCTION

The intertemporal allocation of consumption lies at the core of modern macroeconomics and finance. According to the representative agent framework, consumption decisions are driven by expectations about future returns and preferences, as captured in the Euler equation. While this model has been extensively tested in developed economies, there is less empirical evidence for countries undergoing structural transformation, such as the post-communist economies of Central and Eastern Europe.

This study aims to fill this gap by estimating the Euler equation for nondurable consumption in eleven post-communist EU member states over the period 1995–2019. These countries experienced significant institutional changes, market liberalisation, and eventual integration into the European Union. Such dynamics offer a unique setting for testing the validity of the consumption-based asset pricing model in non-standard macroeconomic environments.

We apply the Generalised Method of Moments (GMM) to account for possible endogeneity between consumption growth and returns. The real return on capital, constructed from internal rate of return (IRR) measures in the Penn World Tables, is used as the intertemporal price signal. This approach allows us to assess whether consumption behaviour in these economies aligns with the optimising behaviour implied by the standard intertemporal model.

Several empirical studies have evaluated the validity of the Euler equation in both developed and emerging economies. Hall [8] pioneered the empirical testing of the consumption-based model, finding limited support for its implications in U.S. data. Campbell and Mankiw [3] introduced models with rule-of-thumb consumers, highlighting deviations from full intertemporal optimisation. In the context of transition economies, studies such as Gourinchas

and Parker [7] and Herrendorf et al. [10] emphasised the role of financial constraints and institutional instability. More recently, applications of GMM estimation in emerging Europe, including work by Égert [5] have revealed mixed support for consumption smoothing, often attributing the heterogeneity to varying speeds of financial development and market liberalisation. Building on this literature, using a consistent return proxy, our paper offers a unified GMM estimation of the Euler equation across eleven post-communist EU countries.

2 MODELS AND METHODS

The Generalised Method of Moments is a robust econometric method used to estimate parameters of economic models, including the Euler equation, which is central to dynamic optimisation problems in macroeconomics and finance. The Euler equation describes the intertemporal optimality condition for consumption derived from a utility maximisation problem. The economic model of individual behaviour, the Consumption-based Capital Asset Pricing Model (CCAPM) developed by Hansen and Singleton [9] and elaborated by Campbell [1], [2] implies a set of moment conditions that can be used to estimate the unknown key parameters.

We consider an individual agent that maximises expected CRRA-type utility in the form:

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma} \tag{1}$$

depending on consumption C with relative risk aversion γ (higher values correspond to a more risk-averse individual).

Solving the problem for the rational behaviour of the agent takes the form:

$$\max E_t \sum_{s=0}^{S} \beta^s \frac{C_{t+s}^{1-\gamma}}{1-\gamma} + \lambda_{t+s}\left((1+r_{t+s})A_{t+s-1} + w_{t+s} - C_{t+s} - A_{t+s}\right) \tag{2}$$

where C_{t+s} denotes consumption at time $t+s$, β is the discount factor, γ is relative risk aversion, E_t denotes the expected value conditional on all information available at time t, A_{t+s} is financial wealth at the end of period $t+s$, r_{t+s} is the returns to financial wealth in period $t+s$, and w_{t+s} is the wage income of period $t+s$.

The first-order conditions take the form:

$$E_t\left(\beta \frac{\partial U(C_{t+1})}{\partial C_{t+1}}(1+r_{t+1})\right) = \frac{\partial U(C_t)}{\partial C_t} \tag{3}$$

where on the right-hand side is the marginal utility of one added euro consumed today, and on the left is the expected marginal utility of the saved euro by the next period when it is consumed. According to the first-order condition of the optimisation problem, these are equal. The Generalised Method of Moments is appropriate for estimating of the Euler equation:

$$E_t\left(\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(1+r_{t+1}) - 1\right) = 0 \tag{4}$$

In the context of GMM, the over-identifying restrictions are tested via the commonly employed J statistic of Hansen (1982). The J statistic is distributed as $\chi 2$ with degrees of freedom equal to the number of over-identifying restrictions. A rejection of the null hypothesis implies that the instruments do not satisfy the orthogonality conditions required for their employment.

3 DATA AND RESULTS

To estimate (4), the consumption of nondurables and interest rate time series is needed. Annual data on real nondurable consumption are gathered from Eurostat. The interest rates used are internal rate of return data reduced by the depreciation rate from the Penn World Table database. The lagged population real GDP interest rate and the share of nondurable consumption were used as instrumental time series. Data from Bulgaria, Croatia, Czechia, Estonia, Hungary, Latvia, Lithuania, Poland, Romania, Slovakia, and Slovenia are used. The database range is from 1995 to 2019. Table 1 displays the estimates of the parameters of the specification (4).

Table 1: Estimates of discount factor (Beta) and relative risk aversion (Gamma).

Country	***Beta***	***Std.Err***	***Gamma***	***Std.Err***	***J-stat***	***Prob-J***
Bulgaria	0.905	0.007	0.596	0.117	6.569	0.584
Czechia	0.995	0.001	0.045	0.013	5.222	0.633
Estonia	0.986	0.002	0.176	0.016	2.581	0.630
Croatia	0.980	0.001	0.070	0.032	4.008	0.548
Latvia	0.983	0.002	0.103	0.024	4.773	0.311
Lithuania	0.938	0.004	0.206	0.082	4.740	0.448
Hungary	0.966	0.001	0.049	0.014	3.912	0.689
Poland	0.960	0.015	0.481	0.187	6.361	0.703
Romania	0.937	0.010	0.314	0.140	5.050	0.752
Slovenia	0.993	0.001	0.128	0.024	6.067	0.532
Slovakia	0.966	0.005	0.172	0.032	5.136	0.526

Table 1 includes the standard error of the parameter estimates calculated by the procedure of Newey and West [11] and *J* statistics of each equation with the corresponding probability value. All the estimates of the discount factor are statistically significant at a 1% level, and all the estimates of the relative risk aversion are statistically significant at a 5% level. The non-orthogonality of the instrument sets used in estimations is not rejected in all cases.

4 CONCLUSION

The values of intertemporal consumer behaviour parameters such as the discount factor and relative risk aversion are important for the quantitative formulation of the consumer utility function, which is vehemently used in current macroeconomic general equilibrium models such as DSGE models, CGE models, and New Keynesian models.

Our GMM-based estimation of the Euler equation reveals partial support for the standard consumption-based model in the examined post-communist EU countries. The estimated coefficients fall within theoretically plausible ranges in several cases, indicating forward-looking consumption smoothing.

Future research could explore time-varying parameters, nonlinearities, or habit formation to better capture the evolution of consumption dynamics in such economies.

Acknowledgement
The authors would like to acknowledge support from the Grant Agency of Slovak Republic 1/0052/24 ” Estimation of key structural parameters of current macroeconomic models”.

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HOW WOMEN MANAGE CAREER CHANGE IN TODAY'S LABOUR MARKETS

Kosovka Ognjenović
Institute of Economic Sciences, Welfare Economics Department
Zmaj Jovina 12, 11000 Belgrade, Serbia
E-mail: kosovka.ognjenovic@ien.bg.ac.rs

Abstract: Occupational segregation is one of the main causes of all gender differences today, especially of the gender earnings gap, along with job characteristics and industry. It is empirically proven that educational segregation, work experience and other personal characteristics of workers make a smaller contribution to explaining the existing inequalities between men and women in the labour market and are even less significant. This paper examines conceptually how women cope with professional challenges, particularly in terms of the impact of technology on the labour market, changes in educational structures and occupations. Drawing on existing theoretical concepts and empirical findings, the conclusions will serve to highlight gaps in current approaches and provide some directions for future empirical research on trends in career change and women's participation in the labour market.

Keywords: career, job positions, occupations, technology impact, women.

1 INTRODUCTION

The professional development of women and the corresponding changes over the course of their productive age depend primarily on how active they are in the labour market. Their status in the labour market is determined by many factors, as the literature suggests, including educational attainment [6, 7], employment history, termination and continuation of employment [12], marital status, geographical factors [11], social norms and cultural differences [3, 7] and other personal characteristics, but also characteristics of firms and employers [1, 10]. Thus, many factors are taken into account, psychological, sociological and even health conditions, but when economic motives are singled out, some of them are considered important, such as expected earnings, promotion and other factors that can affect the economic situation of the individual and their household. At a younger age, they are guided by economic factors and the expected annual salary they can earn from doing a certain job. Thus, they may change their occupation, e.g. start as a shop assistant and continue as a teacher, and at an older age they may even leave the paid labour market and become self-employed [8, 9]. As the occupational level is less complicated, the income gains expected from changing jobs or careers are lower and it is easier to make a change. Women who take on positions of responsibility in organisations and survive in these positions face even greater difficulties, and these situations show marked gender differences [2, 6].

The transition from education to employment in the European Union, for example, depends strongly on prior educational attainment, age groups and the willingness of women to contribute by actively participating in the paid labour market, as trends in the gender structure of secondary and tertiary graduates show that women are less likely to find employment than men. Eurostat data for 2023 [4] show that the employment rates of tertiary graduates (20-34 years old) are between 85% and 82% for men and women (83.5% at the Union level). The employment rates of secondary graduates are lower for both men (81.7%) and women (80%) and average 81% at the EU-27 level. These rates for men and women with secondary and tertiary education lead to a gender employment gap that indicates slow changes in the advancement of women in the labour market and may have further implications for their career progression.

Based on the available data, this paper aims to focus on several research questions that may be relevant in the context of examining how women are coping with challenges that are likely

to affect career change in today's labour markets. Thus, several questions will be examined: 1. how the structure of educational attainment of working women is changing; 2. to what extent do working women take advantage of continuing education programmes; 3. how the occupational structure of working women is changing over time. These questions are analysed using the aggregated survey data and serve as a basis for drawing conclusions about possible directions in women's occupational and career change. In aggregated form, the results will provide approximate values and general insights into the current situation.

2 DATA AND METHODS

As mentioned above, this paper addresses three questions that may be relevant to women's decisions about entering the labour market, remaining in the labour market, and transitioning between jobs and career advancement. The focus is on the European labour market, so that average values at the level of Unions will be examined as well as gradual changes. The first research question relates to the structure of educational attainment of working women and the changes observed. The second research question examines the extent to which working women make use of further education programmes. Finally, the last question examines how the occupational structure of working women changes over time.

At least two Eurostat datasets can be used to answer the above questions: the European Union Labour Force Survey (LFS) data series with annual indicators and the European Union Adult Education Survey (AES) data. The first dataset provides continuous annual data series, while the second dataset contains the data for the years in which the survey was conducted (starting in 2007, then 2011, 2016 and 2022). Thus, the data available at an aggregated level allows the application of descriptive statistical analysis methods.

3 RESULTS AND DISCUSSION

Over the last ten years, the employment prospects of women in the European Union (EU-27) have improved, with the employment rate increasing by an average of four percentage points and reaching 49.5% for women aged 15 and over in 2024. At the same time, the employment rate for men rose by 2.6 percentage points to 59.9 % in 2024, which shows that the gender balance in terms of employment is still a long way off.

Figure 1 shows the development of female employment rates across the educational levels defined by ISCED codes.

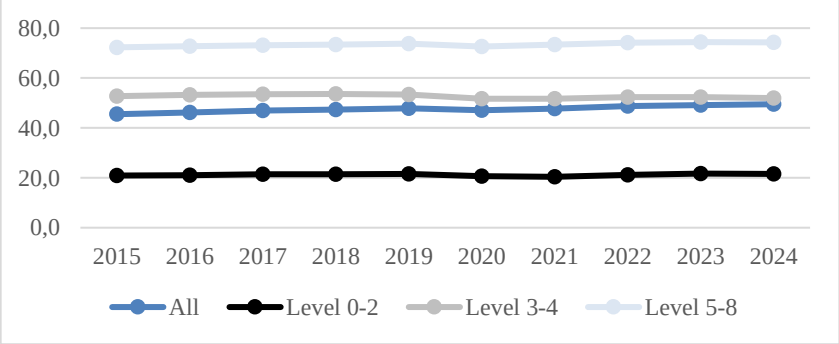


Figure 1: Employment rates of women by educational attainment and years, EU-27

The first research question assumes that the educational structure can be seen as an important factor in increasing the employment rates of women. Over the ten-year period, the employment rate of women with a tertiary education rose by two percentage points (74.3% in 2024) and

that of men by 1.2 percentage points to a rate of 75.6%. This is probably because the employment rates of university graduates (aged 20 to 34 years) are rising faster among women, as already explained. If we look at the distribution of female employment by level of education, two further subtleties emerge. The employment rate of women with upper secondary and post-secondary non-tertiary education is declining, in contrast to men, for whom the employment rate has remained almost the same. This is in line with the higher enrolment rates in tertiary education and the postponement of employment. For both women and men, the employment rates of those with the highest level of lower secondary education increase (for men by 1.7 percentage points to 38.1% in 2024, while the employment rate for women increases by 0.6 percentage points to 21.5%).

Women are more likely to participate in continuing education programmes, as shown by data from the current AES 2022 – the participation rate of women (51.3%) is higher than that of men (50.1%) in the population aged 18-64 in the EU-27. On the other hand, according to the LFS 2024 data, a larger proportion of employed women (36.8%) participate in continuing education than men (32%), showing that women are more aware of the impact of education on their career development despite all the obstacles they face [7, 13].

Table 1: Changes in the occupational structure of employed men and women, EU-27

Occupation	***Men***			***Women***		
	Structure, in %			***Structure, in %***		
	2015	*2024*	*Diff.*	*2015*	*2024*	*Diff.*
Managers	68.1	64.8	-3.2	31.9	35.2	3.2
Professionals	47.0	46.2	-0.8	53.0	53.8	0.8
Technicians	50.4	50.1	-0.3	49.6	49.9	0.3
Clerks	32.8	33.9	1.1	67.2	66.1	-1.1
Service workers	37.1	36.6	-0.5	62.9	63.4	0.5
Skilled agricultural workers	66.3	72.2	5.9	33.7	27.8	-5.9
Craftsmen	88.9	89.3	0.4	11.1	10.7	-0.4
Operators	81.9	82.7	0.7	18.1	17.3	-0.7
Elementary occupations	43.2	46.6	3.4	56.8	53.4	-3.4
Total	54.2	53.6	-0.6	45.8	46.4	0.6

Source: Author on the basis of Eurostat (2025) data.

The greatest changes in the employment structure by occupation can be observed among managers and professionals, which can be directly linked to the changes in women's careers (Table 1). Women are still underrepresented in science, technology, engineering and maths occupations [5], but overrepresented in education, social protection and health. In terms of professional careers, the occupational structure of female managers is developing positively, while the proportion of women who do not pursue a profession is decreasing.

4 CONCLUDING REMARKS

Previous studies have been concerned with reviewing the known and developing new models of how women cope with career challenges over the course of their working lives, using theoretical concepts and empirical research findings. The literature has found that women's career development is primarily determined by individual aspirations and organisational concepts. The linear models of women's and men's career development can no longer be considered conceptual, both because of differences in individual characteristics and because of the influence of technology driven by global progress.

This paper confirmed that women pay more attention to education, but also that, despite changes in the occupational structure, some occupational groups, such as managerial positions

and engineering occupations, are less accessible to them. This paper lacks the analysis based on the individual or microlevel data and monitoring women's distribution through the positions providing the examples of career advancement and changes related to specific jobs or workplaces in organizations and comparison with their male counterpart in order to examine gender balance and specific challenges women experience when moving along the career development path. This paper raises questions that are worth exploring in further research.

Acknowledgement
The research presented in this article was funded by the Ministry of Science, Technological Development and Innovation of the Republic of Serbia on the basis of Contract No. 451-03-47/2023-01/200005.

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LONG-TERM CARE PROGRAMS IN RURAL REGIONS MUST REDUCE THE RISK OF SOCIAL ISOLATION

Željka Ostović[1], **Samo Drobne**[2] and **Marija Bogataj**[1,3]
[1] Alma Mater Europaea University, Maribor, Slovenia
[2] University of Ljubljana, Faculty of Civil and Geodetic Engineering, Ljubljana, Slovenia
[3] Institute INRISK, Trebnje, Slovenia
zeljka.ostovic@almamater.si, samo.drobne@fgg.uni-lj.si, marija.bogataj@guest.arnes.si

Abstract: Home nurses play an important role in supporting older people in rural areas of Croatia. They not only provide medical care, but also psychological support. An essential task is to combat social isolation. We want to investigate how carer visits affect the detection and prevention of social isolation and reduce the other barriers that affect older adults. The Spearman correlation and the test of the percentage of those who consider the importance of appropriate long-term care (LTC) to be important were calculated using Agresti-Caffo's z+4 statistic. In a structured survey, elderly people in rural areas receiving home care were interviewed. The following hypotheses were proven: (a) psychological support and practical advice are significantly more necessary than medical interventions alone for more than 97% of rural care recipients in home care, and (b) a caregiver who is more frequently in contact with older people in rural areas is more likely to recognise signs of social isolation. It follows that LTC programmes need to include more frequent home visits by nurses, which could help to reduce the costs of outpatient and inpatient care.

Keywords: home nursing care, older adults, social isolation, rural healthcare.

1 INTRODUCTION

Population ageing is one of the key demographic challenges of modern society and is particularly felt in rural areas where health and social services are limited [5]. Effective care for older adults in these areas requires a comprehensive approach that includes medical, psychological and social support. Home nurses play a key role in the delivery of these services. They act as intermediaries between patients, the healthcare system and the community, especially in a rural perspective [4].

Older people in rural areas often face problems such as reduced mobility, limited access to health services and an increased risk of social isolation and depression [1]. According to research, the lack of regular medical care can lead to an exacerbation of chronic diseases and a decrease in quality of life [3], which could increase the outflow of funds from health insurance. In this context, nurses play an important role in care by providing not only health but also psychological support, thus reducing feelings of isolation and increasing older people's sense of security [2].

The aim of this study was to examine the role of home care nurses in improving the quality of life of older adults in rural areas of Croatia, focusing on their need for psychological support and access to health care. In addition to psychological support, access to healthcare is an important aspect of maintaining the health of older adults in rural areas. One of the biggest challenges in this context is the difficulty of transport to healthcare facilities in rural areas. The factors that influence the perception of the importance of psychological support and the relationship between the availability of health services and the frequency of medical interventions were analysed. The statistical analyses performed, the Spearman correlation and the Z+4 test, provided deeper insights into these relationships and conclusions that can serve as a basis for the development of better health policies for older people in rural areas.

2 RESEARCH QUESTIONS AND METHODS

The population in this study includes older people (65+ years) who live in rural areas and use home care services. In particular, the study is conducted among older people living in Bjelovar-Bilogora County (see Figure 1), where health and social services are often difficult to access due to lack of transport, distance to health facilities and limited number of medical staff.

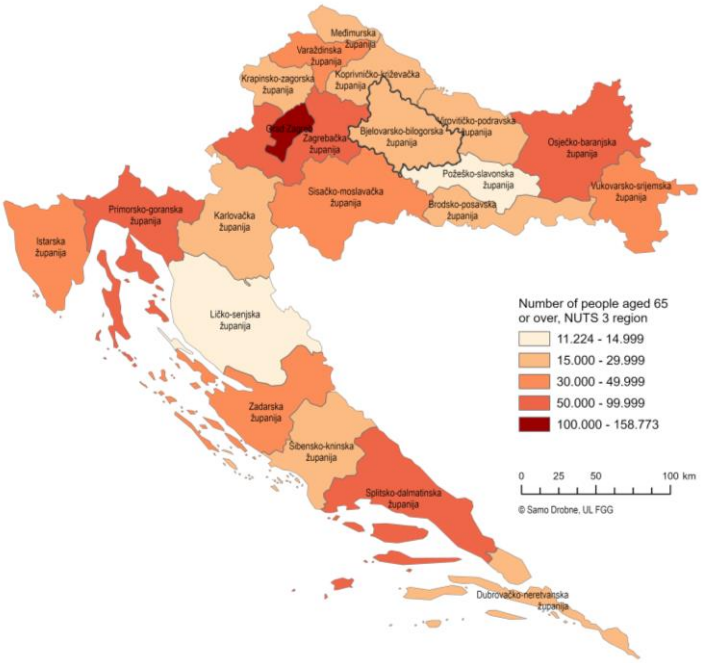


Figure 1: Number of people aged 65 or over, year 2023, Republic of Croatia, NUTS 3 regions
(Source of data: Republic of Croatia, State Statistical Office)

A total of 97 beneficiaries took part. A self-designed questionnaire was used, which was adapted to the specific needs of older people in rural areas. The questionnaire contained closed-ended questions and a 5-point Likert scale that allowed respondents to express their opinion on the frequency of visits by nurses, the need for psychological support and the availability of medical care. Before completing the questionnaire, respondents were informed in detail about the aims of the study, ensuring anonymity of the data, and they agreed to participate in accordance with the principles of ethical research.

We tested the following hypotheses: H1: Older adults in long-term care assume that practical advice from a carer is more necessary than medical interventions in day-to-day care. H2: A nurse who has frequent contact with older people in rural areas is more likely to recognise signs of social isolation. Therefore, the following survey questions were asked: (Q1) To what extent do you think that practical advice from a nurse is more useful than medical interventions in day-to-day care? (from 1- Not useful at all, to 5- Much more useful); (Q2a) How often are you visited by a nurse? (Q2b) To what extent do you think your nurse recognises signs of your social isolation?

To test the statistical significance of the structured responses, the Spearman correlation and the percentage test were calculated using Agresti-Caffo's z+4 statistic.

3 RESULTS

The results of research questions Q1, Q2a and Q2b are shown in Tables 1-3.

Table 1: Number of responses to question Q1
(Source of data: own calculation)

Q1: To what extent do you think that the psychological support and practical advice of a nurse is more necessary than only medical interventions in daily care?	**Number of responses**
1 - Not useful at all	0
2 - A little more useful	1
3 - Moderately more useful	15
4 - More useful	37
5 - Significantly more useful	44

Since the percentage of those who find this more useful or very useful is more than 83% (Table 1), these results confirm H1, using Z+4 test, *p-value* < *0.01*.

Table 2: Number of responses to question Q2a
(Source of data: own calculation)

Q2a: How often does a nurse visit you?	**Number of responses**
1 – Never	17
2 - Rarely (less than once a month)	27
3 - Occasionally (once a month)	21
4 - Often (two to three times a month)	17
5 - Very often (once a week or more)	15

Table 3: Number of responses to question Q2b
(Source of data: own calculation)

Q2b: To what extent do you believe that your nurse recognizes signs of your social isolation?	**Number of responses**
1 - Does not recognize at all	0
2 - Rarely recognizes	2
3 - Occasionally recognizes	19
4 - Often recognizes	38
5 - Always recognizes	38

On the question about frequency of the nurse visits a person in the LTC (Q2a), 97 respondents answered as follows: 1 – Never: n_1 =17; 2 - Rarely (less than once a month): n_2=27; 3 - Occasionally (once a month): n_3=21; Often (two to three times a month): n_4 =17; 5 - Very often (once a week or more): n_5 =15. On the question: "To what extent do you believe that the nurse recognizes the signs of your social isolation" (Q2b), the following frequency of 97 answers are: 1 - Does not recognize at all: n_1 =0; 2 - Rarely recognizes: n_2=2; 3 - Occasionally recognizes: n_3=19; 4 - Often recognizes: n_4 =38 and 5 - Always recognizes: n_5 =38. Table 4 shows the cross-tabulation of these answers.

Table 4: Contingency table: Frequency table of perception of recognition of social isolation and Agresti-Caffo percentages in dependence of frequency of visits

Visits	**Nurse recognizes the signs of social isolation**						
	n_i			p_e		p'	
		1+2	**3+4+5**	**1+2**	**3+4+5**	**1+2**	**3+4+5**
1+2		2	42	0.045	0.955	0.0652	0.9348
3+4+5		0	53	0	1	0.0182	0.9818

Agresti-Caffo's Z+4 test yields the following conclusion: $\Delta p'_e = 4.7\ \%; SE_{\Delta p} = 4.0\ \%; z = 1.175$. The *z-value* obtained is 1.175. Since the *p-value* is greater than 0.12, we can conclude that we cannot find a statistically significant percentage difference between the frequency of nurse visits and the perception of recognising of social isolation using the Z+4 test. Therefore, we also perform a rank correlation test.

Figure 2 illustrates the distribution of data in the contingency table and shows how respondents in different categories of nurse visit frequency rated the nurse's ability to recognise their social isolation. Respondents who receive more frequent visits from a nurse have a higher perception of recognising social isolation, which is why the statistical analysis confirmed this association as significant: the calculation of Spearman's rank correlation coefficient, r_S, resulted in: $\sum d^2 = 408$; $r_S = 1 - 6 * \frac{408}{97^3 - 97} = 0.997, p\text{-}value < 0.01.$

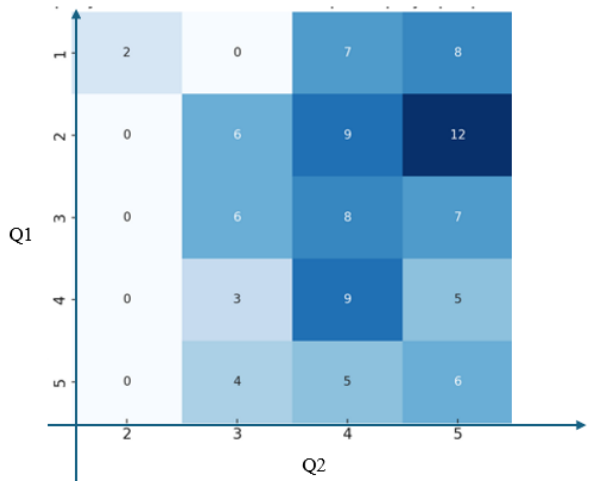


Figure 2: Contingency table of answers on Q1 (How often does a nurse visit you?) and Q2 (To what extent do you believe that your nurse recognizes signs of your social isolation?)
(Source of data: own calculation)

4 CONCLUSIONS

The findings of this study confirm the importance of nurses' work in the home care of older adults in rural areas, particularly in relation to recognising and preventing social isolation, providing psychological support and practical advice, and improving the availability of medical care. Psychological support and practical advice from a nurse were seen as much more necessary in daily life than medical interventions, confirming the importance of the counselling and educational role of nurses in the care of older people.

For the future, we recommend a new, more comprehensive study to deepen the understanding of the role of nurses in the home care of older people and the factors that influence their social inclusion and access to health care.

Acknowledgement
Marija Bogataj and Samo Drobne acknowledge the financial support of the Slovenian Research Agency through the research project A Regional Model of Social Infrastructure networks for the Care of Older Adults in the Community and Institutions, including the Planning of Human Resources (L7-60163). Samo Drobne also acknowledges the financial support of the Slovenian Research Agency in the framework of the research programme Earth Observation and Geoinformatics (P2-0406).

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MAPPING AI-ENABLED INNOVATION RESEARCH IN THE DANUBE REGION: A BIBLIOMETRIC AND TOPIC ANALYSIS USING VOSVIEWER

Mirjana Pejić Bach
University of Zagreb, Faculty of Economics and Business, Department of Informatics
Trg J.F. Kennedya 6, 10000 Zagreb, Croatia
Faculty of Commercial and Business Sciences
Lava 5, 3000 Celje, Slovenia
IEDC-Bled School of Management
Prešernova cesta 33, 4260 Bled, Slovenija
E-mail: mpejic@net.efzg.hr

Mile Bošnjak
University of Zagreb, Faculty of Economics and Business, Department of International Economics
Trg J.F. Kennedya 6, 10000 Zagreb, Croatia
E-mail: mbosnjak@net.efzg.hr

Maja Meško
Faculty of Organizational Sciences, University of Maribor
Kidričeva cesta 55a, Kranj, Slovenia
E-mail: maja.mesko@um.si

Abstract: Innovations enabled by artificial intelligence (AI) are reshaping socio-economic development across Central and South-East Europe, yet the dynamics of scholarly activity in the Danube region remain poorly understood, which is relevant due to the digital divide still present in this region. This study employs a bibliometric and topic analysis to map and evaluate AI-enabled innovations research (2010-2025) in nine countries along the Danube river. A corpus of 404 Web of Science records was analysed with VOSViewer to visualise co-keyword co-occurrence networks and to analyse cross-country collaborations. Cross-border collaboration is most intense among Romania, Hungary, and the Czech Republic.

Keywords: artificial intelligence, innovations, Danube region, bibliometric analysis, VOSViewer

Acknowledgement: This work has been supported by the EU Danube Region Programme Project No. RP0200277 'Boosting smart specialization and encouraging Spin-offs in IT across Danube Region - Spin-it' and bilateral Croatian-Slovenian project 'Modeling the driver of innovation using a machine learning approach (INNO-DRIVE)'.

1 INTRODUCTION

Innovation enabled by artificial intelligence (AI) has become one of the most relevant drivers of modern socio-economic advancement globally [10]. Innovations have been significantly enabled by advancements in AI over the last decade, during which AI has played a dual role [2]. On one side, AI enables innovations that utilise the competencies and processes relevant for AI deployment. On the other side, AI enhances innovations by transforming the capabilities of organisations to innovate.

Gama et al. [2] conducted a literature review on the relationship between AI and innovation. This paper aims to build on their work, enabled by two motivations. First, Gama et al. [2] did not focus on separate regions, although many parts of the world lag in terms of AI utilisation [16]. Second, over the last two years, generative AI has become a mature solution, resulting in tectonic shifts in the way businesses operate [6]. Therefore, we focus on the Danube regions, which lag more developed European countries in terms of digital transformation [4], covering the timeline 2010-2025. Additionally, most bibliometric reviews of AI or digital innovation

adopt a global scope [12] or narrow national/sectoral foci in CEE/Balkan contexts [18], typically mapping keywords only. By integrating keywords co-occurrence and country co-authorship within a single pipeline for the Danube macro-region, we bridge this geographic and methodological gap and benchmark thematic maturity and impact across clusters.

The goal of this research is to investigate how, where, and with whom scholars in the Danube region investigate the intersection of AI and innovation. Three guiding questions were formulated: (i) RQ1. What are bibliometric characteristics of papers in the area of innovation enabled by AI?; (ii) Which thematic clusters dominate this research area?, and (iii) How do collaborative patterns at country level shape knowledge production? A multi-stage research design was employed to address these questions, closely mirroring the systematic, step-by-step logic applied in our earlier work on text mining in finance [15].

The paper continues as follows. In the next chapter, the methodology is outlined in a step-by-step manner. The third chapter contains the literature review analysis, organised around the three research questions. The final chapter contains concluding remarks, a summary, results, and practical implications.

2 METHODOLOGY

To provide answers to research questions, bibliometric analysis and topic mining have been employed. The literature review analysis has been conducted according to the following research steps.

First, data were acquired from Web of Science on July 11, 2025, by searching scientific articles published in journals whose topics were identified using one of the following keywords: artificial intelligence, machine learning, generative artificial intelligence, or innovation. This search resulted in 12,900 scientific papers.

Second, the papers that had at least one author from the Danube countries (Bosnia and Herzegovina, the Czech Republic, Croatia, Hungary, Montenegro, Romania, Serbia, or Slovenia) resulted in 404 scientific papers. The used Boolean string is as follows (Figure 1).

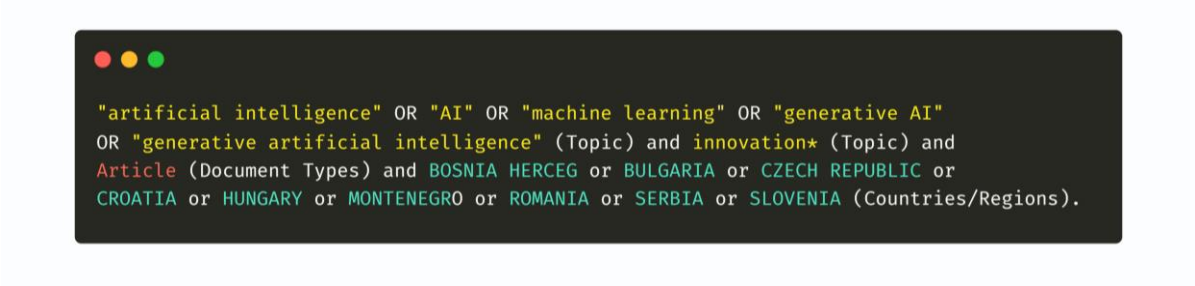


Figure 1: Boolean string for searching Web of Science

Third, a bibliometric analysis has been conducted to identify trends in publications related to the scientific area, the most frequently published journals, organisations, and funding agencies.

Fourth, VOSViewer was used to extract keywords, with a threshold of 5 papers per keyword, which resulted in 89 identified keywords. The thesaurus was developed to remove duplicate keywords, such as digitisation and digitalisation, resulting in 82 unique keywords. Sensitivity analyses were performed by incrementally tightening and loosening threshold values (± 2 occurrences or citations). The stability of the main clusters under these perturbations confirmed the robustness of the mapping.

The VOSViewer mapping identifies the optimal location for a specific term on the map by minimising the weighted sum of squared Euclidean distances across all element pairings. This mapping technique enables the positioning of words on a map so that the distance between each pair graphically represents their similarity. The greater the number of publications in

which two terms co-occur, the stronger the correlation between the terms. Terms that commonly appear in the same publications are clustered in a term map, whereas loosely related terms (with infrequent co-occurrence) are positioned further away.

A similar approach is used to identify cooperation among countries [17]. Although VOSviewer applies the same layout and clustering algorithm as it does for keyword maps, in a country network, the links represent co-authored publications, so clusters are interpreted as collaborative communities rather than thematic groups. Keywords co-occurrence networks were built using full counting and the default association strength normalisation, so each keyword occurrence contributed equally to link weights. Country co-authorship links were fractionally counted and normalised by the total publications of each country to avoid inflating ties from large multi-country papers.

3 RESULTS

3.1 The bibliometric characteristics of papers in innovation enabled by AI (RQ1)

The bibliometric survey reveals that articles are most frequently classified under Management (13.1%), Business (10.9%), and Electrical Electronic Engineering (9.4%), yet they span a broad spectrum of technical, social science, and natural science domains, indicating the interdisciplinarity of innovation and AI research in the Danube region.

Over 60% of all records have been published in the last two years alone (2024: 32.7%; 2025: 27.5%), indicating that authors have focused intensely on this research topic in recent periods, revealing a sudden increase in papers about AI-enabled innovations across the Danube region.

The papers are published the most often in Sustainability journal (4.5 %), Applied Sciences (3.0 %) and Strategies for Policy in Science and Education (2.7 %), followed by open-access technology venues such as IEEE Access and Electronics, with the remainder dispersed across a diverse long-tail of specialised journals, which indicates the field's preference for multidisciplinary, open platforms and its broad thematic reach.

The Bucharest University of Economic Studies contributes the largest share of publications (8.7%), followed by the University of Ljubljana (5.0%), the University of Zagreb (4.7%), Corvinus University Budapest (3.8%), and the University of Belgrade (3.6%). Other universities published less than 3.5% of publications, indicating a research effort that is concentrated in the most prominent universities.

European Union support emerges as the single largest funder, with more than 50 projects funded (12.4% of the papers), exceeding any individual national agency, which demonstrates the EU's central role in supporting AI-innovation research across the Danube region.

3.2 Thematic clusters in innovation enabled by AI (RQ2)

The eight keyword clusters were extracted using VOSviewer's modularity algorithm, which was formed using 82 keywords (Table 1):

- Cluster 1 (Machine learning and infrastructure) focuses on algorithms, data sources, such as big data, and industrial applications.
- Cluster 2 (Business Model and management) is focused on the organisational frameworks needed to adopt artificial intelligence.
- Cluster 3 (Politics, digitalisation and challenges) represents the management and regulatory aspects of rapid artificial intelligence growth, such as risks and infrastructure.

- Cluster 4 (Optimisation and prediction) contains the papers with a narrow methodological focus, such as optimisation and prediction models, e.g., neural networks.
- Cluster 5 (Society and digital transformation) investigates the adoption of generative tools among users about education and ethics.
- Cluster 6 (Impact and energy transition) links artificial intelligence research to measurable outcomes in areas such as renewable energy deployment, energy efficiency, consumption patterns, and health effects.
- Cluster 7 (Sustainability and competitiveness) examines how artificial intelligence enhances organisational efficiency, inter-firm collaboration, and industrial competitiveness, positioning artificial intelligence as a key driver of sustainable economic performance.
- Cluster 8 (Innovation and knowledge) examines artificial intelligence within national and regional innovation systems.

Among the eight clusters, citation impact is highest in Cluster 7 (Sustainability and competitiveness) with an average of 20.1 citations per paper, followed by the two methods-oriented clusters: Cluster 4 (Optimisation and prediction, with an average of 14.6 citations per paper) and Cluster 1 (Machine learning and infrastructure, with an average of 14.4 citations per paper). In contrast, the newest topic area, Cluster 5 (Society and digital transformation), has the lowest citation rate (an average of 6.4 citations per paper), which may be related to the fact that the papers in this cluster have the most recent average publication period.
Publication timing is tightly grouped around 2023, with Clusters 5 and 6 being the most recent (2023.8 and 2023.7), while Clusters 7 and 8 are the earliest (2022.7 and 2022.5).

Table 1: Keywords clusters in AI-innovation literature of the Danube region, with mean publication year and citation impact (VOSviewer analysis)

Cluster	Representative keywords*	Avg. publication year	Avg. citations / item	Example publication
1	machine learning, big data, industry 4.0, automation, information technology, deep learning	2023.1	14.4	[7]
2	management, model, future, business, framework, strategy	2023.3	9.7	[11]
3	technology, challenges, blockchain, internet, digitalisation, governance	2023.5	11.9	[12]
4	performance, neural network, prediction, quality, optimisation, perceptions	2022.9	14.6	[9]
5	artificial intelligence, digital transformation, education, adoption, ChatGPT, behaviour	2023.8	6.4	[1]
6	impact, consumption, renewable energy, technological innovation, energy efficiency, health	2023.7	9.8	[14]
7	sustainability, collaboration, efficiency, industry 4, competitiveness, digitalisation	2022.7	20.1	[8]
8	innovation, systems, growth, knowledge, policy, research and development	2022.5	10.7	[3]

Source: Authors' work using VOSViewer

The co-occurrence map of keywords presents how the Danube region frames and connects its research about innovations enabled by AI (Figure 2). Node size reflects the frequency of a keyword; link thickness reflects co-occurrence strength. Colours indicate clusters detected by VOSviewer's modularity algorithm (thematically related terms).

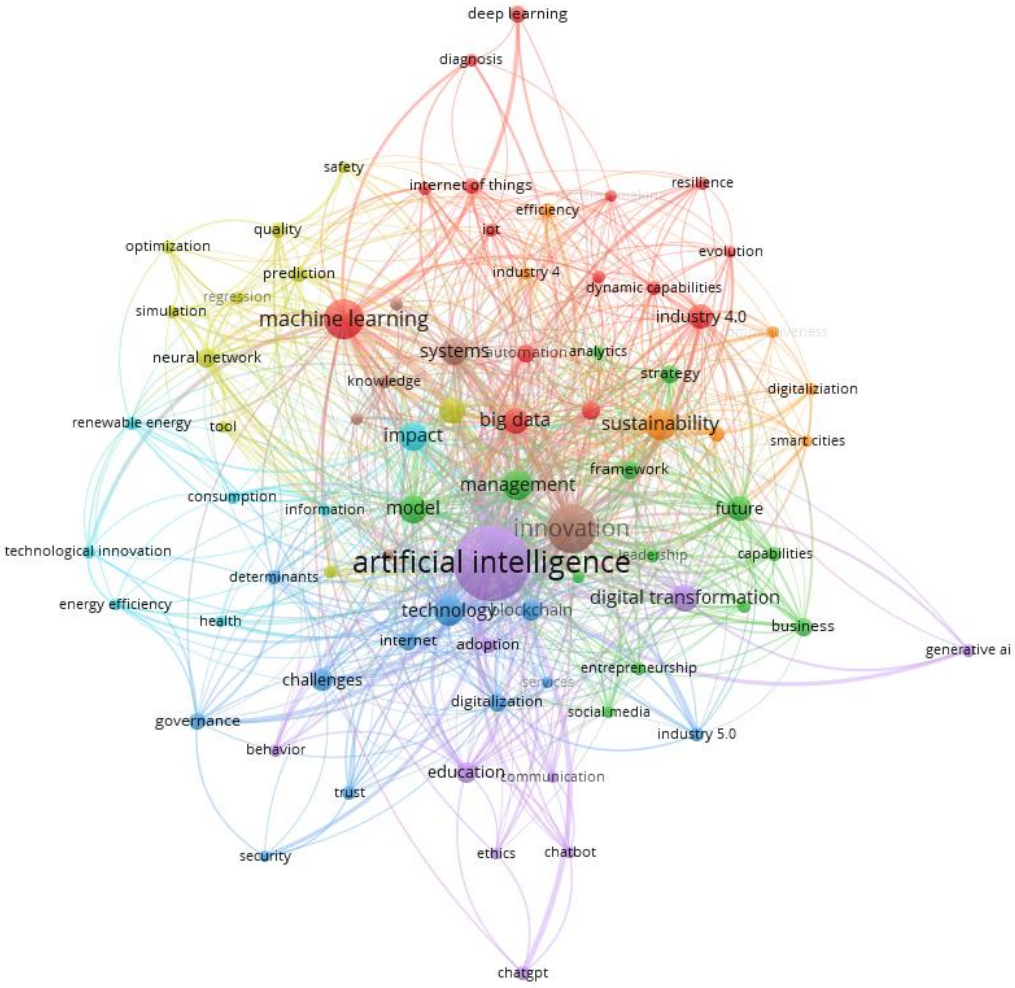


Figure 2: Keywords co-occurrence network of AI-innovation literature in the Danube region (VOSviewer).

3.3 Collaborative patterns in innovation enabled by AI (RQ3)

Using VSOViewer, we modelled the co-authorship landscape using the nations indicated in the authors' affiliations (Figure 3). Node size denotes each country's publication count; link thickness shows the intensity of co-authored papers (fractional counting). Colours mark collaboration communities (clusters).

Nearly two-fifths of all joint outputs are produced by a dominant Central-Eastern hub, which is concentrated in Romania, Hungary, and the Czech Republic. Romania is focusing its cooperation on partners outside of Europe, including China, Iran, and Vietnam. High internal density and weak exterior links characterise a South-Eastern nexus led by Serbia. The British-Irish pair and Poland serve as the anchors of an Atlantic-Nordic cluster that is outward-looking but peripheral to the core. Southern Europe and Russia are connected by a Mediterranean-post-Soviet bridge, based in Italy; however, the least cohesive group is a diffuse periphery group centred on Bulgaria and Ukraine.

Nearly half of all the edges in the Danube macroregion connect pairs of riparian states, with the most intense dyads being Romania-Hungary, Serbia-Croatia, and Romania-Serbia. This makes the river basin a vital conduit for scientific interchange. Upstream partners, such as Slovenia and Bosnia-Herzegovina, however, are still restricted to a single hub, which emphasises the necessity of multi-country bridging projects that would diversify their joint portfolios and lessen their reliance on powerful neighbours.

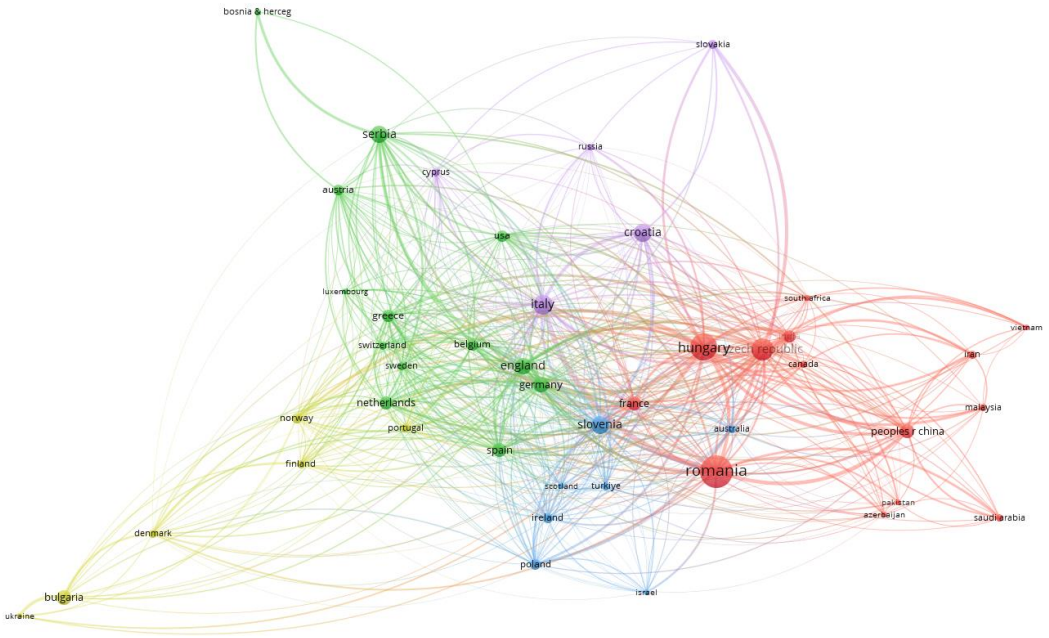


Figure 3: Country co-authorship network in AI-innovation research (VOSviewer).

4 CONCLUSIONS

The study analysed the papers indexed in Web of Science (2010–2025), using VOSviewer to map AI-enabled innovation research in the Danube region. Implications of this research are three-fold. First, the increase in AI-enabled innovation research indicates that the Danube region has transitioned from being a passive follower to an actively engaged participant in the global AI transformation. The rapid growth of publications since 2024 indicates that regional academics, organisations, and financiers are rallying around AI as a catalyst for competitiveness and development. The Management/Business area of research, in conjunction with Engineering, indicates that AI is perceived not merely as a technical instrument but as a strategic, organisational, and policy concern, which is particularly relevant for underperforming regions in terms of digital transformation, such as the Danube region [5]. Second, the topic map is significant as it indicates the current research focus and citation impact: sustainability- and performance-oriented applications gather the highest citation count, while societal and transformational enquiries are still in development. This disparity indicates both a strength (distinct economic significance) and a deficiency (insufficient evidence regarding ethics, behaviour, and governance). The policymakers and universities can use it as a guideline for future initiatives and curriculum development. Third, the cooperation among Danube region researchers is still limited to three countries, while others remain marginal or dependent on individual partners. Therefore, future funding streams, such as Interreg, Erasmus, and Horizon, could be utilised to fill this gap by fostering strong multi-country collaborations in research, as well as entrepreneurship and start-up initiatives.

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MULTICRITERIA SPATIAL ANALYSIS FOR THE LOCATION-ALLOCATION OF FOREST BIOMASS TRANSFER STATIONS

Sandra Silva
proMetheus, Instituto Politécnico de Viana do Castelo
Rua Escola Industrial e Comercial Nun'Álvares, 4900-347 Viana do Castelo, Portugal
E-mail: sandrasilva@esa.ipvc.pt

Leonel Nunes
proMetheus, Instituto Politécnico de Viana do Castelo
Rua Escola Industrial e Comercial Nun'Álvares, 4900-347 Viana do Castelo, Portugal
E-mail: leonelnunes@esa.ipvc.pt

Helder Amorim
Agrarian School of Polytechnic Institute of Viana do Castelo
Rua D. Mendo Afonso, 147 Refóios do Lima, 4990-706 Ponte de Lima, Portugal
E-mail: h.amorim@ipvc.pt

Renato Silva
Agrarian School of Polytechnic Institute of Viana do Castelo
Rua D. Mendo Afonso, 147 Refóios do Lima, 4990-706 Ponte de Lima, Portugal
E-mail: renato.silva@esa.ipvc.pt

Joaquim Alonso
proMetheus, Instituto Politécnico de Viana do Castelo
Rua Escola Industrial e Comercial Nun'Álvares, 4900-347 Viana do Castelo, Portugal
E-mail: malonso@esa.ipvc.pt

Abstract: Forest biomass transfer stations serve as crucial logistical hubs within the forest-based supply chain, facilitating the temporary storage and transportation of biomass while enhancing operational efficiency, reducing costs, and delivering environmental benefits. The selection of optimal sites for these facilities involves a complex, multi-criteria decision-making process that must consider proximity to biomass sources, accessibility via transport infrastructure, environmental and social impacts, and economic viability.

This study integrates Geographic Information Systems (GIS), Spatial Multi-Criteria Decision Analysis (MCDA) and Multiobjective Decision Making (MODM) to identify and evaluate suitable locations forest biomass transfer stations in the Alto Minho region of northwestern Portugal. Chosen for its high forest biomass potential and renewable energy development opportunities, the region is analyzed through a spatially explicit framework incorporating the Analytic Hierarchy Process (AHP) and various GIS-based spatial analysis techniques.

Results provide strategic insights into optimal forest biomass transfer stations siting, addressing both location and allocation problems to enhance supply network efficiency and sustainability. This framework contributes to regional bioenergy planning and supports sustainable forest management by promoting integrated, evidence-based decision-making.

Keywords: Multi-Criteria Decision Analysis; Location-Allocation; GIS; Forest Biomass; Renewable Energy; Spatial Planning.

1 INTRODUCTION

Forest biomass or transfer stations are essential infrastructures designed to temporarily store and support the efficient transportation of forest-based materials such as timber, pulpwood, and other residues. These facilities serve as critical links between forest production areas - both natural and managed forests - and processing plants like sawmills or bioenergy units, improving operational logistics and reducing both economic and environmental costs [1], [2]. The displacement and commercialisation of forest biomass in local markets, in addition to

being of a small scale, is generally controlled by a large number of intermediaries that do not coordinate, leading to a lower efficiency of resource use. The implementation of biomass logistic centres is a simple and innovative method to develop and organise local markets for the use of forest biomass. [3]

Selecting suitable locations for such stations involves multidimensional spatial problems, often characterized by conflicting objectives such as minimizing transportation costs while maximizing accessibility and minimizing environmental impact. In this context, integrating Geographic Information Systems (GIS) tools with Multicriteria Decision Analysis (MCDA) and Multiobjective Decision Making (MODM) has emerged as a powerful approach in spatial planning for logistics systems [4], [5].

Spatial decision-making frameworks allow stakeholders to evaluate and prioritize location alternatives by incorporating economic, environmental, and social criteria [6]. This is particularly relevant in forest-rich regions such as Alto Minho, where biomass resources are abundant, but logistical challenges remain due to topographical and infrastructural constraints.

Furthermore, a location-allocation model can be used to determine not only optimal locations for transfer stations but also to allocate biomass sources and processing facilities efficiently [1]. Such a model supports the development of resilient and sustainable biomass supply chains.

2 METHODOLOGY

The methodological framework is structured in three main phases:

- Biomass Potential Estimation: Quantification of available residual agroforestry using spatial data on forest cover and productivity.
- Suitability Analysis: Application of a spatial MCDA considering the following steps:
    - Criteria Definition: Relevant criteria for the location of intermediate forest terminals were identified and defined. The considered criteria included economic, environmental, and social aspects.
    - Data Collection: Geographic data and relevant information were collected for the study area. This included information on the distribution of forest production areas, the existing road network, terrain information, and other data relevant to spatial analysis.
    - Spatial Analysis: The GIS-based spatial analysis incorporated multiple layers, including land use, transportation networks, urban and industrial areas, terrain features, environmental hazards, and planning regulations. Each factor was standardized and weighted to produce composite suitability maps.
    - Multicriteria Analysis: The defined criteria were weighted and evaluated using multicriteria analysis techniques such as the Analytic Hierarchy Process (AHP).
    - Modeling and Evaluation of Alternatives: Based on the data, criteria, and weights defined, decision models allow the assessment and classification of location alternatives according to the established criteria.
    - Sensitivity Analysis: To analyze and evaluate the influence of varying criteria weights and data used in location decisions.
- Location-Allocation Optimization: Identification of optimal facility locations based on the spatial correlation between biomass availability and suitability zones, incorporating objectives such as minimizing transport costs, reducing environmental impacts, and maximizing access to forest biomass, using network analyst tool (location-allocation).

This methodology was applied to the Alto Minho region, selected due to its abundant forest resources and strategic relevance for decentralized bioenergy infrastructure. The model aimed to determine not only where logistics stations should be located, but also how biomass sources should be allocated among them for maximum efficiency.

3 RESULTS AND DISCUSSION

The results highlight several high-suitability zones for the installation of forest biomass logistics stations. These zones demonstrate a favourable balance between biomass resource availability, transport accessibility, low environmental risk, and socio-economic integration. Additionally, an accessibility analysis was conducted and correlated with the suitability map, highlighting three out of the ten counties under study as containing locations with the highest suitability.

The application of the location-allocation model enabled the identification of an optimal configuration of logistics stations that minimizes transport distances while maximizing biomass collection efficiency, supported by scenario analysis: implementing two, three, five, seven or ten forest biomass logistics stations. According to the various scenarios, it is possible to identify that the optimal location for the installation of transfer stations is near the main access points to the road networks. Furthermore, it was observed that an increase in the number of transfer stations resulted in a reduction of the average distance traveled, thereby minimizing transportation costs and optimizing the efficiency of the system. It was also noted that a greater number of stations allowed for a more equitable distribution of biomass, ensuring a more stable and consistent supply to each facility.

The analysis suggests a decentralized network of facilities that supports the region's bioeconomy and contributes to sustainable forest management.

4 CONCLUSIONS

This study proposes an integrated spatial decision support framework for the optimal siting and allocation of forest biomass logistics stations. By combining Geographic Information Systems (GIS), Multi-Criteria Decision Analysis (MCDA), and Multi-Objective Decision-Making (MODM) techniques, the research offers a methodology to improve the efficiency of biomass logistics, facilitate regional energy transitions, and support environmentally and socially sustainable land-use planning.

Applied to the Alto Minho region of Portugal, the framework identifies strategic locations for logistics infrastructure that balance technical feasibility with spatial, environmental, and socio-economic considerations. Future research may expand this model by incorporating dynamic biomass supply scenarios and comprehensive cost-benefit analyses, thereby strengthening its applicability in decision-making under conditions of uncertainty.

Acknowledgement
This work was developed within the scope of the project proMetheus – Research Unit on Materials, Energy and Environment for Sustainability, FCT Ref. UID/05975/2020, financed by national funds through the FCT/MCTES.

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SOCIO-ECONOMIC DETERMINANTS OF LIFE SATISFACTION IN EUROPEAN COUNTRIES

Blanka Škrabić Perić
University of Split, Faculty of Economics, Business and Tourism
Department of Quantitative Methods
Cvite Fiskovića 5, Croatia
E-mail: bskrabic@efst.hr

Abstract: This article analyses the socio-economic determinants of life satisfaction in European countries. Since the outbreak of COVID 19, life satisfaction scores have declined. Most of existing study assume income as the most important determinant of life satisfaction. Some studies point importance of life expectancy and tertiary education. Our results provide somewhat different findings. Income is no longer an important factor. It is changed with unemployment. From education variables tertiary education no longer dominates over secondary education. Life expectancy is not an important determinant, whereas healthy life expectancy and perceived health status are. Finally, marital status plays significant role.

Keywords: life satisfaction, education, unemployment, health

1 INTRODUCTION

Before COVID-19, income was assumed as the main determinant of life satisfaction. However, the empirical papers did not provide clear results. [6] summarised the results of studies that investigated the relationship between life satisfaction and income and divided the results into two groups. The first group indicates that economic growth has no statistically significant influence on well-being, while the other group of studies confirms a positive and statistically significant influence of economic growth. Based on these contradictory results, they assume that the relationship between life satisfaction and GDP growth depends on social trust and inequality. They found that GDP growth has a direct influence on life satisfaction in transition countries, while the relationship is not statistically significant in developed countries. However, they found that social trust has a moderate effect on the relationship between GDP growth and life satisfaction for all countries, while reducing income inequality has a moderate effect only for developed countries. This is consistent with [5] who suggests that for inclusion in OECD countries, inequalities need to be reduced. It explains awareness of the importance of equality in developed countries.

A similar conclusion is drawn by [2]. In industrialised countries, they confirmed that the positive relationship between GDP per capita and life satisfaction is moderated by social support and freedom of choice in countries, while in developing countries the relationship is mainly deepened by social support. They also found negative effects of air pollution for developed countries.

[4] investigated the influence of education on the well-being of Europeans. It confirms a significant influence of income, marriage and physical health as significant determinants of life satisfaction. They found that tertiary education is significant in all models, regardless of which indicators of well-being are used. Somewhat smaller effect is found for vocational education, while secondary education has no significant influence. He also finds that unemployment is one of the most important determinants of life satisfaction.

On the other hand, health and work were confirmed as important factors for life satisfaction in [1]. However, before the crisis, work situation and health risk were the most important predictors of life satisfaction, while after the outbreak of COVID-19, job insecurity and lack of family support are the main reasons for the decline in life satisfaction.

The literature review shows that there is no consensus on the main indicators of life satisfaction. This paper attempts to develop a model of life satisfaction for European countries that takes into account the results of previous research. It follows the model proposed in [4] and extends it to include the health status variable, which is recognised as an important determinant in [1,7], as well as the control of corruption and income inequality variables from [2].

Therefore, this paper contributes to the existing literature on life satisfaction in several ways. First, it extends the life satisfaction model from [4] with the new variables: health, control of corruption and GINI index. Second, it covers a new time period that includes the period before and after the outbreak of COVID-19. Furthermore, our time period starts with the year 2013, while [4] finished with 2012, therefore it can be seen as a continuation of their research. In addition to the COVID-19 outbreak, other changes have occurred. For example, the proportion of the labour force with tertiary education has increased by almost 10 percentage points in the last decade, and it is possible that tertiary education no longer plays the same role as before.

In [8], tertiary education was found to have a negative effect on life expectancy, so it would be interesting to re-examine the role of tertiary education on life satisfaction in recent times. Therefore, this research will provide new evidence on the importance of tertiary education on life satisfaction.

The rest of the paper is organised as follows. The Data and methodology section describes the data and the methodology used. It is followed by the Empirical results and discussion section. The Conclusion summarises the main findings and provides policy recommendations and directions for future research.

2 DATA AND METHODOLOGY

The dataset consists of life satisfaction data for the years 2013, 2018, 2021, 2022 and 2023 for for 37 European countries. Data are obtined from the Eurostat database. All other variables are collected for the corresponding years. Most of independent variables are obtined also from Eurostat. Only Health percived status is taken from the OECD database.

Income is measured by median housholed equivalised net income in EURO. Secondary education is measured by the percentage of the population with upper secondary and post-secondary non-tertiary education (levels 3 and 4) between the ages of 15 and 64, tertiary education is measured by the percentage of the population with tertiary education (levels 5-8) between the ages of 15 and 64. Unemployment is measured by percentage of unumployment population in the labour force. The health status of the population is measured using three indicators. Life expectancy at birth is defined as the average number of years a newborn child can expect to live. Healthy life expectancy is the average number of years a person is expected to live in a healthy state (i.e. without any activity limitations). Perceived health status is the percentage of the population who consider their health status to be good or very good.

For the institutional variables, we used the Corruption perceptions index, where a score of 0 indicates very high corruption and a score of 100 indicates a very clean country.As a higher value means a lower level of corruption, the variable was renamed control of corruption for easier interpretation.

The GINI index is a measure of inequality in income distribution. Finally, two indicators are also taken into account for marriage status. Firstly, the marriage rate, which indicates the number of marriages in a year in relation to the average population in that year, expressed per 1000 inhabitants, and the divorce rate, which indicates the number of divorces in a year in relation to the average population in that year, expressed per 1000 inhabitants.

Due to the fact that the dependent variable is not available for every year and there are many missing values for other independent variables, static panel data models are used. According

missing values in the dependent variable for some years, the use of a dynamic panel data estimator leads to an additional loss of observations. Therefore, use of dynamic panel data for such a sample would lead to additional bias.

According [9], three diagnostic tests are performed to find a suitable estimator for our data. F-test is performed to determine whether the fixed-effects model (FE) is more appropriate for the estimation than the pooled model (PM). The second test is the LM test. It compares the pooled model with the random effects model (RE). If the F-test and the LM-test reject the null hypothesis, the Hausman test is performed. If the null hypothesis is rejected, the fixed-effects model is suitable for the estimation. If the null hypothesis is not rejected, the random-effects model provides more efficient results. Results of all three tests are shown in Table 1 and Table 2.

3 EMPIRICAL RESULTS AND DISCUSSION

Our empirical model is based on [4] and includes income and two variables from secondary and tertiary education. In addition, it is extended by the variable health because recent studies indicate it as one of the most important determinants of life satisfaction [7]. Our basic model is

$$Life\ satisfaction_{it} = \mu + \beta_1 Economic_{it} + \beta_2 Secondary_{it} + \beta_3 Tertiary_{it} + \quad (1)$$
$$+\beta_4 Health_{it} + \alpha_i + \varepsilon_{it}, i = 1, \dots, N; t = 1, \dots, T$$

where $Life\ satisfaction_{it}$ is the dependent variable, two indicators are used for $Economic_{it}$ variable $Income_{it}$ and $Unemployment_{it}$. $Secondary_{it}$ is percentage of people with secondary education in country i in period t, while $Tertiary_{it}$ is percentage of people with tertiary education in country i in period t, and $Health_{it}$ is the value of one of the three indicators $Life\ expectancy_{it}$, $Healthy\ life\ expectancy_{it}$, $Percived\ health\ status_{it}$. $\beta_1, \beta_2, \beta_3, \beta_4$ are coefficients to be estimated. μ is constant term, $\alpha_i$is fixed or random effect and ε_{it} is error term. The results of model specifications of model (1) are shown in Table 1.

Table 1: Results of the panel data analysis of basic model

	(1)	*(2)*	*(3)*	*(4)*	*(5)*
***Income**it*	0.00000190	-0.000000541	0.00000605		
	(0.0000124)	(0.0000139)	(0.00000990)		
***Unemployment**it*				-0.0465***	-0.0334***
				(0.0107)	(0.00949)
***Secondary education**it*	0.0283***	0.0350***	0.0258**	0.0205*	0.00475
	(0.0106)	(0.0115)	(0.00991)	(0.0115)	(0.00843)
***Tertiary education**it*	0.0319***	0.0313***	0.0184**	-0.00288	0.00418
	(0.0102)	(0.0113)	(0.00925)	(0.0105)	(0.00917)
Life expectancyt.	0.0154				
	(0.0280)				
Healthy life expectancy it		0.0361***		0.0368***	
		(0.0119)		(0.0115)	
Perceived health status it			0.0432***		0.0322***
			(0.00721)		(0.00607)
cons	3.573	2.392**	2.430***	4.333***	4.947***
	(2.318)	(1.048)	(0.691)	(1.127)	(0.814)
N	159	114	138	103	121
F test	0.000	0.000	0.000	0.000	0.000
LM test	0.000	0.000	0.000	0.000	0.000
Hausman test	0.0224	0.0023	0.0122	0.0037	0.1855
Model	FE	FE	FE	FE	RE

Notes: values in parentheses are standard errors, *, **, *** notes statistical significance at 10%,5% and 1%
Source: Authors' calculations

In the second step, the model is expanded with other variables that have been identified as important drivers of life satisfaction in previous studies. $Marriage_{it}$ and $Divorce_{it}$ are used for marital status. According results of [2] model is extended with $Control\ of\ corruption_{it}$ and with $GINI\ index_{it}$ to measure income inequality. Results of estimated model specifications are shown in Table 2.

Table 2: Results of the panel data analysis of the extended model specifications

	(6)	***(7)***	***(8)***	***(9)***
Income $_{it}$	-0.00000894	-0.0000122	0.0000234***	0.0000163**
	(0.0000140)	(0.0000260)	(0.00000706)	(0.00000804)
Secondary education$_{it}$	0.0401***	0.0264**	0.00805	0.0184***
	(0.0127)	(0.0128)	(0.00693)	(0.00602)
Tertiary education$_{it}$	0.0397***	0.0338*	0.00544	0.0191***
	(0.0128)	(0.0182)	(0.00865)	(0.00736)
Healthy life expectancy$_{it}$	0.0361***	0.0361***		
	(0.00766)	(0.00956)		
Marriage$_{i\ t}$	0.162***			
	(0.0577)			
Divorce$_{it}$		-0.00754*		
		(0.00429)		
GINI index$_{it}$			-0.0542***	
			(0.0144)	
Control of corruption$_{it}$				0.0202***
				(0.00507)
cons	1.284*	3.248***	7.814***	4.111***
	(0.752)	(1.029)	(0.766)	(0.489)
N	106	97	123	160
F test	0.000	0.000	0.000	0.000
LM test	0.000	0.000	0.000	0.000
Hausman test	0.000	0.006	0.0573	0.0685
Model	FE	FE	RE	RE

Notes: values in parentheses are standard errors, *, **, *** notes statistical significance at 10%,5% and 1%
Source: Authors' calculations

Table 1 and Table 2 show that model specifications (1)-(7) are estimated with the fixed-effects estimator (FE), while models (8) and (9) are estimated with the random-effects estimator (RE). It can also be seen that unemployment and the two health indicators, healthy life expectancy and perceived health status, are the most important factors influencing life satisfaction. They are followed by the two education indicators secondary and tertiary education. Income and life expectancy have no statistically significant influence on life satisfaction. Control of corruption and the GINI index have a significant but weak influence on life satisfaction. They become statistically significant when the health indicators are excluded from the model.

These results offer a different perspective on the drivers of life satisfaction. In previous studies, income was very often emphasised as the main determinant of life satisfaction [2]. On the other hand, this result is in line with [6], who found that income is not correlated with life satisfaction in developed countries. The reason for this could be that people are relatively satisfied with their income and that additional growth has no statistically significant influence on satisfaction growth. It is interesting to note that unemployment has almost the strongest influence on life satisfaction. This means that insecurity has the strongest influence on the decline in life satisfaction. A similar conclusion is also reached by [1], who found that job insecurity is the most important predictor of life satisfaction in COVID 2019.

Furthermore, both education indicators are not statistically significant when unemployment is included in the model, similar to [4]. It is also interesting to note that this study confirms

secondary and tertiary education as important determinants of life satisfaction. These results are not consistent with those of [4], as they confirm a significant effect for tertiary education, while secondary education is not significant. Our results show that secondary education is as important as tertiary education. This result is in line with [3], who found that secondary education maximally reduces the risk of depression. In our model, secondary education is significant in more model specifications than tertiary, confirming that its influence is more robust.

Previous research has established a relationship between life expectancy and life satisfaction. Our results show that life expectancy is no longer a significant determinant. This may be due to increasing longevity and the decline in life satisfaction among older adults [7]. However, healthy life expectancy and perceived health status have a strong positive influence on life satisfaction. These two indicators confirm that health is an important factor for life satisfaction.

Marital status, as measured by marriages and divorces, also has a significant influence on life satisfaction. The control of corruption and GINI index, only become statistically significant, if we exclude the health indicator from the model. It can therefore be concluded that although they have a significant influence, this is less than the influence of unemployment, health and education.

4 CONCLUSIONS

This paper analyses the determinants of life expectancy for European countries in five waves from 2013 to 2023. The results cover the periods before, during and after COVID-19 and provide new insights into the main determinants of life satisfaction in European countries. The results show that unemployment, physical health and perceived health are the main determinants of life satisfaction. This is followed by secondary and tertiary education, which means that tertiary education is not a dominant predictor of life satisfaction. Furthermore, income is not a statistically significant determinant of life satisfaction in European countries.

This research confirms that marriages are a significant determinant with positive effects, while divorce are also statistically significant and have negative effects. Finally, the institutional variable control of corruption and GINI index also have statistically significant effects, but their effects are weaker than those of employment, health and education.

The results of this study provide policy implications for policy makers. They need to reduce labour market insecurity and increase the quality and availability of health services in order to increase the life satisfaction of the population. However, a new question arises. Tertiary education does not lead to more life satisfaction than secondary education. More attention needs to be paid to this finding and more research is needed on the relationship between tertiary education and different indicators of well-being.

However, this study has some weaknesses. The main weakness is the unavailability of data on life satisfaction on an annual basis. Given the lack of data, only a static panel data analysis was conducted. For future research, it will be necessary to use other indicators of well-being whose data are collected annually. This data will allow more sophisticated modelling and applying most advanced panel data estimators. By using higher quality data and advanced econometrics, the results are more meaningful and provide a clearer and stronger directions for policy makers.

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The 18th International Symposium on
Operational Research in Slovenia

SOR '25

Bled, SLOVENIA
September 24 - 26, 2025

Session 3:

Finance and Investments

FISCAL POLICY RESPONSES TO INFLATIONARY SHOCKS IN CROATIA: AN ANALYSIS OF THE POST-COVID PERIOD

Frane Banić
Croatian National Bank, Economic Analysis Department
Trg hrvatskih velikana 3, Zagreb, Croatia
E-mail: frane.banic@hnb.hr

Irena Palić
University of Zagreb, Faculty of Economics and Business Zagreb
Trg J.F. Kennedya 6, Zagreb, Croatia
E-mail: ipalic@efzg.hr

Abstract: This paper examines the fiscal response to the inflationary shock in Croatia following the Covid-19 crisis. Using a disaggregated approach, we have estimated the inflationary impact on both revenue (indirect and direct taxes) and expenditure (final consumption expenditure and gross fixed capital formation). The findings indicate a positive inflationary effect on revenue side, while the impact on expenditure is not significant or negative and significant. These results highlight the automatic adjustment of revenues to inflation, particularly in a consumption-driven tax system, but also raising potential concerns for bracket creep effect in terms of positive personal income tax reaction. Expenditure adjustments is expected to occur with a time lag.

Keywords: Croatia, disaggregated approach, fiscal policy, inflationary shock.

1 INTRODUCTION

In recent years, inflationary shocks have created major challenges for economies worldwide, including Croatia. Policymakers have introduced various fiscal measures to reduce the negative impact on households, businesses, and overall economic stability. Policymakers in advanced economies have not encountered high inflation rates since the early 1980s [19].

The impact of inflation on public finances is twofold and influenced by several factors. Typically, higher inflation is expected to benefit public finances in the short term, as it increases government revenue, while public spending tends to rise more slowly. However, the fiscal impact of an inflationary shock is influenced by the nature and magnitude of the inflation shock, the fiscal response to the shock, institutional features of the government budget, such as the indexation of public expenditures and tax system characteristics [4].

Since the implementation of the Stabilization Program in October 1993, the Croatian National Bank has achieved its primary objective of price stability [8]. However, following the unexpected rise in public spending during the Covid pandemic in 2020-21 [22], Croatia and other EU countries were hit in 2022-23 by a sharp increase in energy prices, which was related to Russian invasion of Ukraine [1]. Various income and price measures were implemented at the EU level to mitigate the harmful effects of higher energy prices on business operations and living standards [12]. The fiscal response to inflationary shocks has gained in importance in the context of recent global economic events, as governments had to adjust their fiscal policies in response to inflationary shocks, in order to ensure the sustainability of public finances [4].

Figure 1 shows debt dynamics in Croatia from 2018 to 2023. Debt dynamics in Croatia in 2022-23 is largely driven by inflationary shock, as well as by real GDP growth due to strong domestic and external (export of services in tourism and EU funds absorption) factors. Positive primary balance also contributed to decline in debt-to-GDP ratio, while implicit interest rate affected debt position in the opposite way.

Debt dynamics equation is given by (1):

$$\Delta d_t = \frac{i_t - (\pi_t + g_t)}{(1+\pi_t)(1+g_t)} \; d_{t-1} - pb_t + dda_t \qquad (1)$$

where: d_t *– debt in period t,* i_t*- implicit interest rate on lagged government debt (t-1) in period t,* π_t *- GDP deflator growth in period t,* g_t *- real GDP growth in period t,* pb_t *- primary balance in period t,* dda_t*- deficit debt adjustment in period t.*

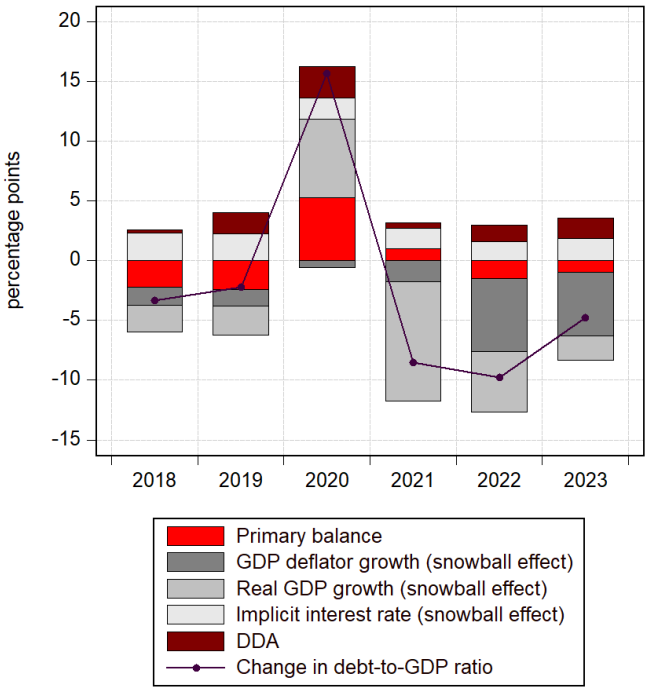


Figure 1: Debt dynamics in Croatia from 2018 to 2023
Sources: Eurostat (2024a;2024b;2024c)

This paper aims to assess Croatia's fiscal reaction to the inflationary shock in the aftermath of the Covid-19 crisis, with a focus on both the revenue and expenditure sides of fiscal policy. We will try to outline how inflation affects direct and indirect taxes, as well as current and capital expenditures.

Previously, [2] estimated the fiscal reaction function with structural breaks and suggested that inflation has a favourable effect on the primary balance in the short term. This finding reflects mostly positive effect of inflation on nominal tax revenues (VAT due to tourism, CIT due to extra-profit and PIT due to increase in wages and salaries), while the effect on expenditure side of budget is expected with lag. Therefore, in the medium run, inflation is likely to have a negative effect on the primary balance by increasing government expenditure more than tax revenues. The primary contribution of this research lies in employing a disaggregated approach to extend Banić et al. (2023) in analysing fiscal reactions in Croatia. On the revenue side, the study examines how direct and indirect taxes respond to inflationary shock. On the expenditure side, the research focuses on the reaction of both current expenditures, such as wages and salaries and intermediate consumption, as well as of capital expenditures, like government investments and infrastructure spending, to inflation. Thus, a disaggregated approach offers a more detailed insight into how inflationary shock influences fiscal policy in Croatia.

2 LITERATURE REVIEW

Recent economic fluctuations. The economic consequences of the COVID-19 pandemic, combined with the surge in energy prices, have revived interest in the relationship between inflation and public finances. Inflationary shocks create great challenges for economic stability, and fiscal policy responses can mitigate inflation effects by influencing fiscal positions, and overall macroeconomic stability. Fiscal policy responses to economic shocks usually include automatic stabilizers that adjust to economic changes, and discretionary measures, which are specific actions taken by governments in response to shock, like inflation (Dolls et al., 2012).

[4] outline the differing views on the role of fiscal policy in price determination. The early monetarist perspective asserts that inflation is driven by money growth, dismissing fiscal policy's

influence unless it is money-financed. In contrast, the fiscal theory of the price level argues that an unbacked fiscal expansion, where public debt increases without matching future primary surpluses, raises perceived real wealth, boosting consumption and prices. If future surpluses do not cover outstanding nominal debt, the price level must rise to ensure fiscal solvency. New Keynesian models highlight the role of discretionary fiscal policy in stabilizing the economy, especially when monetary policy is constrained or during deep recessions to prevent deflation. Empirical studies generally suggest that expansionary fiscal policy contributes to higher inflation, as governments use inflation to decrease debt burden.

In the short term, higher inflation increases government revenues [21], [2]. [18] suggests that a 1-percentage-point increase in CPI inflation improves budget balances by approximately 0.5% of GDP in the short term, with the effect peaking within two quarters. This occurs as revenues grow in line with nominal GDP, while expenditures remain relatively fixed due to budget constraints and annual spending limits. Interest expenses, however, tend to rise gradually due to the prevalence of fixed-rate, long-maturity government debt [11], [10]. On the expenditure side, categories such as public sector wages and social benefits adjust with a delay. However, in the medium term, inflation is expected to negatively influence the primary balance, as increased government expenditures outpace the growth in tax revenues [2]. Expected inflation increases often trigger higher spending and rising borrowing costs, offsetting any fiscal gains [6]. [5] also outlined that, while inflation may provide temporary fiscal relief, its long-term effects depend on broader macroeconomic conditions and policy responses. Same finding was highlighted by [17], using panel data on EU countries from 1976 to 2024, showing that inflation usually increases revenues higher than expenditures and lowers debt-to-GDP ratio on the account of favourable effect of GDP deflator, but in medium-term restrictive monetary policy could affect the economy in opposite way, taking also into account the increase of implicit interest rate, i.e. interest expenditures paid on debt.

3 EMPIRICAL FRAMEWORK AND ANALYSIS

Using Bohn (1998) fiscal reaction function, popularized in domestic literature (Arčabić, 2018; Arčabić & Banić, 2021) with structural breakpoints (Rebić i Arčabić, 2023, Banić et al., 2023), we will estimate the effect of inflation, measured by Harmonised Index of Consumer Prices (HICP) on disaggregated fiscal positions.

3.1 Data and methods

Our analysis covers data period from 2002q3 to 2023q3.
Simple fiscal reaction function model with breakpoints with 4 different fiscal categories is expressed as**:**

$$dv_t = \alpha + \phi_1 dv_{t-1} + \beta_1 \tilde{y}_t + \beta_2 \pi_t + \varepsilon_t, \quad (2)$$

with different dependent variable (dv_t)- 4 models [8]:

- Model 1- growth rate of general government indirect taxes
- Model 2- growth rate of general government direct taxes
- Model 3- growth rate of general government final consumption expenditure
- Model 4- growth rate of general government gross fixed capital formation.

Independent variables- for all models:

- lagged dependent variable (dv_{t-1}),
- output gap ($\tilde{y}_t$), estimated with Cobb-Douglas production function approach (Banić et al., 2024),
- inflation rate – HICP (π_t) (Eurostat, 2025d),

and

- ε_t *- error term.*

3.2 Results

Tables 1,2,3 and 4 show results regarding the inflationary shock on revenue and expenditure side of general government in Croatia using breakpoints models.

In more detail, inflation does not have a statistically significant effect on indirect taxes developments until strong economic recovery in post-pandemic period. Higher domestic and external consumption also reflected in nominal macroeconomic bases like personal consumption and exports of services related to tourism, which had favourable effect on indirect taxes, especially VAT. One may conclude that strong domestic and foreign demand contributed to strong growth of indirect taxes in the aftermath of pandemics. Stronger consumption is also related to previous higher propensity to save during the pandemics.

Table 1: Fiscal reaction function – General government indirect taxes

OLS with BP	2002Q3-2021Q1	2021Q2-2023Q3
Indirect taxes (-1)	0.311**	-0.151
	(0.013)	(0.339)
Inflation rate	-0.211	0.742*
	(0.659)	(0.061)
Output gap	1.364***	-1.212
	(0.000)	(0.434)
R^2	0.678	

Note: *,** and *** indicate significance at 10%,5% and 1%.
Sources: authors' calculations

Regarding the inflationary effect on direct taxes, statistically significant result is also recorded in the post-pandemic period. First round effect can be linked to corporate income taxes due to energy crises which also reflected on higher gross operating surplus in national accounts, which can be linked to higher companies' mark-ups (Lukinić-Čardić & Zrnc, 2023). To mitigate this effect, Government also introduced one-off extra-profit tax in 2022. Inflationary effect on personal income taxes is usually recorded with time lag, which is reflected in increase in wages and salaries in private and public sector. Also, inflation-induced increase in wages and PIT could initiate bracket creep effect, lowering citizens' living standard. Hence is necessary to adjust tax brackets to inflationary developments.

Table 2: Fiscal reaction function – General government direct taxes

OLS with BP	2002Q3-2020Q2	2020Q3-2023Q3
Direct taxes (-1)	0.477***	-0.569***
	(0.000)	(0.000)
Inflation rate	-0.644	5.246***
	(0.409)	(0.000)
Output gap	1.205***	-0.789
	(0.000)	(0.239)
R^2	0.670	

Note: *,** and *** indicate significance at 10%,5% and 1%.
Sources: authors' calculations

When it comes to reaction of expenditures to inflationary shock, results are different from those on revenue side. During energy crisis, the relationship between current expenditures and inflation is negative, in line with Staehr et al. (2023). In other words, discretionary reaction to inflation in terms of social transfers, wages and salaries in public sector respond with time lag.

Table 3: Fiscal reaction function – General government final consumption expenditure

OLS with BP	2002Q3-2019Q4	2020Q1-2023Q3
Final consumption exp. (-1)	0.571*** (0.000)	-0.696*** (0.000)
Inflation rate	-0.403 (0.123)	-1.368*** (0.000)
Output gap	0.402*** (0.009)	-0.04 (0.842)
R^2	0.769	

Note: *,** and *** indicate significance at 10%,5% and 1%.
Sources: authors' calculations

Breakpoint in government investment is not detected, reaction to inflation is not statistically significant and R^2 is substantially lower comparing to previous models. Usually in small economy, integrated in EU, dynamics is driven to large extent by EU funds absorption. Also, when economy is hit by large economic shock, investment plan realization is postponed due to higher-than-expected inflation affecting initial price of input and materials in process.

Table 4: Fiscal reaction function – General government gross fixed capital formation

OLS with BP	2002Q3-2023Q3
Gross fixed capital formation (-1)	0.619*** (0.000)
Inflation rate	0.355 (0.123)
Output gap	0.027 (0.977)
R^2	0.386

Note: *,** and *** indicate significance at 10%,5% and 1%.
Sources: authors' calculations

Potential risk for fiscal sustainability could occur if current expenditures increase substantially and permanently in medium-term, when the inflationary and cyclically positive effect on revenues is expected to fade away, generating pressures on borrowing needs and public debt. Also, insignificant output gap in all models most likely suggests that traditional approach captures only demand, and not supply shocks, which were important during covid and energy crisis.

4 CONCLUSIONS

In this paper we assess fiscal reaction to inflationary shock in Croatia, focusing on recent energy crisis with disaggregated approach. We use breakpoints regression to capture the reaction of both revenue and expenditure side on inflation surge. Results for Croatia are broadly

the same comparing to previous research related to EU, indicating automatic positive reaction of revenue side (VAT, CIT, PIT) to inflation, while we find no evidence regarding positive reaction on expenditure side, which is expected to react with time lag (current expenditures).

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GREEN BONDS: SOME BUY-SIDE ASPECTS

Ana Marija Ćoko
University of Split, Faculty of Economics, Business and Tourism
Cvite Fiskovića 5, 21000 Split, Croatia
E-mail: amcoko@efst.hr

Zdravka Aljinović
University of Split, Faculty of Economics, Business and Tourism
Cvite Fiskovića 5, 21000 Split, Croatia
E-mail: zdravka.aljinovic@efst.hr

Abstract: As sustainability becomes a central theme in global finance, green bonds have emerged as a critical vehicle for environmentally responsible investment. This study examines the function of green bonds in a diversified portfolio, assessing their risk-return characteristics in comparison to traditional and alternative asset classes from January 2022 onward. The analysis demonstrates that green bonds have low volatility, which makes them a good choice for Environmental, Social, and Governance (ESG) - aligned investment strategies. Despite small or negative returns driven by the "greenium", their defensive characteristics and alignment with sustainability goals make them more appealing to both institutional and retail investors. This paper further examines the implications of ESG scoring variability, market accessibility, and greenwashing risks in shaping investor behavior and market development.

Keywords: Green Bonds, ESG, Risk, Return, Portfolio diversification

1 INTRODUCTION

The rapid evolution of sustainable finance has placed ESG criteria at the forefront of investment decision-making, particularly within the realms of sustainable finance and green bonds [7]. Green bonds have emerged as key sustainable finance instruments since their 2007 introduction [1], growing from $3 billion annual issuance in 2012 to exceeding $500 billion in 2021 [2].

Unlike conventional bonds, green bonds feature dedicated use-of-proceeds disclosures [10] and often benefit from a pricing premium known as the "greenium" [12], which can lower issuer borrowing costs [5]. As relatively low risk instrument [17], with risk profiles closely aligned with treasury and investment-grade corporates [8], green bonds often show lower volatility than conventional bonds [1] offering considerable diversification benefits.

The market's expansion, supported by frameworks like the Green Bond Principles [4] and EU Green Bond Standards [16], demonstrates how ESG integration is transforming investment practices [17] while regulatory efforts continue to address standardization challenges in both ESG scoring and green bond verification [13]. The development of green bond indices and Exchange-Traded Funds has further facilitated accessibility, broadening participation among institutional and retail investors [3].

This paper evaluates green bonds from an investor's perspective—assessing their financial characteristics, diversification benefits, and carried risks. Using daily return data from 2022 onward, we compare green bonds to a representative sample of asset classes, including equities, commodities, cryptocurrencies, and conventional bonds. The analysis aims to determine whether green bonds offer competitive advantages in terms of volatility, returns, and alignment with sustainable investing mandates, while also addressing challenges like greenwashing and limited market accessibility.

2 DATA AND METHODOLOGY

The asset selection for this study was guided by Modern Portfolio Theory (MPT), which assumes investors seek optimal risk-adjusted returns [11]. Following Markowitz's standard deviation (σ) measure we quantified an asset's total risk by measuring the dispersion of returns around their mean. It serves as a proxy for volatility, where higher standard deviation implies greater uncertainty in expected returns [9]. This study utilizes daily adjusted closing prices for a diversified set of financial instruments that represent major asset classes, as detailed in Table 1.

The dataset covers the period from January 1, 2022 to present and was collected using R's quantmod package, which sources historical market data from Yahoo Finance [18]. The analysis is conducted in U.S. dollars (USD), and no currency conversion was applied. Each asset's adjusted daily closing price was obtained and aligned to ensure a consistent time series across all instruments. Any observations with missing values were removed using na.omit() to maintain data integrity and comparability.

Table 1: Asset Classes and Selected Proxies

Asset Class	Index	Ticker	URL
Gold	SPDR Gold Shares	**GLD**	https://finance.yahoo.com/quote/GLD/
Green Bonds	iShares USD Green Bond ETF	**BGRN**	https://finance.yahoo.com/quote/BGRN/
Commodities	S&P GSCI Index	**^SPGSCI**	https://finance.yahoo.com/quote/%5ESPGSCI/
S&P 500	Standard's & Poor 500	**^GSPC**	https://finance.yahoo.com/quote/%5EGSPC/
Real Estate	Vanguard Real Estate ETF	**VNQ**	https://finance.yahoo.com/quote/VNQ/
Cryptocurrencies	Etherium	**ETH-USD**	https://finance.yahoo.com/quote/ETH-USD/
Broad Bond Market	Vanguard Total Bond Market Index	**BND**	https://finance.yahoo.com/quote/BND/
Renewable Energy	iShares Global Clean Energy ETF	**ICLN**	https://finance.yahoo.com/quote/ICLN/

Source: Own calculations

3 RESULTS

Daily expected returns were computed using the ROC() function, representing the percentage change in asset prices from one day to the next. From these return series, the following statistics were calculated for each asset: Daily Standard Deviation as a measure of daily price volatility, calculated using the sd() function. Annualized Standard Deviation computed by scaling the daily standard deviation by the square root of 252 (the approximate number of trading days in a year). Daily and Annualized Expected Returns calculated as the mean of daily returns and its annualized counterpart (multiplied by 252), providing a proxy for average performance over time. All return and risk metrics were quantified as percentage values to ease comparative analysis and interpretability across the dataset.

Table 2: Expected daily and annualized standard deviations and returns for selected assets (2022-Present)

Asset	Daily_SD	Annualized_SD	Daily_Return	Annualized_Return
Gold	0,96%	15,29%	0,07%	18,41%
Green Bonds	0,39%	6,23%	0,00%	-0,54%
Commodities	1,40%	22,29%	0,01%	1,28%
S&P 500	1,19%	18,84%	0,03%	7,90%
Real Estate	1,31%	20,83%	-0,01%	-1,86%
Cryptocurrencies	4,48%	71,19%	0,06%	15,24%
Conventional Bonds	0,43%	6,84%	0,00%	-1,18%
Clean Energy	1,77%	28,13%	-0,04%	-10,15%

Source: Own calculations

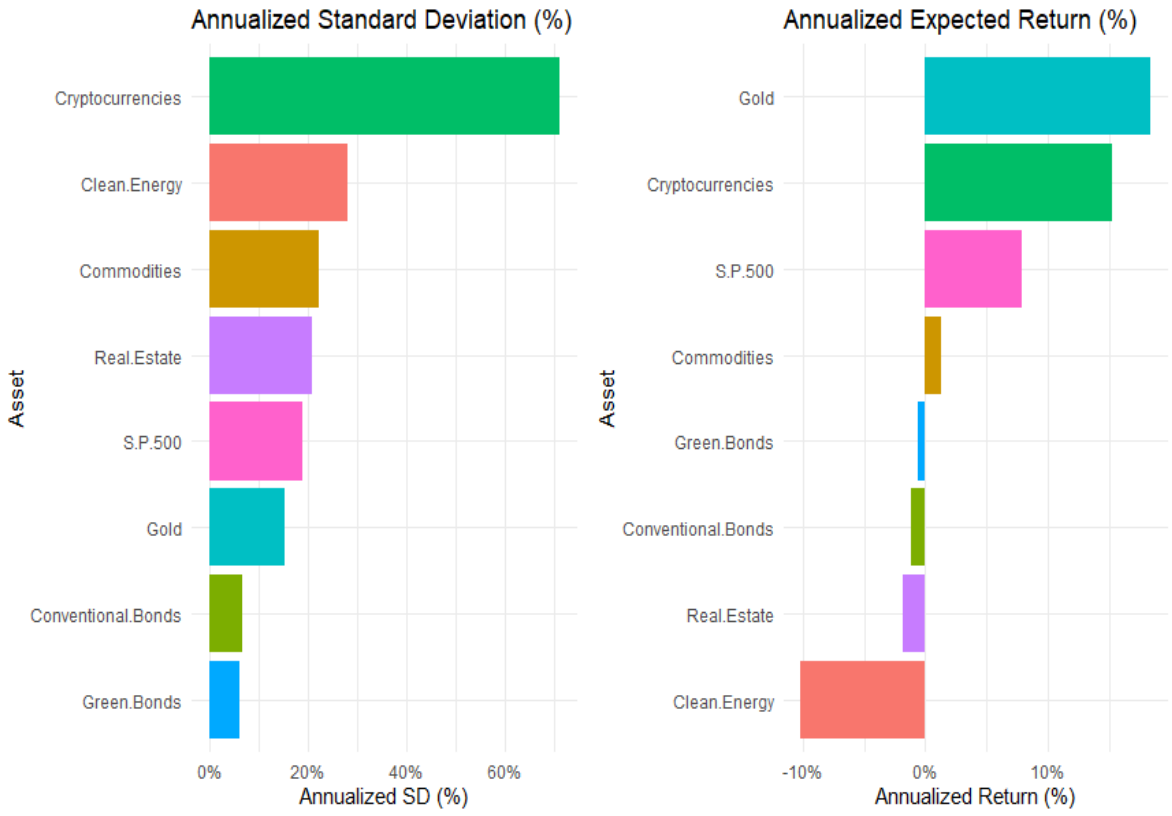


Figure 1: Visualization of expected annualized standard deviations and returns for selected assets (2022-Present)
Source: Own calculations

As evidenced in Table 2 and Figure 1, green bonds again prove the nature of low volatility, making them the least risky asset class in this sample. The annualized return of -0.54% is slightly negative, suggesting capital preservation rather than growth. However, this stability came at the cost of negative returns, likely reflecting the low-yield environment and possible "greenium" pricing effects (where high demand for ESG assets reduces yields).

Strategically, green bonds serve distinct roles across portfolios. For risk-averse investors, they function as a core, low-volatility asset that simultaneously advances ESG objectives—a dual benefit not to be overlooked in contemporary portfolio construction. Within balanced asset allocations, green bonds serve as a stabilizing force, often demonstrating minimal or even

negative correlation with equities during periods of market stress. This characteristic enhances diversification and risk mitigation.

Because green bonds tend to stay stable in price and don't react strongly to market swings, active traders might not find them as appealing for short-term speculation. Instead, these securities are better used as a steady source of carry than as tools for aggressive trading strategies. Their ultra-low volatility is particularly advantageous for liability-driven investors, who require predictable returns to meet future obligations. In summary, green bonds present a compelling proposition as they combine the traditional security of fixed-income instruments with quantifiable environmental benefits, establishing themselves as a reliable pillar for portfolios seeking both financial and sustainable outcomes.

Green bonds demonstrate a slight outperformance over conventional bonds in terms of returns, while also exhibiting reduced volatility (see table 2). Although these instruments do not dominate in performance metrics, their defensive characteristics and alignment with ESG principles position them as prudent components within sustainable, low-risk investment portfolios. Particularly during periods of economic uncertainty or inflationary pressure, green bonds offer investors an appealing balance of capital preservation and ethical consideration.

4 CONCLUSION AND FUTURE RESEARCH STEPS

From an investor’s standpoint, green bonds present advantages such as relative stability, consistent, although modest returns, and diversification potential. Nevertheless, policy risks such as reliance on tax incentives [4], and a concern over greenwashing or fund misallocation [2] pose challenges. Despite their benefits, green bonds remain less accessible than conventional fixed-income securities due to limited supply [14]. Challenges also include market scarcity, the risk of greenwashing, and inconsistencies in regulatory standards across authorities [13].

Despite green bonds' demonstrated potential, several limitations limit their current effectiveness, including market immaturity with limited issuance and liquidity challenges, data insufficiency from the asset class's relative novelty, and regulatory fragmentation that creates inconsistent verification standards and cross-border compliance complexities [15]. These structural constraints are compounded by methodological limitations in existing studies, particularly the reliance on post-2022 data that may not reflect full market cycles.

Nevertheless, the increasing inclusion of these instruments in investment portfolios reflects a broader commitment to harmonizing financial objectives with environmental stewardship. As regulatory frameworks progress and transparency standards strengthen, green bonds are poised to play an even more integral role in sustainable finance. They offer an opportunity for investors who seek both measurable environmental impact and resilience within their portfolios.

Future research should address these gaps through long-term performance analyses across diverse economic conditions, investigations into how regulatory harmonization could improve market functioning, and behavioral studies of investor decision-making, while additional priorities include comparative assessments of ESG methodologies, performance benchmarking against other sustainable assets, and portfolio integration studies to optimize green bonds' role in diversified investment strategies.

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MICE OR NO MICE?
MACHINE LEARNING MODELS FOR FINANCIAL DISTRESS PREDICTION IN CROATIAN MANUFACTURING SME

Tea Šestanović
University of Split, Faculty of Economics, Business and Tourism, Department of Quantitative Methods
Cvite Fiskovića 5, Split, Croatia
E-mail: tea.sestanovic@efst.hr

Marko Čular
University of Split, Faculty of Economics, Business and Tourism, Department of Accounting and Auditing
Cvite Fiskovića 5, Split, Croatia
E-mail: marko.cular@efst.hr

Abstract: This paper proposes the optimal model for financial distress (bankruptcy, rescue plan or liquidation) prediction of Croatian manufacturing SME in pre-Covid, Covid and post-Covid period. For missing data the Multiple Imputation by Chained Equations (MICE) interpolation technique is used and compared to raw data. Linear regression (LR), decision trees and feedforward neural networks (FNNs) are employed using firm size, number of employees, profit margin, solvency and current ratio. The results reflect the superiority of FNN, while MICE only helps LR to extract information in the models.

Keywords: financial distress prediction, financial ratios, neural networks, linear regression, decision trees, SME, MICE

1 INTRODUCTION

Predicting corporate bankruptcy is a critical issue in accounting and finance, with substantial importance for various stakeholders, such as investors, creditors, suppliers, and other key players in the financial ecosystem, who are concerned with a company's financial stability. Bankruptcy is the legal status of a company that is unable to repay its debts, leading to court proceedings for the settlement of obligations, often resulting in liquidation or reorganization [7]. Bankruptcy in most cases is not a sudden event, but rather a gradual process, where the signs of financial distress (FD) can be observed years before the event [19]. In traditional models, the predictive ability of financial variables is high for 1-year horizon, but their predictive ability decreases in time, while for 3-year horizon most approaches give inaccurate prediction results [2]. While 83% of the studies predict default events within a year [10], this paper tries to find the roots of FD even before Covid-crisis, and seeks to find its consequences for manufacturing sector of Croatian small and medium-sized enterprises (SME). SMEs are the pillars of an economy, and their poor performance has a negative impact on living standards of population and country development [14]. Manufacturing is a key sector of the Croatian economy due to its role in employment, export and added value. Having an accurate company default prediction model, which can pick up on time the signs of FD, is vital for all firms, especially for SMEs. Previous research ([1]; [9]; [11]; [14]; [21]) highlighted that specific default prediction models are required to evaluate the risk profiles of SMEs, and only 7% of papers analyse manufacturing sector in FD prediction [20].

Traditional methods like logistic regression (LR) or discriminant analysis (DA) are often used for FD prediction ([18]; [7]). LR produces better results over DA [8] while having less assumptions. Its popularity can be attributed to its robustness, interpretability and the ability to handle various types of predictors, making it a preferred choice among researchers and practitioners. A limiting characteristic of LR is the assumption of linear separability of the observed space of predictors [16]. Recently, most research employ machine learning (ML) models, i.e. support vector machines (SVMs), decision trees (DTs), random forests (RFs) and

neural networks (NNs) [23]. DT has advantage over LR since it allows nonparametric representations of the relationships between the variables in the model. Compared to NN, DT has good interpretability, while NNs can be viewed as *black box*. Contrary, NNs can model nonlinear relationships and interactions between variables in the model and often have higher predictive power, especially for larger datasets, while DT can easily overfit the data. NNs require pre-processing, while DT can deal with raw data. [2] find LR and NNs superior to other approaches for FD prediction. [19] report that ML models systematically outperform LR and DA, with NNs causing the largest average cost reduction. [17] compared the performance of NNs with bootstrap aggregation, SVM, RF, boosting algorithm, k-NN, DT and LR. The results suggest that, NNs and DTs achieve better performance compared to other models. This paper therefore compares results of LR, DTs and NNs for FD prediction.

One of the primary challenges in FD modelling is the availability and quality of data. Namely, financial data is often complex, heterogeneous, and prone to errors, requiring careful preprocessing and feature engineering to extract meaningful insights [12]. Therefore, the results using raw data are compared to interpolated data using Multiple Imputation by Chained Equations (MICE) technique. [15] concluded that MICE improves LR predictive ability, while for DT and NN has equally good predictive performances as simple mean interpolation. MICE is a powerful, model-based method for handling missing data. The advantages of MICE lay in the preservation of the relationship between the variables, while accounting for the uncertainty in imputations and being appropriate for both numeric and categorical data [5]. Moreover, during the evaluation of FD prediction models, it is essential to consider a range of metrics, including sensitivity and precision to obtain a comprehensive assessment of model performance [12]. However, most research present only accuracy, which can be misleading, especially for imbalanced datasets [22]. Although it is important to address the challenge of imbalanced datasets in FD prediction by exploring and developing specialized techniques for handling such data [12], most research use 50-50 rule or different methods to balance the datasets ([8]; [23]), while only few papers tried to replicate the distribution of defaults to non-defaults in the population ([1]; [3]; [9]) as in this paper. Moreover, [10] report that 37% of research do not include or report any variable selection techniques, 25% use only in-sample validation techniques which is methodologically unjustified and more than 50% of studies do not report performance measures. This paper therefore specifies the independent variables *a priori*, based on Welch two sample t-test and Pearson's correlation coefficient lower that 0.6 in order to control for multicollinearity in the model, while comparing models according to both out-of-sample accuracy, recall, precision and specificity.

This paper contributes to the existing literature in several ways. Firstly, by defining the appropriate model for FD prediction of Croatian manufacturing SME through Welch-based variable selection and through proposal of the appropriate ML model which yields better prediction performances in terms of accuracy, precision, recall and specificity. Secondly, by handling the imbalanced dataset with missing data, i.e. by using interpolated and raw datasets and prespecified cut-off value. Finally, by comparing the results through different periods, while trying to find the roots of financial distress 1- to 4-years ahead.

2 DATA AND METHODOLOGY

The sample consists of Croatian SMEs in manufacturing sector, based on NACE (Nomenclature of Economic Activities), retrieved from Moody's Orbis Europe database. Using the homogenous sample has the advantage in better prediction accuracies since in heterogeneous samples the values of financial indicators can differ significantly, although some of the papers include full sample of Croatian companies regardless of the sector [8]. Out of the total of 132 167 SMEs in Croatia from different sectors, only 2 477 are in financial

distress in 2023, i.e. only 1.87%. In manufacturing sector this percentage is 2.56% (365 out of 14 243 companies). However, the original database consists of missing data which are row-wise omitted, yielding to a different number of SMEs with FD status in 2023 with available data from 2022 to 2019 (i.e. 192, 181, 184 and 187 respectively). Since some studies suggest interpolation of missing data, the same is done using MICE approach. Firstly, all missing values are initially filled with mean estimates for numerical or mode values for categorical variables. Secondly, for each variable with missing values a regression model is fit using other variables as predictors, where missing values in that variable are predicted using this regression model. The process is repeated sequentially for each variable with missing data, i.e. in a form of a chain. Entire process is repeated 5 times to generate multiple full datasets. Each iteration introduces random variation, i.e. predictive mean matching (PMM) to reflect the uncertainty in the missing data. PMM fills missing values with real observed values closest to the predicted. Finally, after the imputation, analyses are run on each of the 5 full datasets and the results are pooled using Rubin's rules to get final estimates with valid standard errors. 50% of the full dataset is used for further calculations which corresponds to 182 companies in FD out of 7 121 companies, i.e. similar to raw data. Both raw and MICE datasets are divided randomly into 70% for training and 30% for testing, while keeping the ratio of unhealthy firms as in population.

Table 1: Variables in the models

Variable	***Acronym***	***Calculation***
Firm Status	FS	Binary variable (1-bankruptcy, rescue plan or in liquidation process; 0-healthy)
Firm Size	SIZE	Natural Logarithm of Total Assets
Solvency ratio	SOL	Equity/Assets
Employees	EMP	Number of Employees
Profit Margin	PRM	(Net Income/Revenue)x100
Current ratio	CUR	Current Assets/Current Liabilities

Table 2: Welch two-sample t-test

		raw data						***MICE data***				
	FS	***N***	***μ***	***|t|***	***N***	***μ***	***|t|***	***N***	***μ***	***|t|***	***μ***	***|t|***
		2022			***2021***			***2022***			***2021***	
PRM	***0***	9374	8.27	2.24**	8725	7.66	2.71***	6939	5.65	0.77	5.22	3.21***
	1	192	4.47		181	2.84		182	3.90		2.04	
CUR	***0***	9374	3.64	5.07***	8725	3.61	3.89***	6939	3.91	1.87*	3.65	0.65
	1	192	2.31		181	2.42		182	2.83		3.19	
SOL	***0***	9374	49.45	6.41***	8725	48.80	4.87***	6939	37.82	4.67***	36.58	4.82***
	1	192	37.39		181	39.52		182	22.19		20.55	
SIZE	***0***	9374	5.25	7.10***	8725	5.21	7.41***	6939	4.54	5.43***	4.55	5.04***
	1	192	6.18		181	6.23		182	5.51		5.44	
EMP	***0***	9374	10.47	2.62***	8725	10.63	3.23***	6939	8.29	1.47	8.06	1.87*
	1	192	15.17		181	17.29		182	10.12		10.38	
		2020			***2019***			***2020***			***2019***	
PRM	***0***	8140	5.94	1.68*	7921	6.60	1.61	6939	3.50	1.48	4.86	0.9
	1	184	3.25		187	4.51		182	0.49		3.35	
CUR	***0***	8140	3.63	5.70***	7921	3.33	4.93***	6939	3.55	1.92*	3.16	2.05**
	1	184	2.22		187	2.22		182	2.42		2.38	
SOL	***0***	8140	48.88	5.65***	7921	46.47	4.93***	6939	35.82	3.88***	34.78	3.22***
	1	184	38.42		187	36.99		182	23.69		24.38	
SIZE	***0***	8140	5.20	7.20***	7921	5.16	7.16***	6939	4.49	6.16***	4.39	5.97***
	1	184	6.20		187	6.17		182	5.52		5.43	
EMP	***0***	8140	10.94	3.46***	7921	11.13	3.33***	6939	7.88	2.66***	7.92	2.57**
	1	184	18.63		187	20.05		182	11.98		14.37	

Note: * p < 0.1, ** p < 0.05, *** p < 0.01

LR, DT and FNN are employed to predict whether a company is experiencing FD (bankruptcy, involvement in a rescue plan or liquidation) in 2023. Therefore, *Firm Status* is a binary dependent variable, coded 0 for a healthy, and 1 for an unhealthy firm in 2023. Predictors in the models are firm size, number of employees, profit margin, solvency and current ratio from years 2019 to 2022 (Table 1), respecting heuristic procedures for variable selection. Pearson's correlation coefficient between the independent variables is lower than 0.55, therefore controlling for multicollinearity. Financial ratios are the most commonly used explanatory variables in FD prediction models. They help standardize data, which is important for all ML models. In line with previous studies, this paper uses financial ratios covering major aspects of a firm's operations (profitability, liquidity and solvency). As a firm moves from its desired (healthy) status to rescue plan, liquidation process and then toward bankruptcy status, a firm's profit deteriorates, and its liquidity and solvency worsen. The lower the value of these indicators, the higher a firm's inability to meet its obligations and continue its operations [18]. Traditional ratios (PRM, CUR, SOL) provide a static view of the financial performance by measuring a single point in time [4] and serve as reliable predictors for identifying potential FD with 84% accuracy [7]. Firms with lower PRM, CUR and SOL exhibit a higher likelihood of FD. SIZE has a crucial role in forecasting FD, however there are ambiguous research results. Some researchers argue that larger firms are more likely to enter in rescue plan that would perform better by restructuring in comparison with smaller firms, while other argue that the likelihood of failure is higher as the SIZE of a company increases ([7]; [18]). Others state that there is a higher probability for smaller and recently established SMEs to experience bankruptcy [11]. Finally, there is a positive relationship between EMP and FD [6].

Welch two-sample t-tests, used to compare the means of two independent groups when the assumption of equal variances is not valid**,** for the selected independent variables through years and datasets are given in Table 2. The results show that there is significant difference in mean values of mostly all variables in all selected periods and datasets between healthy and unhealthy SMEs. On average, financially healthy companies exhibit higher profit margins, solvency and current ratios. Interestingly, and somewhat counterintuitively, firms experiencing FD tend to be larger in size and employ a higher number of people. Namely, the complexity, leverage, and visibility of big firms make FD more observable and protracted. Namely, small companies often fall silently or are excluded from datasets, while larger companies go through formal restructuring, rescue plans or liquidation, making FD more visible in datasets. Although their complexity can make them more exposed to systemic shocks since they have higher fixed costs, they tend to have more access to credit allowing them to delay bankruptcy longer.

LR is estimated using *glm* function in *R*. The logistic function is:

$$P(A)=\frac{e^{\beta_0+\beta_1 x_{t,1}+\beta_2 x_{t,2}+...+\beta_p x_{t,p}}}{1+e^{\beta_0+\beta_1 x_{t,1}+\beta_2 x_{t,2}+...+\beta_p x_{t,p}}} \tag{1}$$

where $P(A)$ is the probability that company will have FD. The goal is to estimate parameters $\beta_i,\quad i=1,2,...,p$. Denoting $g(x)=\beta_0+\beta_1 x_{t,1}+...+\beta_p x_{t,p}$ and applying logistic transformation, a linear relationship between the log-odds and independent variables is:

$$\ln\frac{P(A)}{1-P(A)}=\ln\frac{\frac{e^{g(x)}}{1+e^{g(x)}}}{1-\frac{e^{g(x)}}{1+e^{g(x)}}}=\ln e^{g(x)}=g(x)=\beta_0+\beta_1 x_{t,1}+\beta_2 x_{t,2}+...+\beta_p x_{t,p} \tag{2}$$

DT is a ML model that predicts the FD by applying a sequence of decision rules learned from the data. These rules split the dataset into subsets based on specific inputs and threshold values, ultimately assigning a prediction to each subset. Each decision rule involves one input

and divides the data into two groups: those with values below and those above threshold. Model resembles a binary tree, where the root is the entry point for all data, internal nodes represent decision rules and branches guide the data through splits until it reaches a leaf node, where the final prediction is made ([16]; [13]). DTs are defined by several parameters within function *rpart* in R: minimum number of samples required to split an internal node set to 10; minimum observations allowed in a terminal node set to 5; complexity parameter used for pruning set to 0.0001; while maximum depth of the tree, which limits the maximum number of levels the tree can grow set to 25, allows higher complexity.

FNN forwards information from inputs though hidden neurons to output, using weights and activation functions to connect consecutive layers. Backpropagation learning algorithm is used to estimate the weights. FNN in (3) is estimated using *nnet* function in *R:*

$$y_t = f\left(w_{co} + \sum_{h=1}^{q} w_{ho} f\left(w_{ch} + \sum_{i=1}^{p} w_{ih} x_{t,i} \right) \right) + \varepsilon_t, \tag{3}$$

where y_t is the binary dependent variable, x_t is the input matrix with $p=5$ independent variables, while $f(\cdot)$ is logistic activation function. Weights w_{co} and w_{ch} are constant terms of output and hidden neurons respectively. Weights w_{ih} and w_{ho} are the connections between inputs and hidden neurons and between hidden neurons and output respectively. ε_t is an error term. The grid search of 2, 5, 7, 10, 15 and 20 hidden (*h*) neurons is used to find the optimal FNN. Other specifications are held constant, i.e. maximum of 500 iterations and 0.01 weight decay which adds moderate regularization. Namely, it controls L2 regularization while preventing overfitting by penalizing large weights.

Additionally, most papers deal with 50-50 rule, which is a decision threshold or cut-off rule used when interpreting probabilities of FD as output from ML models. It assumes equal cost of false positives and false negatives, which may not be realistic since false negatives, i.e. missed FD, have higher costs than false positives. Therefore in this paper this threshold is set up to 12.5% for better balance between risk and detection [3]. Additionally, papers mostly compare 50% of bankrupt with 50% of similar non-bankrupt companies, which is methodologically valid although practically irrelevant. Sampling approach that resembles the population is kept throughout the paper, while also comparing models performance according to accuracy (*acc*), precision (*prec*), recall (*rec*) and specificity (*spec*) measures from 2019 to 2022 with raw and MICE data (Table 3).

3 RESULTS

The results of LR (Table 3) indicate that firm size, solvency ratio and profit margin in most years are significant predictors of FD in 2023[1], confirming that the higher the profit margin and solvency ratio, the lower the probability of FD. Additionally, the higher the SME, the higher probability of FD. Although raw-LR produces the highest accuracy (97.84% for 2022), it fails to predict unhealthy SMEs, resulting in 0% precision and recall using raw data. MICE improves prediction performance of LR in terms of precision and recall. Namely, in the context of FD prediction the most important measure is recall (sensitivity of true positive rate). It measures how effectively the model identifies all companies in FD, due to the high cost of missing a true FD (false negatives). High recall means the model successfully detects most firms in FD, minimizing missed unhealthy firms. MICE-DT and MICE-FNN increase recall and precision compared to raw models only for 1- or 2-year ahead prediction, while for 3- or

[1] Estimated parameters of LR and performance of FNNs with more than 10 hidden neurons (as there is no increase in predictive ability) are omitted due to lack of space. They are available from the authors upon request.

4-years ahead predictions MICE adds too much noise into ML models and lowers their predictive abilities. FNNs have on average comparable accuracies to DT or LR, but significantly higher recall and precision. FNNs with only 2 or 5 hidden neurons yield optimal results reaching recall of 22.6% for 1-, 31.8% for 2-, 25% for 3- and 19.3% for 4-year ahead prediction with 2 or 5 hidden neurons. Only FNNs succeeded to extract the information for future FD even before Covid. In response to Covid-crisis, Croatia introduced measures to support SMEs like wage subsidies, tax relief and deferrals, favourable liquidity loans, loan guarantees and interest subsidies, grants and microloans, streamlined procedures, aimed to preserve jobs, maintain liquidity, and support the recovery of SMEs during the pandemic.

Table 3: Out-of-sample results of the ML models with raw or MICE-interpolated data for period 2019-2022

	raw data				***MICE data***			
	2022	***2021***	***2020***	***2019***	***2022***	***2021***	***2020***	***2019***
LR								
acc	97.84%	97.53%	97.64%	97.57%	97.43%	97.19%	97.38%	97.38%
prec	0,00%	20.00%	20.00%	0.00%	50.00%	14.29%	40.00%	33.33%
rec	0,00%	1.59%	1.79%	0.00%	3.64%	1.82%	3.64%	1.82%
spec	97.84%	99.85%	99.84%	99.92%	99.90%	99.71%	99.86%	99.90%
DT								
acc	97.56%	97.34%	97.47%	97.04%	97.05%	96.81%	96.53%	96.91%
prec	16.66%	10.00%	26.67%	10.52%	27.77%	0.00%	4.76%	7.69%
rec	3.22%	1.58%	7.14%	3.51%	9.09%	0.00%	1.81%	1.81%
spec	99.64%	99.65%	99.54%	99.28%	99.37%	99.37%	99.04%	99.42%
FNN (h=2)								
acc	97.04%	86.52%	93.27%	94.16%	92.37%	93.96%	91.99%	96.21%
prec	23.26%	5.93%	6.92%	10.28%	8.46%	12.24%	6.72%	13.89%
rec	16.13%	31.75%	16.07%	19.30%	20.00%	21.82%	16.36%	9.09%
spec	98.82%	87.85%	95.04%	95.96%	94.28%	95.87%	93.99%	98.51%
FNN (h=5)								
acc	95.05%	96.14%	92.23%	95.56%	95.37%	94.15%	95.46%	94.57%
prec	12.96%	14.29%	8.43%	9.52%	15.63%	13.54%	11.11%	7.04%
rec	22.58%	12.70%	25.00%	10.53%	18.18%	23.64%	10.91%	9.09%
spec	96.65%	98.16%	93.77%	97.60%	97.41%	96.01%	97.69%	96.83%
FNN (h=7)								
acc	96.27%	94.98%	94.59%	96.01%	92.65%	90.87%	95.08%	95.83%
prec	11.86%	11.83%	10.89%	10.00%	8.87%	6.79%	10.94%	14.58%
rec	11.29%	17.46%	19.64%	8.77%	20.00%	20.00%	12.73%	12.73%
spec	98.15%	96.86%	96.31%	98.11%	94.57%	92.74%	97.26%	98.03%

4 CONCLUSION

This paper reveals great possibility of ML models to accurately predict the FD of Croatian SMEs. While MICE improved LR performances, it is not suitable for ML models. ML models have better recall and precision in out-of-sample forecasting, while FNNs with low number of hidden neurons yield optimal results regardless of the time span. This research contributes to the field of financial risk management by improving risk assessment, providing a model for an early warning system, enhancing risk management practices and by giving the framework for informed regulatory considerations. Future research may add non-financial predictors and other ML models for enhanced FD prediction, while dealing with missing and imbalanced data.

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THE EFFECT OF MARKET ATTENTION FLUCTUATIONS ON PORTFOLIO STRATEGY PERFORMANCE

Jialei Xiong
Technical University of Ostrava, Department of Finance
17. listopadu 2172/15, 708 00 Ostrava-Poruba, Czech Republic
E-mail: jialei.xiong@vsb.cz

Diana Barro
Ca' Foscari University of Venice, Department of Economics
San Giobbe, Cannaregio 873, 30121 Venice, Italy
E-mail: d.barro@unive.it

Martina Nardon
Ca' Foscari University of Venice, Department of Economics
San Giobbe, Cannaregio 873, 30121 Venice, Italy
E-mail: mnardon@unive.it

Abstract: This study explores the integration of Google Trends data to enhance investment portfolios. The utilization of search interest fluctuations as a metric for market attention has resulted in the formulation of three distinct strategies, each exhibiting a unique sensitivity to attention. Each strategy adjusts asset weights dynamically based on market attention, compared against a benchmark and naive strategy via risk-adjusted returns. An analysis of Hong Kong stock data reveals that strategies focusing on attracting attention outperform benchmarks, a finding that is further supported by the predominance of Google Trends in search traffic patterns within this context. The enhancement of these skills is contingent upon their capacity to respond expeditiously to alterations in attention. This research establishes a correlation between behavioural finance and portfolio optimization, providing evidence that real-time attention data can refine models. The proposed framework offers practical methodologies for the management of attention-driven risks.

Keywords: Market attention, Google Trends, Portfolio Strategy, Behavioural finance

1 INTRODUCTION

Traditional financial models commonly assume investors are rational and markets are efficient (Fama, 1970). Yet behavioural finance offers a different lens, highlighting how psychological factors shape decision-making and market outcomes (De Long et al., 1990; Barberis and Thaler, 2003). Notably, researchers have paid less attention to how shifts in market attention affect investment performance.

The internet age and big data advancements now allow us to measure this phenomenon quantitatively. As shown by Da et al. (2011), Google Trends data serves as a real-time indicator of market attention. Studies (Preis et al., 2013; Joseph et al., 2011; de Castro et al., 2023) have linked these trends to short-term price movements and trading activity.

This study tests whether integrating Google Trends-based attention metrics can enhance portfolio construction. We develop attention factors from search interest changes, incorporating them into a diversified strategy that rebalances to capture attention shifts. We compare three portfolio scenarios against a standard benchmark.

Our analysis aims to examine how market attention signals impact risk-adjusted performance, using selected metrics for evaluation. By merging behavioural insights with practical portfolio management, we seek to determine if tracking attention dynamics outperforms traditional methods.

2 DATA DESCRIPTION

The initial phase of the study involves defining the scope of the transaction. It has been established through preceding research that the Hang Seng Index comprises 82 constituent stocks. Due to temporal limitations, the focus was directed toward the top 40 stocks based on their current market capitalization. Following a thorough examination of the available data, 30 stocks were identified as having sufficient information to be included in the study. The search data for these stocks was manually collected using Google Trends every week, with the period covered ranging from January 2020 to December 2024. This five-year span denotes the maximum timeframe for which Google Trends provides weekly data. For this study, it is assumed that transaction costs are negligible, and portfolios are rebalanced every week.

To apply Google Trends effectively, it is first necessary to understand how the data works. It is important to note that the platform does not provide exact search volumes. Instead, it employs a relative format to illustrate trends in search interest over time. All data is standardized and scaled proportionally to reflect the total number of searches within a given period and region. Search interest is measured on a scale from 0 to 100, with 100 representing the peak popularity of a keyword within the selected time and location.

The primary objective of employing Google Trends in this study is to meticulously monitor search activity pertaining to the selected stocks. According to the findings of preceding studies, the most reliable method for retrieving relevant search data is the use of stock codes as keywords. Consequently, the format "HKG: [stock code]" was utilized for each stock. For instance, when searching for Tencent Holdings Limited, the term "HKG:0700" was entered. To enhance the precision of the findings, the "Theme" category in Google Trends was also selected when available.

3 METHODOLOGY

Google Trends constitutes a real-time analytical tool that measures daily variations in search query volumes on the Google search engine (Choi and Varian, 2012). The scale utilized for the purpose of measurement ranges from 0 to 100, with 100 representing the maximum query frequency within a specified time frame and geographic region.

As Google Trends provides relative rather than absolute measures of search interest, this study introduces the change in search interest metric to assess shifts in market attention. Specifically, the $\Delta GSV_{t,i}$ is calculated by comparing the current week's search interest to the median value of the preceding three weeks, as follows:

$$\Delta GSV_{t,i} = \frac{|GSV_{t,i} - Med(GSV_{t-1,i},\ldots,GSV_{t-3,i})|}{Med(GSV_{t-1,i},\ldots,GSV_{t-3,i})}. \quad (1)$$

Where $GSV_{t,i}$ represents the Google Trends search interest for stock *i* in week *t* This approach is capable of capturing short-term fluctuations in search interest, thus providing a quantifiable basis for analysing market behaviour.

This study investigates the application of market attention in portfolio strategy, drawing on the diversification strategy proposed by Kristoufek (2013). Kristoufek's "Google Trends strategy" employs search interest for stock-related keywords as a proxy for market attention. The core premise is that increased search interest correlates with higher stock risk. Consequently, within the portfolio optimization, stocks exhibiting higher search interest receive lower weights, while those with lower interest receive higher weights. In this study, we will replace Google Trends search interest to the changed Google Trends search interest. The weight for stock i at week t is given by:

$$w_{t,\mathrm{i}} = \frac{\Delta GSV_{t,i}^{-\beta}}{\sum_{j=1}^{n} \Delta GSV_{t,i}^{-\beta}}. \quad (2)$$

The exponent β (a power-law parameter) quantifies the sensitivity of the weighting to search frequency: $\beta > 0$ reduces weights for highly searched stocks, $\beta < 0$ increases them, and $\beta = 0$ results in equal weighting.

4 RESULTS

In the weighting strategy that was defined, the power-law parameter β was specified. This power-law parameter β varies in increments of 0.1 within the range of -1 to 1, and by incorporating it into Equation (2), the stock weights under different power-law parameters can be obtained. A positive value of the power-law parameter β is consistent with the expected application of the risk diversification method. To enable the comparison of performance changes under different settings of the power-law parameter β, a selection of four values was made from the 21 available options for analysis. In addition, to assess the possible consequences of shifts in market attention on investment strategies, three distinct investment strategies were devised:

- Investment Strategy A: Regardless of whether the change in Google Trends search interest for stock *i* in week *t* is positive or negative, it is directly incorporated into the Equation (1).
- Investment Strategy B: When the change in Google Trends search interest for stock *i* in week *t* is negative, it is used in Equation (1); otherwise, it is set to 0.
- Investment Strategy C: When the change in Google Trends search interest for stock *i* in week *t* is positive, it is used in Equation (1); otherwise, it is set to 0.

To this end, a naive strategy was implemented as a benchmark against which the performance of different investment strategies could be compared. (see Tab. 1)

Table 1: Investment portfolio performance results under different investment strategies

	Investment Strategy A				***Investment Strategy B***				***Investment Strategy C***				***Navie***
	β				*β*				*β*				
	-1	*-0.5*	*0.5*	*1*	*-1*	*-0.5*	*0.5*	*1*	*-1*	*-0.5*	*0.5*	*1*	
E(Ri)	34.804%	36.332 %	9.629%	7.988%	-6.186%	-6.632%	-6.886%	-6.840%	51.577 %	52.072 %	21.811 %	17.028 %	10.639%
STD	32.397 %	31.553 %	19.467 %	19.579 %	20.448 %	19.720 %	19.454 %	19.974 %	35.770 %	33.893 %	22.716 %	23.013 %	19.689%
Sharpe Ratio	1.003	1.079	0.376	0.289	-0.415	-0.453	-0.472	-0.458	1.378	1.468	0.859	0.640	0.423
MDD	37.113 %	33.900 %	25.611 %	24.677 %	51.139 %	49.128 %	46.489 %	47.111 %	46.723 %	41.399 %	24.393 %	24.966 %	29.119%
Carmer Ratio	87.576 %	100.389	28.613 %	22.963 %	-16.597 %	-18.183 %	-19.763 %	-19.404 %	105.465%	120.224%	79.980 %	58.989 %	28.633%
skew	0.819	0.180	0.095	0.171	-0.218	-0.299	-0.349	-0.340	-0.003	0.051	0.047	0.072	0.053
kurt	3.200	3.173	0.794	0.945	1.984	2.059	1.956	1.592	2.008	2.117	0.154	0.541	0.917

E(Ri) is representative of the expected return, *STD* is the standard deviation, and *MDD* is the maximum drawdown rate. As demonstrated in the results section, investment strategy C has been found to demonstrate exceptional performance in terms of expected return and Sharpe ratio but has relatively high standard deviations when compared with another strategy. A

comparison of the three strategies A, B, and C indicates that the differences are also quite noticeable when β is positive or negative.

5 CONCLUSION

Empirical evidence suggests that incorporating market attention information into investment strategies can enhance investment performance in most cases, particularly when market attention is trending upward, as investment portfolios often achieve higher returns. This phenomenon is consistent with the behavioral finance hypothesis of 'increased market attention leading to price increases,' suggesting that changes in attention can be an important signal of market sentiment and potential value.

Nevertheless, it is crucial to acknowledge that heightened attention frequently corresponds with increased market volatility and increased investment risk. This suggests that while market attention can offer opportunities for excess returns, it is also associated with significant uncertainty and risk. Consequently, a strategy that relies on market attention indicators as the sole basis for constructing investment portfolios may not be rational.

In terms of risk control, a diversified diversification adjustment strategy was introduced to decrease the impact of changes in attention on volatility. The findings indicate that when β is positive, the risk diversification strategy reduces volatility but also significantly decreases the return space.

The findings of the present study lend support to the incorporation of behavioural finance variables, with market attention being of relevance, into the portfolio optimization process, as a valuable supplementary information source. The dynamic adjustment mechanism, which is based on Google Trends data, provides new directions for traditional investment models and empirical evidence for the application of behavioural finance in actual asset management.

Acknowledgement
The research was supported by the Student Grant Competition of the Faculty of Economics, VSB - Technical University of Ostrava, project number SP2025/043.

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The 18th International Symposium on
Operational Research in Slovenia

SOR ’25

Bled, SLOVENIA
September 24 - 26, 2025

Session 4:

Location and Transport, Graphs and their Applications

GRAPH-BASED MODELING OF WAREHOUSE LAYOUTS BASED ON DATA FROM RELATION DATABASE FOR OPTIMIZING ORDER PICKING PATHS

Nejc Čelik
University of Maribor, Faculty of Organizational Sciences
4000 Kranj, Slovenia
E-mail: nejc.celik1@um.si

Andrej Škraba
University of Maribor, Faculty of Organizational Sciences
4000 Kranj, Slovenia
E-mail: andrej.skraba@um.si

Abstract: In this paper, we present a straightforward method for modelling warehouse layouts as graphs from data from relational database to enable efficient optimization of order picking paths. Order picking is a critical and time-consuming operation in warehouse logistics, often accounting for a significant portion of operational cost. By representing the warehouse as a graph structure, we simplify the pathfinding problem and create a flexible foundation for optimization techniques. The feasibility of the method is evaluated based on ability of GA to generate near optimal path and thus is evaluated demonstrating the effectiveness of graph-based modeling for order picking optimization.

Keywords: Graph Modeling, Warehouse, Optimization, Order Picking

1 INTRODUCTION

Order picking is one of the most labour-intensive and costly operations in warehouse logistics, often accounting for a significant portion of total operational expenses. While many optimization algorithms and methods have been proposed they usually focus on particular use cases and their effectiveness depends also on how the warehouse environment is modeled and set up [1]. A simple, general, flexible, data-driven, and efficient representation of the warehouse layout is therefore essential to support optimization in real world.

In this paper, we present an approach for modeling warehouse layouts with the goal of optimizing order picking paths. Our method uses data usualy available in relational database (or that can be interpreted from data automatically) that describes elements of a warehouse such as aisles, storage location, intersections (corners), and entrances. From this data, we construct a graph in which nodes represent key elements within the warehouse and edges denote traversable paths. Each node and edge include attributes such as, type (e.g., storage location, intersection, entrance, aisle), size (for storage locations) and distance (for edges). While many aproaches calculate distances between storage locations use Euclidean measuremen and rectilinear distance measurement [4] we aim to calculate distances by finding the shortest path between two nodes in graph. Such approach would be beneficial in more dynimac and random warehouse layouts compered to streight paralel and linear block aisles. With this we aim to get more robust method of finding shortest paths for use in real world enviroments. While graph theory has already been applied [2] in this paper we focus on how to extract vaible graph from relational database.

We demonstrate how different types of warehouse elements can be encoded as graph nodes and how distances can be represented as edges. Once the layout is modeled as a graph, we apply Genetic Algorithm (GA) [6]—to compute near-optimal paths for randomly generated order sets. We validate graph representation by evaluating generated paths.

This work highlights the value of graph-based modeling as a practical and extensible foundation for warehouse optimization tasks.

2 METHODOLOGY

This study follows the Design Science Research (DSR) [5] methodology to develop and evaluate an artifact for modelling warehouse layouts as graphs using data usually already available in WMS systems. The artifact integrates two main components: a graph construction derived from relational database schemas (including racks, aisles, intersections, and entry points), and an optimization module that applies Genetic Algorithm (GA) on the graph to solve find shortest order picking paths to validate effectiveness of graph representation. The warehouse layout is modelled as a weighted undirected graph where nodes/vertices represent locations (e.g., storage positions, intersections) and edges capture navigable paths with associated distances. The artifact is demonstrated using synthetic warehouse data and randomly generated order sets. This paper contributes a flexible, generalizable methodology for warehouse path optimization and highlights how graph modelling can be used for real-world logistics processes optimization.

3 RESULTS

The warehouse graph model is based on data that can typically be extracted from a WMS or approximated (e.g. distance between intersections, aisle width …) if not directly available. Distances between storage locations are derived from their physical dimensions—locations are spaced horizontally by their width, and vertically stacked locations by their height. In this model, stacked locations are connected only to the ground-level location directly below. Within each aisle, only ground-level (y=0) locations are horizontally interconnected, including connections between opposite sides of an aisle where the distance is equal to width of aisle. Aisle-to-node distances are set to the full aisle length, making such paths less likely to be optimal compared to those through intersections and storage nodes. Intersections are represented as nodes on aisles, connected to both the intersecting aisle and the nearest storage locations on those aisles. Consecutive intersections on the same aisle are also connected, forming continuous paths along aisles. Figure 1 represents very small warehouse with 60 storage locations, 4 intersections, 4 aisles and 1 entrance.

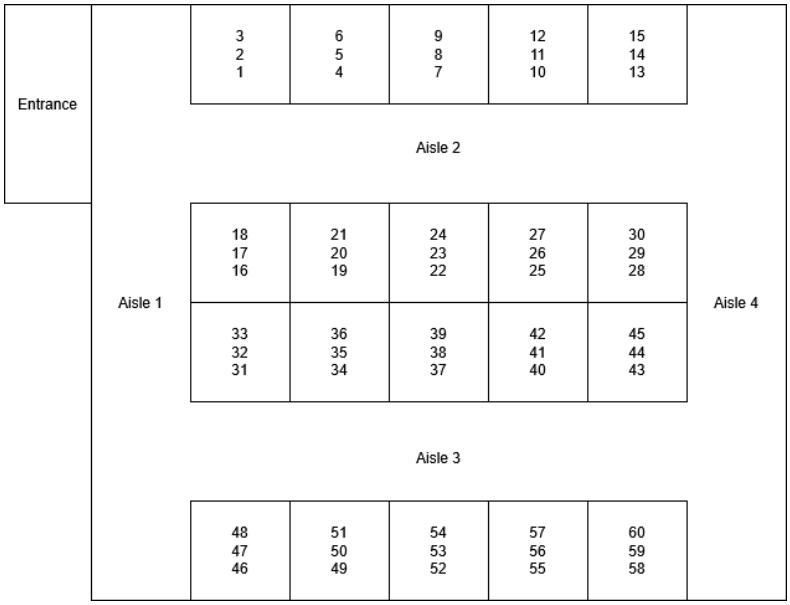


Figure 1: Sample 1 block warehouse

To generate efficient order picking routes, our approach first computes the shortest paths between all storage locations using Dijkstra's algorithm [3]. This step establishes the pairwise shortest distances required for the optimization phase. As the underlying graph structure is static for a given warehouse layout, these distances can be precomputed and reused across multiple picking scenarios, significantly reducing computational overhead during repeated optimizations. Once the distance matrix is obtained, we model the order picking task as a Traveling Salesman Problem (TSP), where the objective is to determine the most efficient sequence in which to visit the required locations. To solve this, we apply a Genetic Algorithm (GA).

Figure 2 presents visual representation of shortest path between 10 randomly selected locations generated with GA (generated sequence E->3->6->23->26->16->31->32->57->56->53->E).

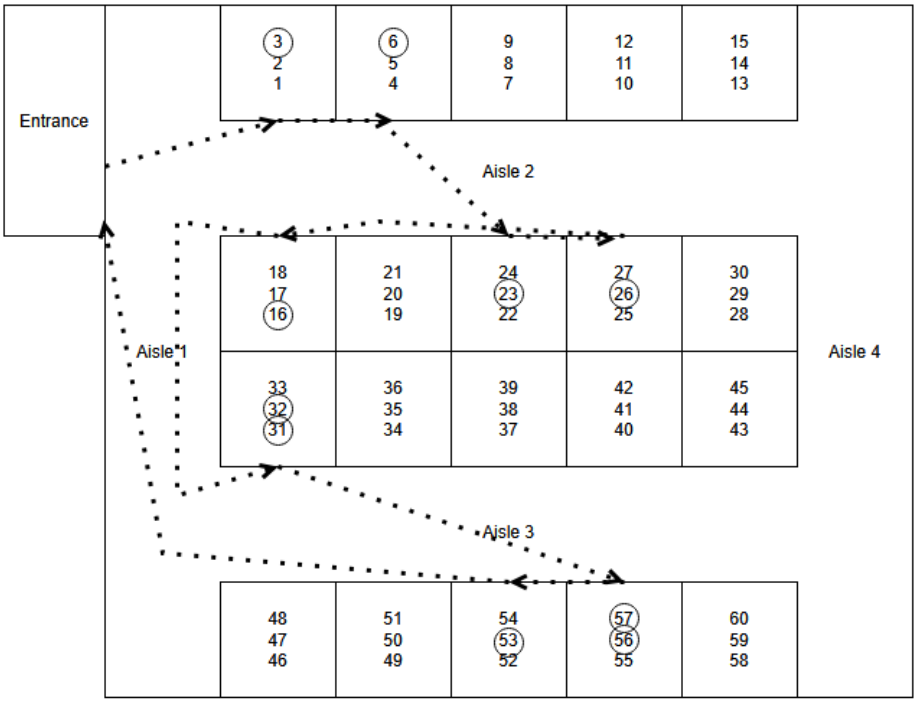


Figure 2: Example path by GA

4 CONCLUSIONS AND FUTURE WORK

This paper presents a practical and extensible approach to modeling warehouse layouts as graphs for the purpose of optimizing order picking paths. By leveraging structured warehouse data to construct a weighted graph model, we enable the application of advanced optimization techniques, such as Genetic Algorithm (GA), to solve the order picking problem efficiently. The graph-based representation captures key spatial and structural relationships within the warehouse, providing a reusable and computationally effective foundation for logistics optimization. While graph representation and advanced optimization algorithms are not needed in 1 block warehouses it may provide benefit in bigger warehouses.

The proposed method demonstrates promising results in terms of route efficiency and computational performance using synthetic data, highlighting its potential applicability in real-world warehouse environments. Importantly, the modular design of the system supports future expansion and integration with existing Warehouse Management Systems (WMS).

Future work will focus on extending this model in several directions: incorporating Graph Neural Networks (GNNs) and Deep Reinforcement Learning (DRL) techniques to enhance adaptive and learning-based path optimization; supporting order batching and priority-based picking strategies; modeling congestion dynamics and multi-agent coordination for more realistic warehouse operations; and enabling real-time updates to reflect dynamic changes in inventory or picker status. Future work should also aim to also model warehouse goods and orders as graph. These enhancements aim to move the system closer to deployment-ready solutions capable of operating in complex, high-throughput warehouse environments.

Acknowledgement

The authors acknowledge the financial support from the Slovenian Research and Innovation Agency (research core funding No. P5-0018), the Ministry of Higher Education, Science, and Innovation of the Republic of Slovenia as part of the Next Generation EU National Recovery and Resilience Plan (Grants No. 3330-22-3515; NOO No: C3330-22-953012), and the Erasmus+ program of the European Union (Grant No. 2021-1-MK01-KA220-HED-000027646

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GRAPH-BASED MODELLING OF HIGH-SKILLED INTERNAL MIGRATION PATTERNS IN SLOVENIA

Samo Drobne[a], **Alberto Garre**[b] and **Eloy Hontoria**[c]
[a] University of Ljubljana, Faculty of Civil and Geodetic Engineering, Jamova cesta 2, Ljubljana, Slovenia
[b] Universidad Politécnica de Cartagena, Department of Agronomical Engineering & Institute of Plant Biotechnology, Paseo Alfonso XIII 48, Murcia, Spain
[c] Technical University of Cartagena, Department of Business Economics, Calle Real 3, Cartagena, Spain
samo.drobne@fgg.uni-lj.si, alberto.garre@upct.es, eloy.hontoria@upct.es

Abstract: This study uses a network-based approach to analyse the internal migration flows of highly skilled persons in Slovenia from 2015 to 2023. Using migration data at the municipality level, directed weighted networks were constructed, with the nodes representing the municipalities and the edges the migration flows. Functional clusters (FCs) were identified using the *spinglass* community detection algorithm, which was selected for its robustness in sparse network structures. To assess regional attractiveness, Kleinberg's authority scores were calculated annually for both general and highly skilled networks. This methodological framework enables the detection of stable migration patterns and the assessment of temporal dynamics of regional pull factors and provides a scalable model for migration analyses in other contexts.

Keywords: high-skilled population, network theory, functional regions, internal migration, Slovenia.

1 INTRODUCTION

In the context of a knowledge-based economy, the internal migration of highly skilled individuals plays a pivotal role in shaping regional development and innovation potential. While international migration has received considerable scholarly attention, internal migration - especially of the highly educated - remains underexplored in smaller European countries such as Slovenia. This study aims to address this gap by identifying functional clusters (FCs) of high-skilled internal migration in Slovenia and examining their spatial and temporal dynamics.

Theoretical frameworks on internal migration often distinguish between forced and voluntary migration, with the latter driven by aspirations for improved socioeconomic status. According to disequilibrium theories, migration acts as a mechanism to balance regional disparities in employment and income [9]. However, empirical evidence suggests that migration patterns are shaped by a complex interplay of economic, social, and cultural factors, which vary significantly across countries and demographic groups.

In the European context, studies have shown that employment opportunities are a primary driver of internal migration, particularly among young and educated individuals [3, 6]. In Slovenia, internal migration rates are relatively modest compared to larger EU countries, yet regional disparities in economic development, infrastructure, and access to services persist. These disparities may influence the mobility of high-skilled workers, who are often more responsive to labor market signals and more likely to relocate multiple times throughout their careers [4].

The migration of highly educated individuals is of particular interest due to its potential to enhance regional innovation capacity, productivity, and entrepreneurship [7, 10]. However, the lack of disaggregated migration data by education level in many national statistics - including Slovenia - poses a challenge for empirical analysis. To overcome this, researchers have increasingly turned to alternative data sources and network-based methodologies to model migration flows and identify FCs and/or functional regions (FRs) [2, 5].[1]

This paper builds on these methodological advances by applying network theory to analyse internal migration patterns of highly skilled individuals in Slovenia. By identifying FCs based on migration flows, we aim to uncover spatial structures that are not necessarily aligned with

[1] Functional clusters (FCs) are groups of functionally related, not necessarily neighbouring, basic spatial units. Functional regions (FRs) are groups of functionally related, neighbouring basic spatial units. Both the FCs and the FRs homogeneously cover the entire area analysed.

administrative boundaries but reflect actual socio-economic linkages. The findings contribute to a better understanding of regional labour dynamics and provide insights for policy measures aimed at attracting and retaining talent in less developed areas.

2 METHODOLOGY

To analyze the internal migration of highly skilled individuals in Slovenia, we constructed directed weighted networks where nodes represent municipalities and edges represent migration flows between them. The edge weights correspond to the number of individuals moving from one municipality to another within a given time period. Two types of networks were created: one for the general population and another filtered to include only high skilled migrants.

Migration data were obtained from the Statistical Office of the Republic of Slovenia [13]. To protect personal data, some edge weights were masked (i.e., weights 1, 2, 3, and 4). For each year, about 75% of the edges were masked. We replaced these data with a weight of 2. The networks were constructed for each year from 2015 to 2023 to capture temporal dynamics. All network analyses were conducted in R (version 4.3.0) using the *igraph* package [1].

FCs were identified using community detection algorithms from network theory. After testing several methods, the *spinglass* algorithm was selected for its robustness in detecting modular structures in sparse networks [11]. This algorithm models the network as a spin glass system and optimizes the Hamiltonian function to assign each node to a community. The algorithm was applied to each annual network, and the resulting clusters were named after the municipality with the highest population within each cluster. To ensure temporal consistency, each municipality was assigned to the FC it was most frequently associated with over the study period.

To quantify the attractiveness of each region, we computed Kleinberg's authority scores for each node in the network [8]. This metric reflects the extent to which a region attracts migrants from other regions, with higher scores indicating stronger pull factors. Authority scores were calculated for both the general and high-skilled networks across all years, allowing us to track changes in regional attractiveness over time.

3 RESULTS AND DISCUSSIONS

The comparison of the results of the FC modeling shows interesting features: FCs over the years 2015–2023 are generally stable and there are no major differences between FCs of highly skilled migrants and FCs of the total population. For both analyzed groups of internal migrants, similar FCs are formed for Maribor (70), Slovenj Gradec (112), Celje (11), Ljubljana (61) and Nova Gorica (84). The only peculiarity in the generalization of the flows of highly skilled migrants is the municipality of Krško (54), which belongs to the "virtual" FC of Novo mesto in the period under study, and FC of Kranj (52), which is only formed if all internal migrant flows are considered. The latter reveals the peculiarities of migration patterns in the FC of Ljubljana (61): More highly educated migrants were located in the southeast and moved mainly to the area of the FC of Ljubljana (61), while the general pattern of migrants emphasized a particular FC of Kranj (52) in northwestern Slovenia.

Fig. 1 shows the FCs in general for highly skilled and all internal migration patterns in Slovenia in the period 2015–2023. The only years in which there are major differences are 2019 and 2020 for highly skilled migrants. This can also be seen in the graphs of the number of municipalities in the FCs as a function of time; see Fig. 2. The problem of the sudden increase in internal migration flows in 2020 compared to a year earlier is a known methodological problem in monitoring internal migration data in the Republic of Slovenia. The high number of internal migration in 2020 was largely related to the measures taken to contain the spread of Covid-19. Namely, after the inter-municipal relocation bans in March and October 2020, a significant number of residents decided to register a change of residence (mostly temporary residence in another municipality), which was statistically recorded as internal migration [12].

To assess municipal attractiveness, we calculated Kleinberg's authority scores for general and highly skilled migration networks across all years. These scores indicate how strongly each municipality attracts migrants and allow us to track changes in attractiveness over time. The most important municipalities are - in this order - Ljubljana (61), Maribor (70), Domžale (22), Koper (50) and Celje (11), for both the highly skilled and the general population. Fig. 3 shows the authority score of highly qualified internal migrants compared to the general population.

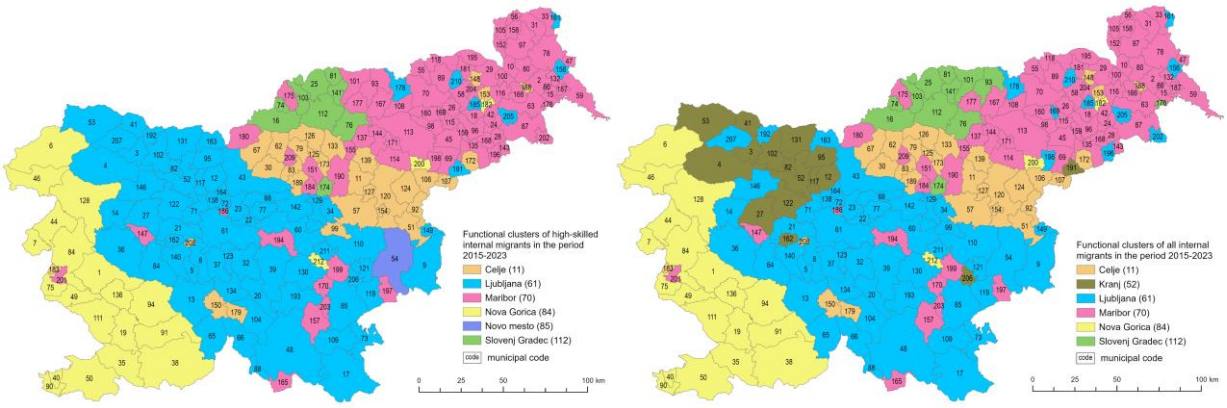


Figure 1: Functional clusters in general of highly skilled (left) and all (right) internal migration patterns in Slovenia in period 2015–2023

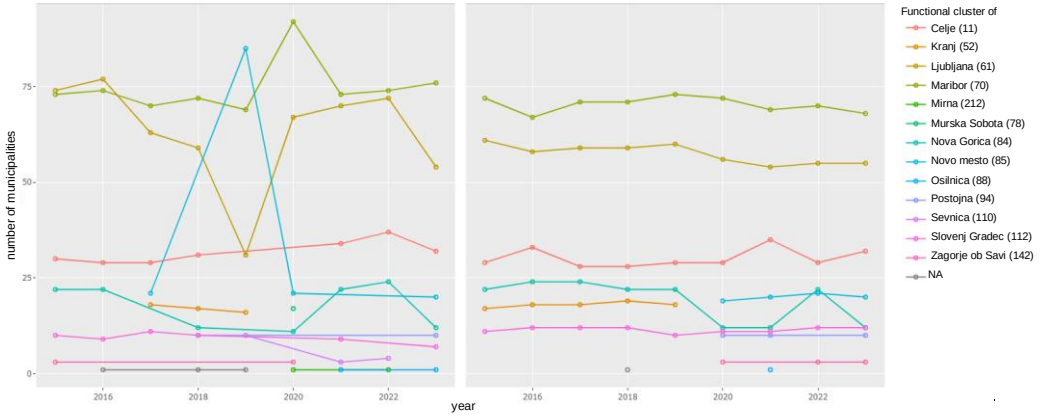


Figure 2: The number of municipalities in a functional cluster as a function of time for highly skilled migrants (left) and for all migrants (right) in the period 2018–2023

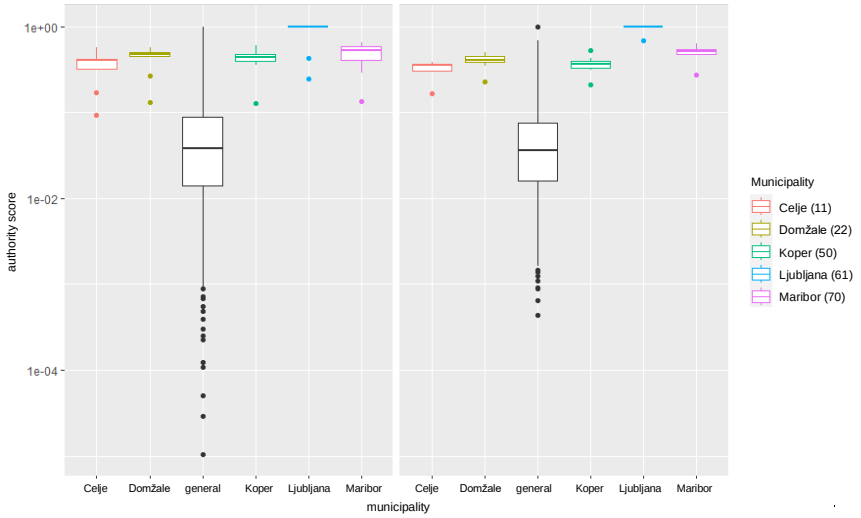


Figure 3: The authority score of the five most important municipalities in Slovenia by number of inhabitants in the networks calculated for highly skilled migrants (left) and for all migrants (right) in the period 2018–2023

4 CONCLUSIONS

This study demonstrates the effectiveness of graph-based modelling in uncovering stable and nuanced patterns of high-skilled internal migration in Slovenia. By identifying functional clusters and calculating authority scores, we reveal persistent regional structures and shifting attractiveness dynamics that transcend administrative boundaries. The findings highlight Ljubljana, Maribor, and Celje as consistent migration hubs, while also exposing temporal anomalies linked to external events such as the COVID-19 pandemic. The methodological framework offers a scalable approach for analysing internal migration in other national contexts and provides valuable insights for regional development and talent retention strategies.

Acknowledgement
Samo Drobne acknowledges the financial support from the Slovenian Research Agency through the research project A Regional Model of Social Infrastructure networks for the Care of Older Adults in the Community and Institutions, including the Planning of Human Resources (L7-60163) and from research core funding Earth Observation and Geoinformatics (P2-0406). Alberto Garre acknowledges being funded by a Ramon y Cajal Fellowship (RYC-2021-034612-I). Eloy Hontoria acknowledges the financial support from the project PID2020-116318RB-C32 by the Spanish Ministry of Science and Research, MCIN/AEI/10.13039/501100011033, as well as by the AGROALNEXT programme and was supported by MCIN with funding from European Union NextGenerationEU (PRTR-C17.I1) and by Fundación Séneca with funding from Comunidad Autónoma Región de Murcia (CARM).

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DESIGNING AN EFFICIENT NETWORK UNDER COST UNCERTAINTY SUBJECT TO CONSTRAINED BUDGET AND MULTIPLE OBJECTIVES

Robert Hlavatý
Charles University, Faculty of Mathematics and Physics, Ke Karlovu 2027/3, 121 16 Praha 2, Czech Republic

Abstract: We discuss a specific instance of the maximum flow problem in a network with multiple objectives. The edge capacities of the network are not explicitly given and are subject to predefined interval bounds. Moreover, the unit costs are known for the edge capacity building, and the total costs are constrained by the available budget. The building costs, however, are subject to uncertainty and may vary by predefined deviations, too. We intend to propose an efficient network capacity design that respects all objectives (one of which is always maximising the network flow from source to terminal vertex) and a given budget. The proposed design is robust to cost changes under a given degree of uncertainty provided by a decision-maker. We take advantage of the DeNovo optimisation framework and Γ-robustness principles. We compare our approach with classic multiobjective optimisation techniques through numerical experiments on an artificial example. It occurs that none of the approaches is superior to the others in a multiobjective sense, but the one we propose yields somewhat better results in average efficiency score.

Keywords: DeNovo optimisation, maximum flow, network design, robust optimisation

1 INTRODUCTION

The classic network flow problem originally introduced by Ford and Fulkerson [6] has seen countless modifications and has been extensively studied from various perspectives. The survey paper by [4] shows that the topic is still significant nowadays. In this contribution, we follow our previous research on the topic [7], [8] and enrich our methodology by incorporating robustness towards uncertainty into the optimisation framework and also comparison with other multiobjective optimisation techniques.

The presence of uncertainty in the network flows originates in practical applications. There are various possibilities for capturing the uncertainty in the literature. A stochastic approach is taken by [10], who combine it with robust optimisation. Robust network flows are also discussed by [9]. Apart from the above, the fuzzy framework can also be implemented, as seen in [5]. Since we work with interval-based data, the interval analysis is also a logical approach to dealing with uncertainty, as shown by [1].

Whichever approach is taken, the papers in the literature are focused on solving the network flow problems under uncertainty and seeking particular solutions. We provide a different perspective on the network flow problem, and we aim to compute proper edge capacities respecting the available budget so that the resulting network has the properties desired by a decision-maker in the multiobjective sense. Thus, our approach is more related to metaoptimisation, through which we seek an efficient design of the network optimisation problem itself. Practically speaking, we propose a way to set the edge capacities in a way that the best possible objective values are achieved while the budget for doing so is minimised, even under uncertain costs. The approach in this paper draws from Denovo optimisation principles introduced by Zeleny [11] in 1986, the Γ-robust optimisation approach by Bertsimas and Sim [3] in 2003, and they are combined to serve this paper's purposes.

2 METHODS

A generalised network flow problem with r objectives is formulated as a linear programming problem (1). Given a directed graph $G = (V, E)$, unknown edge capacities $y_{ij} \geq 0$ for each edge $(i, j) \in E$, source vertex $s \in V$ and terminal vertex $t \in V$, the following formulation maximises the flow from s to t and the flow on the edge $(i, j) \in E$ is x_{ij}. Additional $r - 1$ generic objectives are considered and are maximised (or minimised) over all edges $(i, j) \in E$ for given edge weights w_{ij}^k for every k-th objective.

$$\begin{array}{ll} \max \zeta_1 = \sum_{j:(s,j)\in E} x_{sj} & \text{s.t.} \quad \sum_{j\in V} x_{ji} - \sum_{j\in V} x_{ij} = 0 \quad \forall i \in V \setminus \{s,t\} \\ \max \zeta_k = \sum_{(i,j)\in E} w_{ij}^k x_{ij} \quad k = 2,\ldots,r & \qquad 0 \leq x_{ij} \leq y_{ij} \quad \forall (i,j) \in E \\ & \qquad l_{ij} \leq y_{ij} \leq u_{ij} \\ & \qquad u_{ij} > l_{ij} \geq 0, \forall (i,j) \in E \end{array} \tag{1}$$

In addition, each edge capacity y_{ij} is subject to unit cost q_{ij}, and we have a limited budget B for building the edge capacities, thus $\sum_{(i,j)\in E} q_{ij} y_{ij} \leq B$. In the spirit of DeNovo optimisation, we first solve the problem (1) as a single-objective problem with every objective $\zeta_1, \ldots, \zeta_r$ under a given budget and robust setting:

$$\begin{array}{ll} \max \zeta_k \quad k = 1,\ldots,r & \text{s.t.} \quad \sum_{j\in V} x_{ji} - \sum_{j\in V} x_{ij} = 0 \quad \forall i \in V \setminus \{s,t\} \\ & \qquad 0 \leq x_{ij} \leq y_{ij} \quad \forall (i,j) \in E \\ & \qquad l_{ij} \leq y_{ij} \leq u_{ij} \\ & \qquad u_{ij} > l_{ij} \geq 0, \forall (i,j) \in E \\ & \qquad \sum_{(i,j)\in E} q_{ij} y_{ij} + \sum_{(i,j)\in E} p_{ij} + \Gamma z \leq B \\ & \qquad p_{ij} + z \geq \delta_{ij} y_{ij} \quad \forall (i,j) \in E \\ & \qquad z \geq 0, p_{ij} \geq 0 \quad \forall (i,j) \in E \end{array} \tag{2}$$

where Γ represents the number of unspecified coefficients q_{ij} that may deviate by δ_{ij} and p_{ij}, z are auxiliary variables originating in the robust formulation of [3]. Solving (2) r times yields individual ideal solutions $\zeta_1^*, \ldots, \zeta_r^*$. Next, we solve the problem (3) to minimise the budget for edge capacity building while achieving ideal objective values, under the robust setting again:

$$\begin{array}{ll} \min \sum_{(i,j)\in E} q_{ij} y_{ij} + \sum_{(i,j)\in E} p_{ij} + \Gamma z & \text{s.t.} \quad \zeta_k \geq \zeta_k^* \quad k = 1,\ldots,r \\ & \qquad \sum_{j\in V} x_{ji} - \sum_{j\in V} x_{ij} = 0 \quad \forall i \in V \setminus \{s,t\} \\ & \qquad 0 \leq x_{ij} \leq y_{ij} \quad \forall (i,j) \in E \\ & \qquad l_{ij} \leq y_{ij} \leq u_{ij} \\ & \qquad u_{ij} > l_{ij} \geq 0, \forall (i,j) \in E \\ & \qquad p_{ij} + z \geq \delta_{ij} y_{ij} \quad \forall (i,j) \in E \\ & \qquad z \geq 0, p_{ij} \geq 0 \quad \forall (i,j) \in E \end{array} \tag{3}$$

Solving (3) provides a robust network design respecting all objectives. Problem (3) may not have a feasible solution due to the first constraint set, which is generally very strict. In that case, it is necessary to replace all ζ_k^* by some achievable values, say, ζ_k^{**} that can be obtained, i.e. by the STEP method [2]. Detailed implementation of the STEP method in the DeNovo framework can be found in [7].

3 RESULTS

3.1 Example

We use the following network topology for demonstration, as in the Figure 1. The network is a graph $G(V, E)$ with $|V| = 8, |E| = 15$ a source vertex S, terminal vertex T and all edges have the same (wlog) predefined lower and upper bounds $l_{ij} = 10, \forall(i,j) \in E$ and $u_{ij} = 50, \forall(i,j) \in E$.

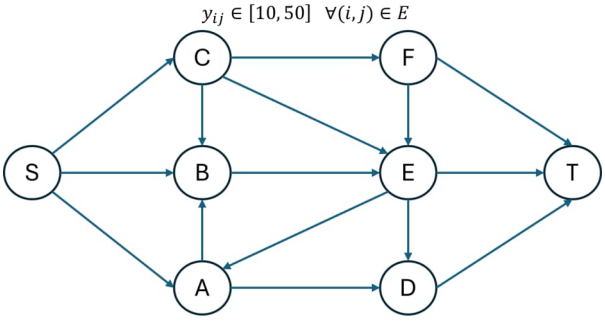


Figure 1: Network topology

We assume only two objectives for simplicity, while the first one is maximising the flow from S to T, which is represented by the following objective function:

$$\zeta_1 = \max\ (x_{sa} + x_{sb} + x_{sc}) \tag{4}$$

and the second objective minimising the risk score over all edges:

$$\begin{gathered} \zeta_2 = \min\ \boldsymbol{w^T x} \\ \boldsymbol{w^T} = (2, 6, 10, 8, 5, 10, 1, 10, 6, 4, 7, 10, 1, 1, 5) \\ \boldsymbol{x^T} = (x_{sa}, x_{sb}, x_{sc}, x_{ab}, x_{ad}, x_{be}, x_{cb}, x_{ce}, x_{cd}, x_{dt}, x_{ea}, x_{ed}, x_{et}, x_{fe}, x_{ft}) \end{gathered} \tag{5}$$

The components of $\boldsymbol{w}(w_{ij})$ were randomly generated over uniform distribution on $[1, 10] \cap \mathbb{Z}$. The goal is to compute appropriate edge capacities y_i within the predefined intervals with respect to both objectives and the constrained budget B:

$$\begin{gathered} \boldsymbol{q^T} = (64, 1, 16, 4, 67, 18, 67, 37, 61, 17, 65, 14, 13, 67, 63) \\ \boldsymbol{q^T y} \leq B \end{gathered} \tag{6}$$

The costs $\boldsymbol{q}(q_{ij})$ were randomly generated over a uniform distribution on $[1, 100] \cap \mathbb{Z}$, and we assume that a certain number Γ of unspecified costs q_i may increase by 10 %. The value of 10 % was chosen at random. Note that a different value of allowed increase can be set for each coefficient q_i individually.

3.2 Computation experiments & Conclusions

The example was computed for different budget availability levels ranging between $[9000, 19000]$ and a given level of uncertainty Γ, a corresponding robust flow and risk value were obtained. We have also applied two classic multiobjective approaches to the problem: goal programming and the weighted sum approach[1]. From the results of all three methods (ours, GP and WSA), two indices *flow/budget* (I_1) and *(risk*budget)* (I_2) were computed and normalised to range between $[0, 1]$ to allow performance comparison of all methods. In Table 1, the values of $I_1 + I_2$ (rounded) are shown for all methods and different levels of uncertainty.

[1]Due to lack of space, the models are not explicitly provided here but can be found in [8]

Uncertainty level	Proposed approach	Goal programming	Weighted sum
$\Gamma = 0$ (0 %)	0.93	0.74	0.76
$\Gamma = 5$ (33.$\bar{3}$ %)	0.86	0.70	0.75
$\Gamma = 10$ (66.$\bar{6}$ %)	0.93	0.69	0.75
$\Gamma = 15$ (100 %)	0.89	0.70	0.75

Table 1: Average performance index

Even though the individual results of the methods are generally non-dominated, our approach shows better average performance with computed solutions. It remains to be seen whether the better performance of our approach can be generalised, and indeed, more general computation experiments are needed.

4 Acknowledgements

This paper was supported by the Czech Science Foundation (GAČR) project No. 25-15714S.

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IMPROVING THE CREDIBILITY OF THE EMERGENCY MEDICAL SYSTEM MODEL

Jaroslav Janáček
University of Žilina, Faculty of Management and Informatics,
Univerzitná 8215/1, 010 26 Žilina, Slovak Republic,
E-mail: jaroslav.janacek@fri.uniza.sk

Marek Kvet
University of Žilina, Faculty of Management and Informatics,
Univerzitná 8215/1, 010 26 Žilina, Slovak Republic,
E-mail: marek.kvet@fri.uniza.sk

Abstract: The standard stochastic model of emergency medical system design assumes that the rate of randomly emerging demands for service is proportional to the population of serviced community. This assumption is reflected by the objective of the optimization method, which minimizes the response time of the emergency system to a request for service by deployment of disposable ambulances over the set of communities of the served region. This paper focuses on credibility of the model based on the above assumption from the viewpoint of the obtained objective function value and the ambulance locations. The associated research is based on the comparison of the optimal system designs, where the first one is obtained by the standard approach and the second one uses the real numbers of calls.

Keywords: Discrete location science, multiple facility locations, optimization model credibility

1 INTRODUCTION

This paper deals with a special class of discrete network location problems with application in emergency services [2, 3]. Common mathematical models [1] are here adjusted into the radial form [4] and extended in such a way that it enables us to locate more facilities at one place making use of generalized objective function minimizing the average response time of the system to random calls [5]. The main scientific question of this study consists in improvement of the model credibility by properly selected weight coefficients for each served community,

2 EMERGENCY MEDICAL SYSTEM MODEL WITH STOCHASTIC ELEMENTS

The Emergency Medical System provides inhabitants of a given region with first aid service. A demand for service emerges randomly at a community location and it is satisfied by an ambulance, which is sent to the community location from a service center. To complete the model, a finite set J of community locations and a finite set I of possible service center locations are assumed. Time necessary for an ambulance to reach the community j from service center location i is denoted by t_{ij}. The EMS design consists in deployment of given number p of ambulances in the set I of possible service center locations. The deployment is modelled by series of integer variables y_i for each $i \in I$. The variable y_i gives the number of ambulances placed at the possible center location i. The associated constraints put on the variables are:

$$y_i \in Z^+ \quad for\ i \in I \tag{1}$$

$$y_i \geq f_i \quad for\ i \in I \tag{2}$$

$$\sum_{i \in I} y_i = p \tag{3}$$

The obligatory constraints (1) assure that only integer numbers of ambulances are considered. The constraints (2) cause centers at densely populated communities have to be granted by a

minimal number of ambulances and the constraint (3) allows to deploy only disposable fleet of ambulances. The design is described by $\mathbf{y}=\{y_i: i\in I\}$, which satisfies constraints (1)-(3). The set of all feasible designs is denoted by $\mathbf{Y}$. The problem consists in minimizing the average response time of the system of service centers to a demand for service. Comparing currently modelled service systems to ordinary systems modelled as *p*-median or weighted *p*-median problems, several substantial differences can be found. Ordinary models assume that a demand is assigned to the nearest service center. This assumption does not hold for the considered emergency system because all ambulances of the nearest service center may be occupied by earlier raised demands for service. Dispatchers of the emergency system solves this situation by assigning the demand to the second or third nearest center. It means that a raised demand is assigned to the first available service center. Probability of the case that an ambulance is at disposal is denoted by q_1 and probability of the case that the *k*-th nearest ambulance is the first available one is denoted by q_k for $k = 1, \ldots, r$. Let us introduce operator $\min_k(M)$ which gives the *k*-th minimal value from the series *M* of possibly repeated values. Symbol $M(t_{ij}, y_i)$ denotes the series of distances t_{ij}, where the value of t_{ij} occurs y_i times. Thus, the result of $\min_k(M(t_{ij}, y_i))$ is the *k*-th minimal member of the series. The last element of the model is the weight w_j of the individual community *j*. It describes the frequency of raised demands for service at the location *j*. After these preliminaries the average response time can be modelled by (4).

$$\frac{\sum_{j\in J} w_j \sum_{k=1}^{r} q_k \min_k \left(M\left(t_{ij}, y_i\right)\right)}{\sum_{j\in J} w_j} \tag{4}$$

The problem of EMS design can be formulated as minimization of (4) subject to $\mathbf{y}\in\mathbf{Y}$.

3 APPROXIMATION OF DEMAND FREQUENCY

Considering randomness of the system users' demands, it can be admitted that the distribution of demands for service plays a substantial role in the accuracy of the model. Two approaches to determination of demand frequency of the individual community can be distinguished. The first approach uses global information about the rate λ of calls (demands for service) in the region and computes it to the number of calls per person and per given time interval. Then the number of expected calls from the location *j* can be estimated by $w_j=b_j\lambda$, where b_j is the number of inhabitants of the community *j*. The objective function (4) can be rewritten to (5).

$$\frac{\sum_{j\in J} w_j \sum_{k=1}^{r} q_k \min_k \left(M\left(t_{ij}, y_i\right)\right)}{\sum_{j\in J} w_j} = \frac{\sum_{j\in J} b_j \sum_{k=1}^{r} q_k \min_k \left(M\left(t_{ij}, y_i\right)\right)}{\sum_{j\in J} b_j} \tag{5}$$

The second approach comes from the observation of the calls generated by an individual community during given period. If the rate of community *j* is denoted by e_j, then the average time distance can be computed according to (6).

$$\frac{\sum_{j\in J} e_j \sum_{k=1}^{r} q_k \min_k \left(M\left(t_{ij}, y_i\right)\right)}{\sum_{j\in J} e_j} \tag{6}$$

The accuracy of the approaches depends on the following factors. Regarding the first approach, one can ask how strong the proportionality is between the number of inhabitants and the

estimated frequency of calls. The confidentialness of the second approach consists in the period in which the observation is made. Comparison of the approaches is reported in the next section.

4 NUMERICAL EXPERIMENTS

Benchmarks for this study were obtained from current EMS servicing the self-governing region of Žilina in Slovakia. The road network of the region includes 315 nodes corresponding to various communities represented by cities, villages and other dwelling places. The region produces average demand for service at 0.036517 per minute per one inhabitant. The total number of inhabitants is 687174. It is considered the number p=36 ambulances to service the region from the network nodes feasible for ambulance location. Different problems were obtained by setting different percentage β of ambulances, which are assigned to the most populated communities directly without any optimization. The basic approach to direct assignment consists in computing the rate of calls per one ambulance. A community generating a bigger rate of calls is assigned by the minimal number of ambulances necessary to cover the calls. The minimal number of ambulances for community j is determined according to (7).

$$f_j = \left\lfloor \frac{w_j p}{\sum_{i \in I} w_i} \right\rfloor \tag{7}$$

If this total number of the preassigned ambulances represents percentage, which is less than, then the numbers of preassigned ambulances is increased according to the rule (8).

$$f_j = \left\lfloor \alpha + \frac{w_j p}{\sum_{i \in I} w_i} \right\rfloor \tag{8}$$

Parameter α is obtained as maximal real number submit the constraints (9).

$$\beta \geq \sum_{j \in J} \left\lfloor \alpha + \frac{w_j p}{\sum_{i \in I} w_i} \right\rfloor / p \tag{9}$$

This way, the set of 8 problems was obtained by changing β from 0.25 up to 0.95 with step of 0.1. The probabilities q_k used in (4) were obtained in the following way: q_1 = 0.798474 was computed from real EMS data. The values of the associated parameters q_k for k = 2, …, 5 follow q_2 = 0.160913, q_3 =0.0324282, q_4 = 0.00653513 and q_5 = 0.00164939. The weights w_j were determined either as b_j or e_j. The problem minimizing (4) subject to $\mathbf{y} \in \mathbf{Y}$ were solved by the optimization software FICO Xpress 8.3 (64-bit, release 2017). The models were run on a PC equipped with the Intel® Core™ i7 8565U 1.80 GHz processor and 128 GB RAM. The concise results of numerical experiments are plotted in Table 1. Each row of the table corresponds with one problem determined by value of β. The columns are divided into three parts denoted by symbols **b**, **e** and **MD**. Part **b** contains results for the objective function (5) and part **e** contains the results for the objective function (6). Each of the parts **b** and **e** consists of three columns denoted by avgRT, CT and **f** respectively. The column avgRT reports on the optimal average response time in minutes modelled by the associated objective function. Column CT gives the computation time in seconds and column **f** reports on the number of preassigned ambulances. The part **MD** contains only one column, in which Manhattan distance

between optimal solutions from sections **b** and **e** is reported. The Manhattan distance of associated optimal solutions $\mathbf{y^b}$ and $\mathbf{y^e}$ is computed according to (10).

$$MD\left(\mathbf{y^b}, \mathbf{y^e}\right) = \sum_{i \in I} \left| y_i^b - y_i^e \right| \tag{10}$$

Table 1: Results of numerical experiments

	b			**e**			
β	avgRT	CT[s]	**f**	avgRT	CT[s]	**f**	MD
0.25	6.445	1.857	9	6.413	3.316	10	18
0.35	6.531	1.629	12	6.420	3.372	12	20
0.45	6.665	1.916	15	6.428	1.474	16	20
0.55	6.624	1.430	18	6.542	1.397	19	16
0.65	6.657	1.374	23	6.590	1.291	23	10
0.75	6.698	1.335	27	6.698	1.265	26	20
0.85	6.743	1.231	30	6.698	1.299	26	18
0.95	7.011	1.193	34	6.626	1.266	34	16

Observing the resulting average response times, several characteristics can be noticed. The average times trend to increase with increasing number of preassigned ambulances. It can be explained by the fact that the additional assignment of ambulances is performed at communities, which have originally assigned at least one ambulance. This additionally assigned ambulance has not such an effect as if it is used in the optimization process. It can be stated that the forced assignment based on (8) loses substantiation for percentage β over 35% (0.35) of the number of disposable ambulances. As concerns the differences between the deployments performed by **b** and **e** approaches in the range of the recommendable percentage β, the Manhattan distance is under 20, i.e. at most 10 of 36 ambulances are located differently.

5 CONCLUSIONS

This paper was focused on improving credibility of the emergency medical system model. The obtained results show that credibility can be increased by using observation of the number of calls generated by individual communities during a period that must be large enough instead of its approximation based on the number of community inhabitants.

Acknowledgement
This paper was supported by the Slovak national grant VEGA 1/0220/25 - Regional equality in access to public rescue services provided from service centers on the transport network.

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TRANSFORMING SUPPLY CHAINS THROUGH REVERSE LOGISTICS: SCENARIO PLANNING WITH HYBRID COLLECTION-RECYCLING FACILITIES

Masoud Yavari
PhD Candidate, University of Ljubljana, School of Economics and Business,
Ljubljana, Slovenia
E-mail: yavari.masoud72@gmail.com

Marko Jakšič
Associate Professor, University of Ljubljana, School of Economics and Business,
Ljubljana, Slovenia
E-mail: marko.jaksic@ef.uni-lj.si

Abstract: The rapid growth in electronic waste (e-waste) generation underscores the need for efficient reverse logistics systems. This paper presents a mathematical model for optimizing a reverse supply chain that integrates hybrid facilities, which function as both collection centers and recycling centers. Utilizing a mixed-integer linear programming approach, the model identifies optimal decisions concerning facility locations, vehicle routing, and inventory management under varying levels of recycling participation. We simulate the transition from a traditional forward supply chain to a sustainable closed-loop system and evaluate whether a gradual or abrupt transformation is more cost-effective. The findings provide managerial insights for designing long-term networks and ensuring policy compliance under sustainability-driven pressures.

Keywords: Reverse logistics, E-waste, Hybrid facilities, Supply chain design, Scenario analysis, Closed-loop logistics

1 INTRODUCTION

The global surge in production and the short life cycle of electronic devices have led to a substantial increase in electronic waste (e-waste). According to [1], more than 53 million tons of e-waste were generated in 2019 alone, yet the recycling rate remained at only 17.4%. This presents major challenges for supply chains, particularly in designing cost-effective and environmentally sustainable systems for product collection, refurbishment, and recycling.

Among the various categories of electronic devices, smartphones and laptops constitute a significant share of returned products due to their frequent replacement cycles and high consumption rates. Although the average return rate for these products ranges between 5% and 6.5% of total sales, only 30% to 35% of returned devices are repaired or refurbished and reintroduced into the market. This reveals a notable gap between actual return flows and their potential value, highlighting the need for more efficient reverse logistics and refurbishment strategies in the electronics supply chain [2].

Traditionally, supply chains have been designed for one-way forward flows. However, current sustainability goals and regulatory mandates demand the integration of reverse flows, giving rise to reverse logistics and closed-loop systems [3,4]. In this context, hybrid facilities, where collection and recycling are co-located, have gained prominence. These facilities offer potential benefits such as cost savings, greater operational flexibility, and improved responsiveness to uncertain return volumes [5,6].

This paper builds upon previous research by incorporating practical logistical assumptions and conducting a scenario-based analysis to explore how reverse logistics networks should evolve as recycling participation increases over time. Beyond static optimization, we also examine strategic development pathways, particularly whether transitioning from low to high recycling rates is best approached gradually or through a comprehensive system overhaul.

2 MODEL DESIGN AND METHODOLOGY

We consider a reverse logistics network comprising five main types of facilities: Distribution Centers (DCs), Retailers, Collection Centers (CCs), Recycling Centers (RCs), and optional Hybrid Centers (HCs) that combine CC and RC functions. Retailers sell products and receive returned items, which are then directed to CCs for inspection and sorting. Suitable products are forwarded to RCs for refurbishment and subsequently returned to DCs for reintegration into the supply chain.

The model simultaneously optimizes facility locations, product flows, vehicle routing, and inventory levels to improve both forward and reverse logistics while minimizing overall system costs. The objective function includes fixed costs for opening facilities, variable costs for sorting and refurbishing, transportation costs (both fixed and variable), and inventory holding costs.

Key assumptions include vehicles operating in closed loops, starting and ending at DCs, and being used for both delivery and return flows. Facility locations are predefined, and the model determines which ones to activate. This flexible structure supports both centralized and decentralized configurations and is adaptable to real-world constraints such as fleet availability and cost dynamics. Demand levels, return rates, and cost parameters are predetermined based on previous studies.

The problem is formulated as a Mixed-Integer Linear Programming (MILP) model, which optimizes a linear objective function subject to linear constraints. It accommodates both discrete (integer) and continuous decision variables, making it well-suited for real-world applications that involve a mix of binary and continuous decisions. MILP is widely used in areas such as production planning, scheduling, and logistics.

The model is tested using return data for electronic devices, specifically mobile phones and digital cameras, in the Slovenian market (Figure 1). The network structure includes:

- 4 candidate locations for DCs
- 8 retailers (serving as both demand and return sources)
- 4 candidate locations for CCs
- 4 candidate locations for RCs
- 4 candidate locations for HCs (co-located with RCs)

The model is implemented and solved using the GAMS optimization platform under six main scenarios, with recycling participation rates of 0%, 20%, 40%, 60%, 80%, and 100%. Each scenario represents a stage in the evolution of the supply chain from a forward-only system to a fully closed-loop recycling system.



Figure 1: Facilities location

3 SCENARIOS AND RESULTS

As sustainability awareness grows and regulatory requirements become more stringent, companies are increasingly compelled to adapt their supply chains to accommodate product returns. This section simulates a company's transition toward circularity by increasing recycling across multiple scenarios, with the relevant costs presented in Table 1 and the evolution of the supply chain network in Figure 2.

Table 1: Breakdown of costs by scenario (in €1K)

Recycling Rate Scenario	***Total Cost***	***Facility Setup Cost (Fixed)***	***Transport Cost (Fixed)***	***Transport Cost (Variable)***	***Total Sorting Cost***	***Total Recycling Cost***	***Total Inventory Cost***
0%	312	122	40	132	0	0	18
20%	426	142	41	134	40	40	29
40%	753	247	69	238	59	89	51
60%	949	332	85	256	71	131	74
80%	1150	369	99	301	91	189	101
100%	1440	447	119	378	118	249	129

In the 0% recycling scenario, the network operates as a traditional forward supply chain, comprising only four Distribution Centers (DCs) that directly serve the retailers. With minimal logistical complexity and the absence of reverse flows, the total cost remains relatively low, at €312K. At a 20% recycling rate, the total cost increases to €426K, primarily due to the activation of a Hybrid Center (HC) located near Ljubljana. This marks the onset of reverse logistics integration. Although the establishment of new facilities adds to fixed costs, the primary logistical challenge arises from coordinating both forward and reverse product flows. At the 40% recycling level, total costs rise to €753K, approximately 1.8 times higher than at 20%. However, the increase is less than proportional due to economies of scale and improved capacity utilization. The addition of a second Collection Center (CC) enhances the efficiency of product collection, while recycling continues to be handled centrally through the HC.

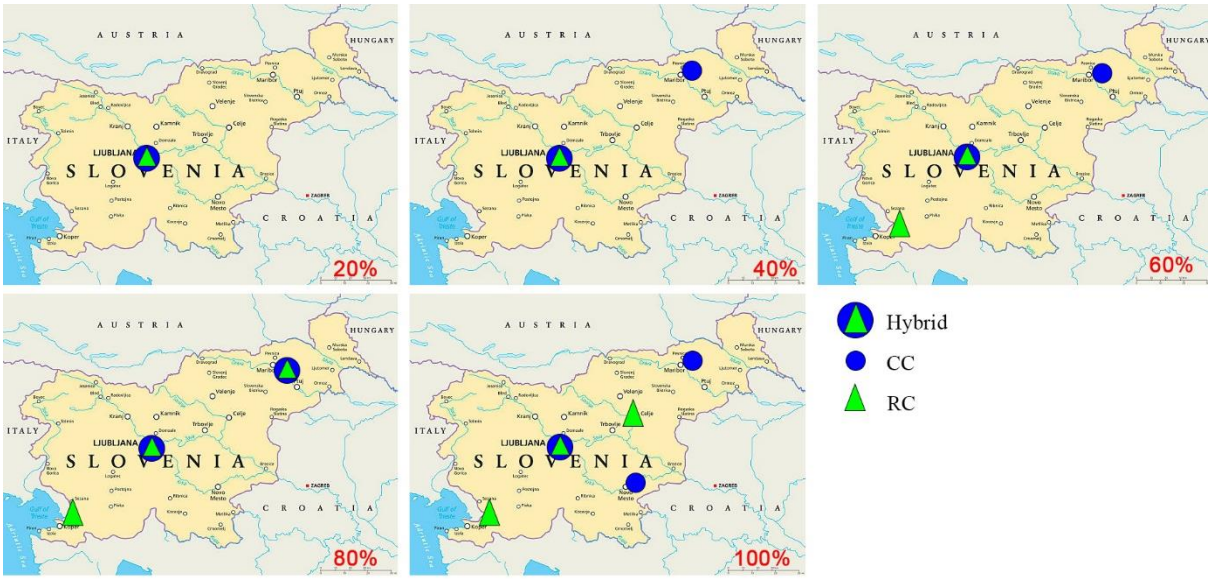


Figure 2: The design of the reverse supply chain network with increasing recycling rates

In the 60% scenario, the total cost reaches €949K. While the growth in reverse logistics volume contributes to cost escalation, the introduction of a dedicated Recycling Center (RC)

significantly enhances logistical efficiency. At the 80% level, total costs rise to €1.15 million. Two HCs and one RC are active, increasing the density of the reverse logistics network. Although fixed and operational costs grow, these are partially offset by reduced transport costs and better utilization of regional facilities. Finally, in the 100% recycling scenario, total costs reach €1.44 million. The additional €250K, compared to the 80% level, is mainly attributed to efforts to cover remote areas through the establishment of regional CCs.

These results indicate that a gradual transition from a forward-only supply chain to a closed-loop reverse logistics system is operationally viable. Cost progression is non-linear, with notable inflection points at recycling rates of 60% and 80%, where the activation of additional recycling infrastructure becomes essential. Early-stage integration of hybrid facilities (at 20%–40%) proves particularly cost-effective, especially in central hubs like Ljubljana. In higher recycling scenarios (80%–100%), deploying dedicated RCs in strategic locations such as Maribor and Ljubljana yields optimal results.

4 CONCLUSION

This study developed a reverse logistics model for managing e-waste recycling in Slovenia, using a scenario-based analysis across varying recycling participation rates. At lower recycling levels (up to 40%), hybrid facilities located in central regions such as Ljubljana provide a cost-effective solution, reducing the need for separate collection and recycling centers.

As recycling increases to moderate levels (around 60%), the establishment of dedicated recycling centers becomes essential to handle the growing volume of returned products efficiently. At high recycling rates (80–100%), a more decentralized network is required—featuring multiple recycling centers, especially in Ljubljana and Maribor—supported by additional collection points, such as those in Novo mesto. Although total system costs rise with increased recycling, unit costs decline due to improved utilization of transportation and processing capacities. From a strategic standpoint, the findings highlight the value of phased infrastructure development in line with policy objectives and rising consumer engagement. Decision-makers are advised to adopt a flexible and adaptive network design that evolves in response to both environmental mandates and operational constraints. Ultimately, a hybrid approach, balancing short-term adaptability with long-term scalability, emerges as the most resilient strategy for transitioning toward a sustainable, circular e-waste supply chain.

These results suggest that a gradual, rate-dependent expansion of the reverse logistics network can enhance cost control, operational efficiency, and alignment with the overarching goals of the circular economy.

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The 18th International Symposium on
Operational Research in Slovenia

SOR ’25

Bled, SLOVENIA
September 24 - 26, 2025

Session 5:

Mathematical Programming and Optimization

A PREDICTOR-CORRECTOR INTERIOR-POINT METHOD WITH NEW SEARCH DIRECTIONS FOR $P_*(\kapa) $-WEIGHTED LINEAR COMPLEMENTARITY PROBLEM

Xiaoni Chi
School of Mathematics and Computing Science,
Guangxi Colleges and Universities Key Laboratory of Data Analysis and Computation,
Guilin University of Electronic Technology, Guilin 541004, Guangxi, P.R. China
Email: chixiaoni@126.com

Goran Lesaja
Corvinus Centre for Operations Research,
Corvinus University of Budapest, Fővám tér 8., H-1093 Budapest, Hungary,
Email: goranlesaja@gmail.com

Guoqiang Wang
School of Mathematics, Physics and Statistics,
Shanghai University of Engineering Science, Shanghai 201620, P.R. China,
Email: guoq_wang@hotmail.com

Abstract: In this paper, we present a new predictor-corrector feasible interior-point method (IPM) for $P_*(\kappa) $-weighted linear complementarity problems (WLCP). The new search directions are derived from the algebraic equivalent transformation (AET) of the Newton system that is based on the positive-asymptotic kernel function. By choosing appropriate parameters, the proposed algorithm is shown to have global convergence and polynomial iteration complexity. Preliminary numerical results indicate the efficiency of the algorithm.

Keywords: Weighted linear complementarity problem, $P_*(\kappa) $-matrix, Interior-point method, Algebraic equivalent transformation, Kernel function.

WEAKLY FEASIBLE AND OPTIMAL SOLUTION SETS IN 0–1 INTERVAL LINEAR PROGRAMMING

Elif Garajová, Milan Hladík
Charles University, Faculty of Mathematics and Physics,
Department of Applied Mathematics, Prague, Czech Republic
Prague University of Economics and Business, Faculty of Informatics and Statistics,
Department of Econometrics, Prague, Czech Republic
elif@kam.mff.cuni.cz, hladik@kam.mff.cuni.cz
Miroslav Rada
Prague University of Economics and Business,
Faculty of Informatics and Statistics, Department of Econometrics,
Faculty of Finance and Accounting, Department of Financial Accounting and Auditing,
Prague, Czech Republic
miroslav.rada@vse.cz

Abstract: We consider an integer linear programming model with binary variables subject to interval uncertainty, where the data can vary independently within specified lower and upper bounds. Our focus is on the properties of the weakly feasible and optimal solutions, i.e. solutions feasible or optimal for at least one realization of the uncertain data. To this end, we formulate a general framework problem for checking whether there exists a scenario admitting prescribed required and forbidden feasible solutions. We also propose a mixed-integer linear programming model for solving the problem. Then, we demonstrate how this framework can be used to describe the set of weakly optimal solutions in 0–1 interval linear programs.

Keywords: integer programming, interval uncertainty, optimal set

1 INTRODUCTION

Optimization under uncertainty plays a crucial role in modeling and solving real-world optimization problems. In this paper, we address the model of an integer linear program with binary variables, in which the coefficients are affected by interval uncertainty and can be independently perturbed within given lower and upper bounds. Linear programming under interval uncertainty has been studied in the literature for continuous variables [3, 7], and, to a lesser extent, also for integer or binary variables [2, 8]. Applications of integer programming models under interval uncertainty include network optimization problems [6] or project scheduling [1].

Here, we address the task of describing the set of optimal solutions of a 0–1 interval linear program. We first formalize the model and formulate the problem of checking the existence of a scenario with required and forbidden feasible solutions. Then, we propose a mixed-integer programming model for deciding the problem. Finally, we illustrate how solving the problem can be used to describe the set of all weakly optimal solutions.

2 0–1 INTERVAL LINEAR PROGRAMMING

Let us first introduce the necessary terminology and notation of interval programming.

Interval data. Throughout the text, the symbol $\mathbb{IR}$ denotes the set of closed bounded real intervals. Given two matrices $\underline{A}, \overline{A} \in \mathbb{R}^{m\times n}$ with $\underline{A} \leq \overline{A}$, an *interval matrix* $\mathbf{A} = [\underline{A}, \overline{A}] \in \mathbb{IR}^{m\times n}$ is defined as the set $\mathbf{A} = \{A \in \mathbb{R}^{m\times n} \colon \underline{A} \leq A \leq \overline{A}\}$, where all inequalities are understood componentwise. An *interval vector* can be defined as a special case of an interval matrix.

Interval optimization. Let an interval matrix $\mathbf{A} \in \mathbb{IR}^{m\times n}$ and interval vectors $\mathbf{b} \in \mathbb{IR}^m$, $\mathbf{c} \in \mathbb{IR}^n$ be given. We define an inequality-constrained *0–1 interval linear program* (abbreviated as 0–1 ILP) as the set of all integer linear programs with binary variables in the form

$$\text{maximize } c^T x \text{ subject to } Ax \leq b,\ x \in \{0,1\}^n, \tag{1}$$

satisfying $A \in \mathbf{A}$, $b \in \mathbf{b}$, $c \in \mathbf{c}$. A particular integer program in the set determined by a triplet $(A, b, c) \in (\mathbf{A}, \mathbf{b}, \mathbf{c})$ is called a *scenario* of the 0–1 ILP. Interval linear systems and their scenarios are defined analogously.

Feasbility and optimality. A given vector $x \in \mathbb{R}^n$ is said to be a *weakly feasible/optimal solution* of the 0–1 ILP if it is feasible/optimal for at least one scenario $(A, b, c) \in (\mathbf{A}, \mathbf{b}, \mathbf{c})$. Similarly, the vector is said to be a *strongly feasible/optimal solution*, if it is feasible/optimal for each scenario $(A, b, c) \in (\mathbf{A}, \mathbf{b}, \mathbf{c})$.

Proposition 2.1 shows the characterization of the weakly and strongly feasible solution set, which directly follows from the corresponding characterizations known for interval linear programs with continuous variables [2, 7].

Proposition 2.1 ([2, 7]) *A given $x \in \{0,1\}^n$ is a weakly feasible solution of the 0–1 ILP if and only if it satisfies the system $\underline{A}x \leq \overline{b},\ x \in \{0,1\}^n$. Moreover, it is a strongly feasible solution of the 0–1 ILP if and only if it satisfies the system $\overline{A}x \leq \underline{b},\ x \in \{0,1\}^n$.*

On the other hand, characterizing the weakly optimal solutions is a more challenging task, even for interval linear programs with continuous variables [4].

3 FEASIBLE AND OPTIMAL SOLUTION SET

In this paper, we formulate a crucial problem of interval linear programming with binary variables (stated in Problem 3.1), propose a mixed-integer linear program for solving it, and demonstrate how it can be used to characterize the weakly optimal solution set of a 0–1 ILP.

Problem 3.1 *Given an interval linear system $\mathbf{A}x \leq \mathbf{b}$ with binary variables and two disjoint sets $S^+, S^- \subseteq \{0,1\}^n$, decide whether there exists a scenario $(A, b) \in (\mathbf{A}, \mathbf{b})$ such that*

$$x \in S^+ \Rightarrow Ax \leq b, \tag{2}$$
$$x \in S^- \Rightarrow \exists i \in \{1, \ldots, m\} : A_{i*}x > b, \tag{3}$$

where A_{i*} denotes the i-th row of the matrix A. In other words, the problem asks whether there exists a scenario of the interval program, such that the set S^+ forms a subset of the weakly feasible solutions for the scenario, while none of the vectors in S^- are weakly feasible.

Problem 3.1 is closely related to the notions of weak and strong feasibility of solutions, as discussed in Remark 3.2. This connection can be exploited to simplify the solution process when some of the vectors are already known to be strongly (in)feasible.

Remark 3.2 *For any strongly feasible solution $x \in S^+$, condition* (2) *is always satisfied. For any strongly infeasible (not weakly feasible) solution in $x \in S^-$, condition* (3) *is always satisfied.*

As a special case, Problem 3.1 can be used to determine whether there exists a scenario in which the set of all weakly feasible solutions coincides exactly with a prescribed set $S \subseteq \{0,1\}^n$.

Proposition 3.3 *Let a set $S \subseteq \{0,1\}^n$ be given. Then, the set S forms the feasible set of some scenario of the interval linear system $\mathbf{A}x \leq \mathbf{b}$ with binary variables if and only if Problem 3.1 with $S^+ = S$ and $S^- = \{0,1\}^n \setminus S$ admits a feasible solution $(A', b') \in (\mathbf{A}, \mathbf{b})$. If so, the set S forms the feasible set of the scenario $A'x \leq b',\ x \in \{0,1\}^n$.*

We now turn to the question of how to solve Problem 3.1. In Proposition 3.4, we present a finite method for deciding the problem by solving a (non-interval) mixed-integer linear program. Note that the vectors x contained in the sets S^+ and S^- are explicitly enumerated in program (4), so they are not among the decision variables of the program. Furthermore, the interval coefficients are replaced by variables a_{ij} and b_i bounded by their respective intervals. The binary variables y_{ix} introduced in model (4) in combination with the big-M constraints ensure that at most $m-1$ of the m original constraints can be satisfied by each $x \in S^-$ (assuming we attain $\varepsilon > 0$). Thus, the forbidden vectors $x \in S^-$ cannot be feasible for the constructed scenario, as required in Problem 3.1.

Proposition 3.4 *For given sets $S^+, S^- \subseteq \{0,1\}^n$, Problem 3.1 can be solved by the following 0–1 linear program with binary variables $y_{ix} \in \{0,1\}$, continuous variables $a_{ij} \in [\underline{a}_{ij}, \overline{a}_{ij}]$ and $b_i \in [\underline{b}_i, \overline{b}_i]$ and a sufficiently large value M:*

$$
\begin{aligned}
&\text{maximize } \varepsilon \\
&\sum_{j=1}^{n} a_{ij}x_j \leq b_i, && \forall x \in S^+\ \forall i \in \{1,\dots,m\} \\
&\sum_{j=1}^{n} a_{ij}x_j \geq b_i + \varepsilon - My_{ix}, && \forall x \in S^-\ \forall i \in \{1,\dots,m\} \\
&\sum_{i=1}^{m} y_{ix} \leq m-1, && \forall x \in S^-, \\
&y_{ix} \in \{0,1\},\ a_{ij} \in [\underline{a}_{ij}, \overline{a}_{ij}],\ b_i \in [\underline{b}_i, \overline{b}_i].
\end{aligned}
\tag{4}
$$

There exists a scenario solving Problem 3.1 if and only if program (4) is feasible and $\varepsilon > 0$ holds for the optimal objective value.

Finally, let us demonstrate how to use the framework of Problem 3.1 for characterizing the weakly optimal solutions of a 0–1 ILP. Proposition 3.5 states that a solution x is weakly optimal if and only if it is the best feasible solution for a scenario with a particular objective vector that is defined based on the values x_j. Here, the objective vector is chosen from the interval objective vector such that it is most favorable for the tested solution x. Due to space limitations, we omit the derivation and detailed proof of the results.

Proposition 3.5 *A given x^* is weakly optimal for the 0–1 ILP if and only if there exists a set $S \subseteq \{0,1\}^n$ that forms the feasible set of some scenario $(A', b') \in (\mathbf{A}, \mathbf{b})$, such that*

$$c^T x^* = \max\{c^T x : x \in S\}$$

for the objective vector $c \in \mathbf{c}$ defined as

$$
c_j = \begin{cases} \overline{c}_j & \text{if } x^*_j = 1, \\ \underline{c}_j & \text{if } x^*_j = 0. \end{cases}
\tag{5}
$$

Building on Proposition 3.5, we can formulate the problem of checking weak optimality of a given solution within the framework of Problem 3.1. The resulting characterization is stated in Corollary 3.6, which also yields a scenario witnessing optimality of the solution.

Corollary 3.6 *For a given 0–1 ILP and $x^* \in \{0,1\}^n$, let $S^-(x^*)$ denote the set of weakly feasible solutions with a better objective value than $c^T x^*$ for c defined in* (5)*, i.e.*

$$S^-(x^*) = \{x \in \{0,1\}^n : x \text{ is weakly feasible for the 0–1 ILP and } c^T x > c^T x^*\}.$$

Then, x^* is weakly optimal if and only if Problem 3.1 with $S^+ = \{x^*\}$ and $S^- = S^-(x^*)$ admits a feasible solution $(A', b') \in (\mathbf{A}, \mathbf{b})$. If so, x^* is optimal for the scenario (A', b', c).

4 CONCLUSION

We examined the properties of weak feasibility and weak optimality in interval linear programming models with binary variables. Specifically, we addressed the problem of determining whether a scenario exists within a given interval program that admits a prescribed set of required and forbidden feasible solutions. We proposed a mixed-integer programming model for deciding the existence of such a scenario. Utilizing this framework, we established a complete characterization of weakly optimal solutions for 0–1 interval linear programs. As a result, we developed a finite method for describing the set of weakly optimal solutions using enumeration and (non-interval) mixed-integer programming.

5 Acknowledgements

E. Garajová and M. Hladík were supported by the Czech Science Foundation under Grant 25-15714S. M. Rada was supported by the Czech Science Foundation under Grant 23-07270S.

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ON THE GENERALIZATION OF THE FGKLA ALGORITHM

Miroslav Rada
Prague University of Economics and Business,
Faculty of Informatics and Statistics, Department of Econometrics,
Faculty of Finance and Accounting, Department of Financial Accounting and Auditing,
W. Churchill Sq. 4, 130 67 Prague 3, Czech Republic
miroslav.rada@vse.cz
Michal Černý
Prague University of Economics and Business,
Faculty of Informatics and Statistics, Department of Econometrics,
W. Churchill Sq. 4, 130 67 Prague 3, Czech Republic
cernym@vse.cz

Abstract: Consider the problem of maximizing the sample variance over an interval-valued dataset: the goal is to select values from the given intervals such that the sample variance of the chosen values is maximized. From an optimization perspective, this problem can be formulated as the maximization of a special convex quadratic function over a hypercube, implying that a maximizer lies at a vertex of the feasible hypercube. The problem is known to be NP-hard; however, efficient algorithms exist for certain special classes of instances. In this paper, we generalize the FGKLA algorithm to handle a broader class of convex quadratic functions and illustrate its applicability on the problem of maximizing the residual sum of squares in linear regression with interval-valued outputs.

Keywords: QUBO, convex quadratic maximization, robust statistics, interval data

1 INTRODUCTION: STATISTICS OVER INTERVAL DATA

This work is motivated by robust statistics. Consider a statistic S (i.e., a function of some data, e.g., the sample variance, mean, or something else). Assume that instead of crisp data points, only intervals containing the true values are known. In this situation, very little information can be inferred. At a minimum, the so-called *identification region* [5] can be obtained: the tight interval $[\underline{S}, \overline{S}]$ that certainly contain S. Many statistics have been studied in place of S, see, e.g., [6].

In this paper, we initially have the sample variance in place of S. We describe the FGKLA algorithm [4, 7] for computing the tight upper bound on variance over interval data. Then we show that the algorithm can be generalized to handle more general statistics and even a general class of optimization problems.

Assume a dataset of n interval-valued observations, represented by vectors $\boldsymbol{y^c}, \boldsymbol{y^\Delta} \in \mathbb{R}^n$ of centers and radii of the intervals, where the ith observation is the interval $y_i = [y_i^c - y_i^\Delta, y_i^c + y_i^\Delta]$. Define the set of all possible crisp values as

$$C \coloneqq \{\boldsymbol{y} \in \mathbb{R}^n \mid \boldsymbol{y^c} - \boldsymbol{y^\Delta} \leq \boldsymbol{y} \leq \boldsymbol{y^c} + \boldsymbol{y^\Delta}\} \tag{1}$$

The problem of computing the tight upper bound on the sample variance reads

$$\max_{\boldsymbol{y} \in \mathbb{R}^n} \quad V(\boldsymbol{y}) \coloneqq \frac{1}{n} \sum_{i=1}^{n} \Big(y_i - \frac{1}{n} \sum_{j=1}^{n} y_j \Big)^2 = \frac{1}{n} \boldsymbol{y}^T \Big(I - \frac{1}{n} \mathbf{1}\mathbf{1}^T \Big) \boldsymbol{y} \quad \text{s.t.} \quad \boldsymbol{y} \in C. \tag{2}$$

Problem (2) has been widely studied in literature. A specific NP-hardness proof was given by Ferson et al. [3]. In this paper, we build on the FGKLA algorithm from [4]. The presented version is described in Rada et al. [7], where also further references to the problem can be found.

2 THE FGKLA ALGORITHM

Note that $V(\boldsymbol{y})$ is a convex function. The NP-hardness of (2) stems from the fact that the maximum of $V(\boldsymbol{y})$ is attained in a vertex of C, which is an n-dimensional hyperrectangle with 2^n vertices.

The FGKLA algorithm from [4] is a brute force algorithm in its essence. It simply examines (some) vertices of C to find the maximal sample variance. However, for suitable instances, it can reduce the number of vertices to be examined to be much lower than 2^n; actually, there are classes of instances in which the number of vertices is polynomial in n. Furthermore Rada et al. [7] proved that if the input data (radii and centers) come from natural probabilistic distributions, hard instances occur very rarely, and the expected running time of FGKLA algorithm is almost linear. This is what makes the algorithm a suitable candidate for further generalizations, in particular for generalizations motivated by robust statistics.

The reduction of the number of vertices of C is based on Lemma 2.1. Define the *ith narrowed interval* as $y_i^n := [y_i^c - \frac{1}{n}y_i^\Delta, y_i^c + \frac{1}{n}y_i^\Delta]$. Lemma 2.1 works with these narrowed intervals.

Lemma 2.1 (Sufficient condition for not being maximizer) *Define* $\mu(\boldsymbol{y}) := \frac{1}{n}\sum_{i=1}^n y_i = \frac{1}{n}\mathbf{1}^T\boldsymbol{y}$. *Let* $\boldsymbol{y}, \boldsymbol{y}' \in C$ *and suppose there exists* $i \in \{1, \ldots, n\}$ *such that* $y_j = y'_j$ *for all* $j \neq i$ *and one of the following is satisfied:*

- $y_i = \overline{y}_i$, $\mu(\boldsymbol{y}) > y_i^c + \frac{1}{n}y_i^\Delta$ *and* $y'_i = \underline{y}_i$, *or*
- $y_i = \underline{y}_i$, $\mu(\boldsymbol{y}) < y_i^c - \frac{1}{n}y_i^\Delta$ *and* $y'_i = \overline{y}_i$,

Then $V(\boldsymbol{y}) < V(\boldsymbol{y}')$.

Lemma 2.1 can be used as follows: if we knew the value of mean of a maximizer, we could set the value a variable i satisfying one of the conditions in the lemma to one of its bounds. We say that variable y_i is *determinable for mean* μ, otherwise we say that variable y_i is *free for mean* μ. More formally, we introduce $L(\mu) := \{i \in \{1, \ldots, n\} \mid \mu \in y_i^n\}$ as the set of indices of free variables for mean μ. Clearly, for a given μ, only $2^{|L(\mu)|}$ vertices have to be examined.

Although the mean of a maximizer is not known in advance, we can dissect the space of all possible means into regions with the same $L(\mu)$. Indeed, $L(\mu)$ changes only in the endpoints of narrowed intervals. Hence, FGKLA works as follows: it takes these $2n$ endpoints one by one, for each of them (say endpoint μ') it examines all $2^{|L(\mu')|}$ vertices of C corresponding to possibilities induced by free variables (other variables are determined to one of their endpoints).

Define $\omega := \max_{\mu \in \mathbb{R}} |L(\mu)|$. Assume that examining one vertex takes $O(n)$ time. The overall complexity of the FGKLA algorithm is $O(n^2 2^\omega)$. Note that this complexity could be further optimized to $O(n \log n + n 2^\omega)$ by sorting the endpoints to allow amortization of the examination of one vertex to $O(1)$. Note also that the complexity is driven by ω: the maximal number of narrowed intervals intersecting in one point. For more detailed description of the algorithm, see [7]. The FGKLA algorithm is illustrated in Figure 1.

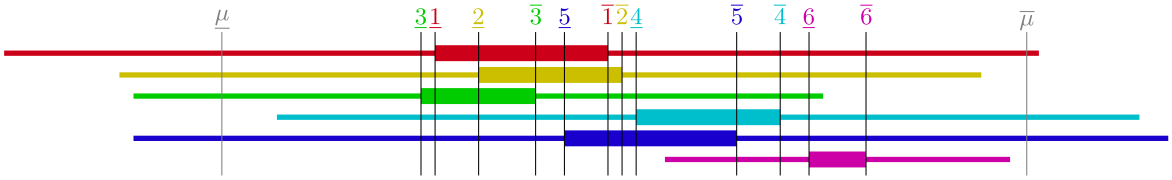


Figure 1: Illustration for the FGKLA algorithm: instance with 6 intervals. Horizontal lines: thiner – original intervals, bolder – narrowed intervals. Vertical lines: endpoints of narrowed intervals. In $\underline{3}, \overline{4}, \underline{6}, \overline{6}$ – $|L| = 1$, in $\underline{1}, \overline{2}, \underline{4}, \overline{5}$ – $|L| = 2$. $\omega = 3$ attained for $\underline{2}, \overline{3}, \underline{5}, \overline{2}$. At most 2^3 vertices will be examined in an endpoint (instead of 2^6).

3 THE GENERALIZATION OF THE FGKLA ALGORITHM

Let $A \in \mathbb{R}^{n\times n}$ be such that $(I - A)$ is positive semidefinite. Consider the optimization problem

$$\max_{\boldsymbol{y}\in\mathbb{R}^n} \quad f(\boldsymbol{y}) \coloneqq \boldsymbol{y}^{\mathrm{T}}(I - A)\boldsymbol{y} \qquad \text{s.t.} \qquad \boldsymbol{y} \in C. \tag{3}$$

Denote by $\mathbf{1}$ the vector of ones of a suitable length. In the special case $A = \frac{1}{n}\mathbf{1}\mathbf{1}^T$, problems (2) and (3) differ only by the multiplicative factor $\frac{1}{n}$.

We now generalize the FGKLA algorithm to handle problem (3). This generalization is particularly useful when $r \coloneqq \operatorname{rank}(A) < n$. Note that $\operatorname{rank}(\frac{1}{n}\mathbf{1}\mathbf{1}^{\mathrm{T}}) = 1$. Firstly, we need a lemma similar to Lemma 2.1. Denote by $\boldsymbol{e}_i$ the ith unit vector.

Lemma 3.1 (Sufficient condition for not being maximizer – generalized) *Let $\boldsymbol{y}, \boldsymbol{y}' \in \mathbb{R}^n$ be given, and let there exist $i \in \{1, \ldots, n\}$ such that $y_j = y'_j$ for all $j \neq i$ and one of the following is satisfied:*

a) $\boldsymbol{e}_i^{\mathrm{T}} A\boldsymbol{y} > y_i^c + y_i^\Delta A_{ii}$ and $\boldsymbol{y}' = \underline{y}_i$, or

b) $\boldsymbol{e}_i^{\mathrm{T}} A\boldsymbol{y} < y_i^c - y_i^\Delta A_{ii}$ and $\boldsymbol{y}' = \overline{y}_i$.

Then $f(\boldsymbol{y}) < f(\boldsymbol{y}')$.

Proof. Provided for item a). Analogous for item b). Define $J \coloneqq \{1, \ldots, n\} \setminus \{i\}$.

$$\begin{aligned} f(\boldsymbol{y}) - f(\boldsymbol{y}') &= \boldsymbol{y}^{\mathrm{T}}(I - A)\boldsymbol{y} - \boldsymbol{y}'^{\mathrm{T}}(I - A)\boldsymbol{y}' = \boldsymbol{y}^{\mathrm{T}}\boldsymbol{y} - \boldsymbol{y}'^{\mathrm{T}}\boldsymbol{y}' - \boldsymbol{y}^{\mathrm{T}}A\boldsymbol{y} + \boldsymbol{y}'^{\mathrm{T}}A\boldsymbol{y}' \\ &= y_i^2 - y_i'^2 - \sum\nolimits_{j\in J} 2y_i y_j A_{ij} - y_i^2 A_{ii} + \sum\nolimits_{j\in J} 2y'_i y'_j A_{ij} + x_i'^2 A_{ii} \\ &= 4y_i^c y_i^\Delta - 4y_i^c y_i^\Delta A_{ii} - \sum\nolimits_{j\in J} 2A_{ij}(y_j y_i - y'_j y'_i) = 4y_i^\Delta (y_i^c - y_i^c A_{ii} + y_i A_{ii} - \sum\nolimits_{j=1}^{n} A_{ij} y_j) \\ &= 4y_i^\Delta (y_i^c + y_i^\Delta A_{ii} - \boldsymbol{e}_i^{\mathrm{T}} A\boldsymbol{y}) < 0. \end{aligned}$$

□

Note that Lemma 2.1 is a special case of Lemma 3.1. The expression $\boldsymbol{e}_i^{\mathrm{T}} A\boldsymbol{y}$ is equal to $\mu(\boldsymbol{y})$ for all i. Both the lemmas provide a way to determine values of some variables, provided that $\boldsymbol{y}$ is in some region. A variable y_i is free (i.e. not determinable) with respect to a given $\boldsymbol{y}$, if $\boldsymbol{y}$ lies within the *band* defined by $|e_i^T A\boldsymbol{y} - y_i^c| \geq A_{ii} y_i^\Delta$. These bands correspond to the narrowed intervals from the FGKLA algorithm. The bands dissect the n-dimensional space to regions with the same sets of free variables. Similarly as for the FGKLA algorithm, some vertices has to be examined for each of the regions, and hopefully the number of bands intersecting in the regions will be lower than n.

The fundamental questions for the generalized algorithm are: (i) how many regions there can be, (ii) how to efficiently enumerate them.

Review the situation through the optics of the well-known geometric concept of arrangements of hyperplanes. Given a collection of hyperplanes, the associated *arrangement of hyperplanes* is a dissection of space to relatively open regions, called *cells*. For an arrangement of m hyperplanes in $\mathbb{R}^d$, the number of cells is $O(m^d)$ by Buck [1]. Furthermore, all the regions can be enumerated using the asymptotically optimal algorithm by Edelsbrunner et al. [2]. In our case, Lemma 3.1 gives $2n$ hyperplanes $e_i^{\mathrm{T}} A\boldsymbol{y} = y_i^c \pm y_i^\Delta A_{ii}, i = 1, \ldots, n$ that form the boundary of the bands. These hyperplanes live in $\mathbb{R}^n$. This is in contrast with Lemma 2.1 and the FGKLA algorithm, where bands and hyperplanes (endpoints of narrowed intervals) acted in $\mathbb{R}^1$ (in the space of means). The key is in the observation that the expression $e_i^{\mathrm{T}} \frac{1}{n}\mathbf{1}\mathbf{1}^{\mathrm{T}}\boldsymbol{y}$ is actually interpreted in the form $e_i^{\mathrm{T}}\mathbf{1}\mu = \mu$ using substitution $\frac{1}{n}\mathbf{1}^{\mathrm{T}}\boldsymbol{y} = \mu$.

For our generalization, we use the same trick. Since $\operatorname{rank}(A) = r$, there exists a decomposition $A = L^{\mathrm{T}}U$ with $L, U \in \mathbb{R}^{r\times n}$. By substituting $\boldsymbol{x} = U\boldsymbol{y}$, we obtain the arrangement of $2n$ hyperplanes $e_i^{\mathrm{T}} L^{\mathrm{T}}\boldsymbol{x} = y_i^c \pm y_i^\Delta A_{ii}, i = 1, \ldots, n$ in $\mathbb{R}^r$. The number of its cells can be bounded by $O((2n)^r)$. Similarly as for the FGKLA algorithm, one can restrict the enumeration to vertices (i.e. 0-dimensional cells), since cells of higher dimension generally correspond

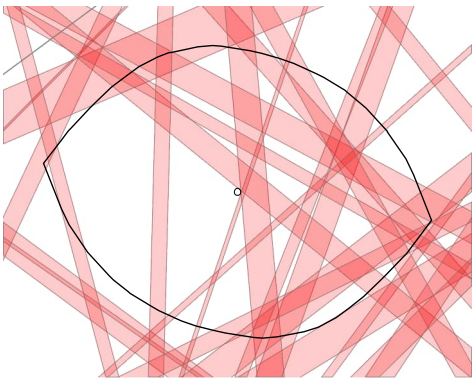


Figure 2: Illustration of bands for the problem of maximization of RSS in linear regression $\boldsymbol{y} = X\boldsymbol{\beta}+\boldsymbol{r}$ with $\boldsymbol{y} \in C$. Input data: $X \in \mathbb{R}^{30\times 2}, \boldsymbol{y^{\Delta}}, \boldsymbol{y^{c}} \in \mathbb{R}^{30\times 1}$. First column of X is $\mathbf{1}$, other data randomly sampled from normal and uniform distributions. Here, $A = X(X^{\mathrm{T}}X)^{-1}X^{\mathrm{T}}$, $r = 2$ and $n = 30$. Bands depicted in the space of OLS-estimates of β. The lemon-like black shape: the set of all possible OLS-estimates for $\boldsymbol{y} \in C$. Despite the fact $n = 30$, $\omega = 5$ only.

to smaller number of intersecting bands. However, restricting the enumeration to vertices does not improve the asymptotic bound for the number of regions to be enumerated.

Overall, the complexity of the generalized algorithm is $O(n \cdot (2n)^r 2^{\omega})$, where ω is the maximal number of bands intersecting at a single point.

4 CONCLUSIONS AND COMPLEXITY CONSIDERATIONS

A natural generalization of the FGKLA algorithm for optimizing more general PSD quadratic forms $\boldsymbol{y}^{\mathrm{T}}(I - A)\boldsymbol{y}$ has been proposed. The resulting algorithm is exponential not only in ω (the maximal number of bands intersecting at a single point) but also in r (the rank of A). The exponential dependence on r arises from the geometric nature of the algorithm and the well-known *curse of dimensionality*. Nevertheless, for a fixed r (see, e.g., Figure 2), the term $O((2n)^r)$ becomes polynomial. The value of ω depends on the widths of the bands (the radii $\boldsymbol{y^{\Delta}}$ of intervals and the diagonal entries of A), as well as their positions (centers $\boldsymbol{y^{c}}$ of intervals) and orientations (rows of L). Characterizing instances for which $\omega = O(\log(n))$ remains an interesting open problem. Another open question is to derive results analogous to those in [7]: by considering a natural probabilistic setup for generating instances (with constant r), one could analyze the expected polynomial-time complexity of the algorithm.

Acknowledgements The first author acknowledges the support of the Czech Science Foundation (GAČR) under the project 23-07270S. The second author acknowledges the support of the Czech Science Foundation (GAČR) under the project 25-04734S.

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The 18th International Symposium on
Operational Research in Slovenia

SOR ’25

Bled, SLOVENIA
September 24 - 26, 2025

Session 6:

Multi-Criteria Decision-Making

INCONSISTENCY THRESHOLDS FOR INCOMPLETE PAIRWISE COMPARISONS: THE ROLE OF THE ASSOCIATED GRAPH

Kolos Csaba Ágoston
Corvinus University of Budapest, 1093 Budapest, Fővám square 8., Hungary
László Csató
Institute for Computer Science and Control (SZTAKI), Hungarian Research Network (HUN-REN), 1111 Budapest, Kende street 13–17., Hungary
Corvinus University of Budapest, 1093 Budapest, Fővám square 8., Hungary

Abstract: Multi-criteria decision making is a widely used approach to evaluate business projects. Several methods are based on pairwise comparison matrices where experts compare pairs of alternatives. Consistency is a required property for pairwise comparison matrices, but experts in their comparison may not fully fulfil this property. Several methods are known to measure the level of inconsistency in the case of pairwise comparison matrices; one of the most widely used among them is Saaty's 10% rule: the level of inconsistency can be tolerated if it is below 10% of the mean of randomly generated matrices. Later, these thresholds were determined for incomplete pairwise comparison matrices, but only the number of missing elements was considered. In this research we have recalculated these thresholds considering not only the number of missing elements but also the associated graph.

Keywords: Multi-criteria decision making, pairwise comparison matrix, inconsistency threshold

1 INTRODUCTION

Pairwise comparison matrices play a crucial role in several multi-criteria decision-making methodology such as the Analytic Hierarchy Process (AHP) (Saaty, 1977, 1980). However, while pairwise comparisons can efficiently decompose complex decision-making problems into simple subproblems, this has a price as there is no guarantee for their cardinal consistency. For example, if alternative A is two times better than alternative B, and alternative B is three times better than alternative C, alternative A is not necessarily six times better than alternative C as suggested by their indirect comparison through alternative B. Inconsistency is measured by inconsistency indices (Brunelli, 2018).

Nonetheless, the numerical value of inconsistency is difficult to interpret without a sharp threshold, which separates acceptable and unacceptable levels of inconsistency. The first inconsistency index and threshold of acceptability has been proposed by the founder of the AHP methodology (Saaty, 1977). It is based on dividing the inconsistency of the given matrix by the average inconsistency of random pairwise comparison matrices: inconsistency can be tolerated if this ratio remains below 0.1, which directly gives a threshold for inconsistency.

Pairwise comparison matrices have a natural generalisation by allowing for some missing entries. In Ágoston and Csató (2022) the thresholds were recalculated taking into account the number of missing elements. In this paper, we use the same methodology as Ágoston and Csató (2022), but we also consider not only the number of missing elements, but also their locations.

2 PRELIMINARIES

Let n denote the number of alternatives. The pairwise comparisons of the alternatives are usually collected into a *pairwise comparison matrix* $\mathbf{A} = [a_{ij}]$, where $a_{ij} > 0$ means that the ith alternative is a_{ij} times more preferred than the jth alternative. They satisfy reciprocity, namely, $a_{ji} = 1/a_{ij}$ for all $1 \leq i, j \leq n$.

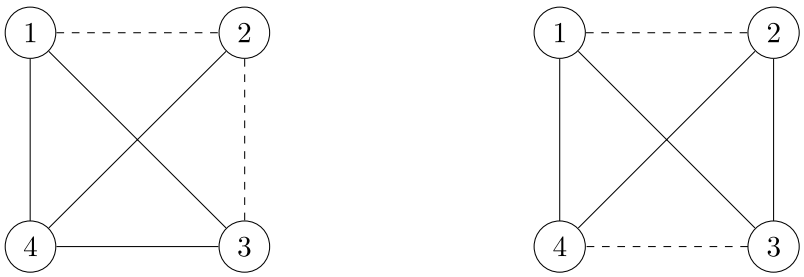


Figure 1: The graph representations of the 4×4 pairwise comparison matrices with two missing elements. The dashed line indicates the missing elements, these edges do not exist in the graphs.

A pairwise comparison matrix $\mathbf{A}$ is *consistent* if and only if $a_{ik} = a_{ij}a_{jk}$ for all $1 \leq i, j, k \leq n$. In other words, the pairwise comparisons are consistent if the direct comparison of any two alternatives coincides with their indirect comparison through any third alternative. Otherwise, the matrix is *inconsistent*.

The Perron–Frobenius theorem implies that a pairwise comparison matrix $\mathbf{A}$ has a dominant eigenvalue $\lambda_{\max}(\mathbf{A})$. The widely used inconsistency index of Saaty (Saaty, 1977) is:

$$CI(\mathbf{A}) = \frac{\lambda_{\max}(\mathbf{A}) - n}{n - 1}.$$

$CI(\mathbf{A}) = 0$ if and only if the pairwise comparisons are consistent.

According to Saaty, $CI(\mathbf{A})$ should be compared to the so-called *random index RI*, which is the average CI of a large number of pairwise comparison matrices where the entries above (below) the diagonal are drawn independently and uniformly from the Saaty scale (details are available in Ágoston and Csató (2022)).

Incomplete pairwise comparison matrices allow for some missing comparisons. Therefore, $\mathbf{A} = [a_{ij}]$ is an *incomplete pairwise comparison matrix* if $a_{ij} > 0$ or $a_{ij} = \star$ such that $a_{ij} > 0$ implies $a_{ji} = 1/a_{ij}$ and $a_{ij} = \star$ implies $a_{ji} = \star$. The number of missing comparisons is denoted by m: $|(i, j) : a_{ij} = \star| = 2m$.

Incomplete pairwise comparisons has a natural graph representation. Let $\mathbf{A}$ be an incomplete pairwise comparison matrix. The associated undirected graph is $G = (V, E)$, where the vertices correspond to the alternatives and $e_{ij} \in E$ is an edge if and only if the pairwise comparison a_{ij} is known.

In this paper, we focus on the case of 4×4 matrices with two missing elements when the associated graph can have two different topologies which as shown in Figure 1. In the figure, the missing elements are marked with a dashed line. We can see that the main difference between the two topologies is whether the edges representing the missing elements are connected or not. We will refer to these two cases as '*connected*' and '*independent*' cases. If we randomly assign the missing elements—or, more precisely, if we randomly pick two edges out of all the six edges—it is quite easy to calculate the probabilities of the two cases: connected: 0.8; independent: 0.2.

3 NUMERICAL RESULTS

We followed the same methodology as Ágoston and Csató (2022): we filled an incomplete pairwise comparison matrix with random element drawn from the Saaty scale. We completed the matrix in a way that minimized the CI measure of the matrix and stored the CI value for the completed matrix, iterating the whole process for 1 million times. In Table 1, we present

some information about the two cases, in Figure 2 we have plotted the histograms for both cases.

	connected	independent
mean CI	0.317	0.265
# instances below the threshold	148210	162960
standard deviation	0.398	0.344
95% confidence interval for mean	0.316 – 0.318	0.264 – 0.265
maximum of CI	2.143	2.370
probabilities	0.8	0.2

Table 1: Summary of connected and independent cases

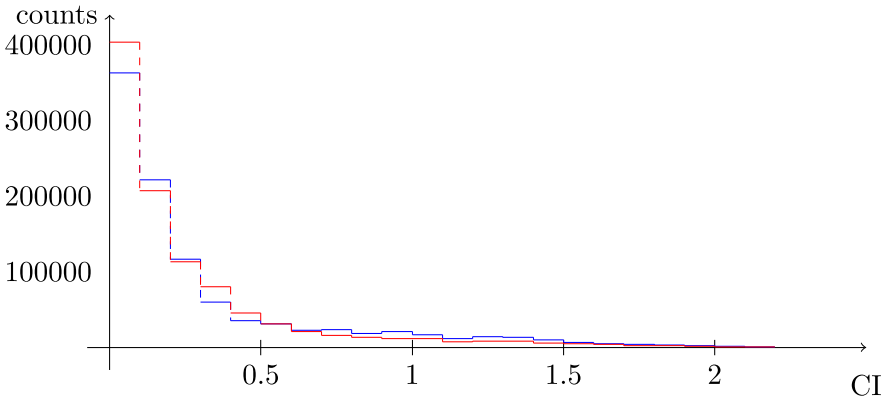


Figure 2: Histograms for the connected (blue) and independent (red) cases

Based on date in Table 1 and in Figure 2 we can conclude that the topology of the associated matrix has a significant effect on the mean CI value: in the case of the independent topology we can complete the pairwise comparison matrix to a less inconsistent matrix. The mean CI for the connected topology is 0.317, and for the independent topology is 0.265; the difference between the means is statistically significant. The mean CI value for 4×4 matrices with 2 missing elements can be obtained by a weighted mean (the weights are the probabilities of the topologies):

$$0.8 \times 0.317 + 0.2 \times 0.265 = 0.3066 \ ,$$

this value can be seen in Ágoston and Csató (2022, Table 2, page 5)

Interestingly, contrary to the fact that the mean CI is less for independent case, the maximum value is also higher for the independent case. There are 356 instances where we obtained a higher CI value in the independent case than the maximum of the CI values in the connected case.

Knowing, that the mean CI is different for the two cases, it is obvious that different thresholds should be used for the two cases. We used the 10% of the mean CI value as the threshold. With this threshold value 14.8% of instances were below the threshold in the connected topology and 16.2% in the independent topology. The difference in the proportions is significant according to the statistical test. Thus, even if we use individual thresholds for the topologies, the probability of considering a randomly filled matrix as a consistent matrix is different.

We found that topology of the associated matrix has a significant effect even in the most simplest case (4×4 matrix, 2 missing elements). It is worth investigating the question in more a general domain: for larges matrices and for larger number of missing elements.

4 Acknowledgements

The research was supported by the National Research, Development and Innovation Office under Grant FK 145838 and the János Bolyai Research Scholarship of the Hungarian Academy of Sciences.

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MULTI-CRITERIA MILP-BASED SCHEDULING OF OUTPATIENT DIAGNOSTIC EXAMINATIONS

János Baumgartner, Ákos Orosz, Tibor Dulai, Ágnes Vathy-Fogarassy, Zoltán Süle
University of Pannonia, Department of Computer Science and Systems Technology
Egyetem str. 10., Veszprém, 8200, Hungary
baumgartner.janos@mik.uni-pannon.hu

Abstract: Scheduling diagnostic and therapeutic examinations in outpatient care requires balancing patient-centered planning with strict medical and operational constraints. This study presents a hybrid modeling framework that integrates the structural modeling capabilities of the P-graph methodology with the optimization power of Mixed-Integer Linear Programming (MILP) to address the personalized scheduling of multiple interrelated procedures for individual patients. The P-graph framework is employed to capture the structural feasibility space of all admissible examination sequences, based on service dependencies, repetition structures, and optional preferences. This graph-based representation provides a complete combinatorial map of process configurations, ensuring that only structurally valid examination plans are considered in the subsequent optimization stage.

Building upon the P-graph structure, we formulate a MILP model that selects an optimal schedule from the feasible set, subject to constraints such as fixed procedure durations and time windows, inter-examination repetition constraints, such as, minimum and maximum days between repeated tests, points of care and optional preferences for specific locations or clinicians. The MILP formulation introduces binary decision variables to encode slot assignments for each procedure instance, and integer or continuous variables to represent temporal positioning. The objective function is flexibly designed to accommodate multiple criteria, such as minimizing the earliest possible completion time, minimizing the elapsed time between the first and last appointment, and minimizing the number of days the patient is required to visit the care provider. These criteria can be tuned via user-defined factors, enabling customized scheduling strategies per institutional or clinical policy.

By combining P-graph-based structural validation with MILP-based temporal and resource optimization, the proposed model guarantees feasible, efficient, and personalized scheduling solutions. This makes the framework particularly suitable for integration into clinical decision support systems, especially in complex outpatient settings involving repetitive diagnostics or therapy regimens.

Keywords: P-graph, mixed-integer linear programming, examination scheduling, healthcare optimization, multi-objective scheduling

1 INTRODUCTION AND PROBLEM DEFINITION

The organization of diagnostic and therapeutic examinations in outpatient care has long been a critical challenge for healthcare providers. Increasingly complex diagnostic pathways, the necessity of repeated procedures, and patient-centered expectations demand planning methods that are both efficient and flexible. Scheduling is not only an operational issue but also a significant factor influencing patient satisfaction and treatment outcomes. Earlier the methodological foundation for optimizing alternative patient appointment schedules by introducing the P-graph framework into healthcare operations has been laid, which work modeled appointment rescheduling as a combinatorial problem, generating all structurally feasible scheduling alternatives while considering resource limitations and patient-specific constraints [1]. Our next result has been built directly upon this methodology by implementing a complete software framework that operationalizes the P-graph-based optimization for real-world use. This next step translated the theoretical approach into a practical tool, capable of supporting automated, optimal appointment planning in complex healthcare settings [2]. In practice, outpatient scheduling often suffers from two extremes: either fragmented appointments spread across too many days, or overly compressed plans that are infeasible due to clinical or resource constraints. This duality underlines the need for robust scheduling

methodologies that can guarantee structural feasibility, adapt to practical requirements, and provide rapid solutions suitable for integration into hospital information systems. The task is to construct feasible and efficient appointment plans for a single patient, considering the required diagnostic or therapeutic procedures and their medical dependencies. Each procedure must be assigned to a concrete time slot in the medical facility's calendar. Since outpatient calendars typically contain a very high number of available slots, even simple problems can quickly escalate into large-scale optimization tasks. For example, a patient with three examinations might have several dozen feasible sequences depending on the calendar structure, the duration of the examinations, and the location or clinician preferences. Three main objectives are considered when defining patient-centered schedules: (i) minimizing the earliest possible completion time, (ii) minimizing the total time elapsed between the first and the last appointment, and (iii) minimizing the number of distinct days the patient is required to visit the healthcare facility. In real applications, one of these goals is chosen as the primary criterion, while the others are applied in a hierarchical manner to further refine the solution. These objectives directly reflect the dual perspective of healthcare operations: efficiency from the provider's side and convenience from the patient's side.

2 MILP-BASED OPTIMIZATION AND MODEL REDUCTION

The structural representation of admissible examination sequences is achieved with the P-graph methodology [3]. Originally developed for process network synthesis, the P-graph framework offers unique capabilities to model combinatorial structures and to guarantee that only structurally valid solutions are considered. In this context, the procedures represent operating units, while the states of the patient and the availability of services are represented as materials. Time-constrained P-graph extensions allow each operating unit and material to be associated with temporal attributes, such as start times, durations, or earliest availability [4]. This explicit structural representation is a key advantage: it avoids the generation of infeasible plans, reduces the search space, and enables efficient enumeration of alternative schedules. Another advantage of the P-graph methodology lies in its rigorous, axiom-based nature. It allows automatic superstructure generation, solution structure generation, and the enumeration of all feasible configurations. For scheduling problems, this means that the combinatorial space of possible appointment orders is mapped before optimization is performed. Consequently, the model avoids being trapped in local optima that may occur when the problem is directly encoded into integer programming variables. On the basis of the P-graph-generated feasibility space, a Mixed-Integer Linear Programming (MILP) model is formulated to determine the temporal allocation of examinations. Binary decision variables encode the assignment of examinations to slots, while integer or continuous variables represent the timing aspects. The constraints ensure respect for procedure durations, time windows, repetition intervals, and optional preferences regarding clinicians or locations. This MILP formulation provides a flexible platform to integrate the multiple objectives outlined earlier.

However, one of the practical difficulties is the explosion of the search space when many free slots are available in the outpatient calendar. A single day may contain dozens of discretized slots, and when several examinations must be scheduled across weeks, the resulting MILP model becomes computationally demanding. To mitigate this, preprocessing techniques are introduced. Slot merging combines consecutive free intervals into larger time blocks, significantly reducing the number of decision variables while still retaining the possibility of fine-grained scheduling. Furthermore, heuristic search space reduction methods divide the overall horizon into smaller intervals and only optimize within those intervals that can realistically contain feasible schedules. This ensures that solutions can be generated within 10–15 seconds, a crucial requirement for real-world clinical decision support applications.

3 ILLUSTRATIVE EXAMPLES

The illustrative test cases cover scenarios from a few examinations with limited slots up to hundreds of options spread over long calendar horizons. Without reduction, solution times can grow from milliseconds to over a million, making direct optimization impractical. By applying interval reduction and slot merging, the heuristic approach consistently cuts computation to below a few seconds while preserving optimality in almost all cases. Even for large-scale instances with more than one thousand slots, the heuristic produced near-optimal results, with deviations limited to a few hours.

No. of examinations	No. of slots	No. of days in whole interval	Original solution time (ms)	No. of days in reduced interval	Heuristic solution time (ms)	Optimal solution value (heuristic result, if different)
6	72	12	12555	1	233	5d 5h (=)
10	10	1	64	1	247	11h 10m (=)
35	40	5	7313	5	6912	4d 6h 5m (=)
6	799	9	1266511	1	17758	4h 35m (=)
3	1196	88	803017	1	635	2h 15m (=)
3	64	10	139	3	390	1d 22h (=)
3	60	7	248	3	236	1d 18h (=)
6	72	12	37162	2	1277	23h 5m (=)
3	1196	211	1235803	3	1276	1d 16h 15m (1d 22h 50m)
3	1196	179	1508726	2	1078	17h 50m (19h 5m)

4 CONCLUSIONS AND OUTLOOK

The integration of P-graph-based structural modeling with MILP-based optimization provides a powerful hybrid framework for outpatient scheduling. It ensures structural feasibility, supports multiple patient-centered objectives, and remains computationally efficient even for large calendars. The combination of preprocessing techniques, such as slot merging, with heuristic reduction strategies guarantees rapid solution times suitable for real-time applications. This research contributes not only to the theory of healthcare scheduling but also to the development of practical clinical decision support systems that can enhance patient care in complex outpatient environments.

Future work will focus on extending the approach to handle multi-patient scheduling scenarios, incorporating stochastic elements such as cancellations or emergency appointments, and validating the framework in larger clinical pilot studies. These developments will further strengthen the potential of P-graph and MILP integration as a foundation for next-generation healthcare optimization systems.

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AN IMPROVED RANKING ALGORITHM IN MULTI-CRITERIA DECISION MODELING METHOD DEX

Marko Bohanec
Jožef Stefan Institute, Department of Knowledge Technologies
Jamova cesta 39, SI-1000 Ljubljana, Slovenia
E-mail: marko.bohanec@ijs.si

Abstract: DEX (Decision EXpert) is a qualitative, hierarchical and rule-based decision-modeling method, particularly suitable for *sorting*: assigning decision alternatives into predefined categories. Under certain conditions and assumptions, DEX can be used also for *ranking*: ordering alternatives from best to worst. There already exists a DEX ranking method called QQ (Qualitative-Quantitative), specifically designed to evaluate alternatives both qualitatively — by assigning them to qualitative classes — and quantitatively — by ranking alternatives within each class. In this paper, we propose a novel method QQ2, aimed at improving some weaknesses of QQ: dependence on assessing criteria weights, and sub-optimal separation of alternatives. QQ2 achieves this by formulating and solving the ranking task as a quadratic optimization problem. An experimental comparison of QQ and QQ2 on 3322 real-life DEX decision tables demonstrated that QQ2 effectively overcomes the limitations of QQ.

Keywords: multi-criteria model, method DEX, decision rules, sorting, ranking, quadratic optimization

1 INTRODUCTION

Multi-criteria decision modeling (MCDM) is an approach used to make decisions when there are multiple, often conflicting, criteria or factors to consider [6]. Typically, MCDM proceeds [1] by defining a set of criteria that are relevant to the decision at hand. These criteria are then incorporated into a structured model that reflects the decision maker's preferences and priorities, and is used to systematically evaluate, compare and analyze different decision alternatives. The outcomes provide valuable information to making a well-informed decision.

According to Roy [8], there are three main types of decision problems: (1) *choosing*: selecting the best alternative from a set of available ones, (2) *ranking*: ordering alternatives from best to worst, and (3) *sorting*: assigning alternatives into predefined categories.

In this paper, we focus on a MCDM method DEX (Decision EXpert). DEX [4] is (1) *hierarchical*: a DEX model consists of hierarchically structured attributes; (2) *qualitative*: all attributes in a DEX model are symbolic, taking values such as "bad", "medium", "excellent"; (3) *rule-based*: decision alternatives are evaluated according to decision rules, acquired from the decision maker and represented in the form of decision tables.

DEX is primarily a sorting MCDM method: the evaluation process assigns each alternative to some distinct final evaluation class [7]. This is because all components of a DEX model are qualitative: attributes and their values, and decision rules that govern the evaluation process. Alternatives are described by qualitative input values and all evaluation results are qualitative.

In practice, however, it is often necessary to address other tasks than just sorting, i.e., choosing and ranking. For example, when there are several alternatives assigned to the same evaluation category, we may still want to distinguish between them and choose the best one. Therefore, it is tempting to think about a method that would rank alternatives using DEX models and would require very little additional effort from the decision maker. The method QQ [2] effectively demonstrates that this is indeed possible, provided that we accept some assumptions about preferential ordering of decision rules and their numerical interpretation. However, QQ exhibits two weaknesses: dependence on assessing criteria weights, and sub-optimal separation of alternatives. In what follows, we demonstrate that these weaknesses can

be effectively addressed by employing the principle of dominance and formulating the ranking task in terms of a quadratic optimization problem. We propose a novel method called QQ2.

2 METHODS

2.1 Method QQ (Qualitative-Quantitative)

QQ is an extension of DEX that adds the ability to rank decision alternatives [2]. It is based on three main ideas:

1. *Combined qualitative-quantitative evaluation*: In addition to qualitative evaluation, which is normally carried out in DEX, it introduces a parallel numeric evaluation of alternatives. Numeric evaluations are used to rank alternatives within each qualitative class.
2. *Consistency of evaluations*: The two evaluations are kept consistent with each other. For each alternative assigned to a qualitative class $\boldsymbol{C}$, the corresponding numerical evaluation must lie in the interval $[\boldsymbol{c}-\mathbf{0.5}, \boldsymbol{c}+\mathbf{0.5}]$, where $\boldsymbol{c}$ is the ordinal number of $\boldsymbol{C}$. The values $+\mathbf{0.5}$ and $-\mathbf{0.5}$ are interpreted as "ideal" and "anti-ideal" evaluations within $\boldsymbol{C}$, respectively.
3. *Automatic construction of the quantitative evaluation function,* which is developed (under some assumptions) from information already available in the DEX model. No additional information is requested from the decision maker.

1.1 QQ constructs the numeric evaluation function $\boldsymbol{Q}: \mathbb{R}^{\boldsymbol{m}} \rightarrow \mathbb{R}$, where $\boldsymbol{m}$ is the number of arguments (input attributes), in the way illustrated in

Figure *1*. The table on the left is an example of a DEX decision table that consists of rows, called elementary decision rules. In this case, the rules map all the combinations of values of attributes PRICE and TECH.CHAR. (technical characteristics) to the qualitative evaluation of a personal CAR.

QQ assumes that rules can be interpreted as points in a multi-dimensional space (

Figure *1*, right) and that they can be sufficiently well approximated (in the least squares sense) by a hyper-plane. Relative attribute weights are inferred from the slopes of this hyperplane along each argument direction (in this case, the weight ratio between PRICE and TECH.CHAR. is 60:40). Finally, QQ partitions the hyperplane to segments corresponding to each class value of the output attribute (CAR). In order to maintain the consistency between qualitative and quantitative evaluation, each segment is scaled so as to lie within the $c \pm 0.5$ interval for each class C.

	PRICE	TECH.CHAR.	CAR
1	high	bad	unacc
2	high	acc	unacc
3	high	good	unacc
4	high	*exc*	unacc
5	medium	bad	unacc
6	medium	acc	acc
7	medium	good	good
8	medium	*exc*	*exc*
9	*low*	bad	unacc
10	*low*	acc	good
11	*low*	good	*exc*
12	*low*	*exc*	*exc*

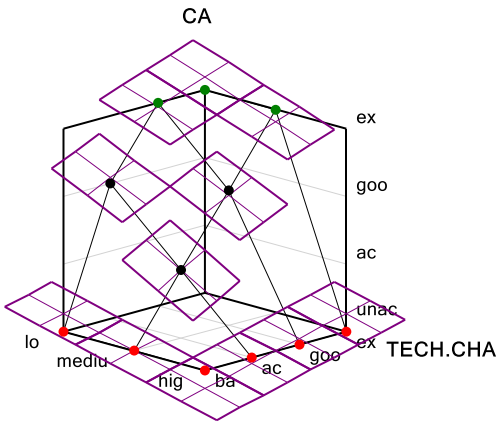


Figure 1: A decision table (left) and the numeric evaluation function Q constructed by QQ (right).

It is important to understand that Q is not used in isolation from other decision tables, but may depend on decision tables positioned at lower levels in a DEX model hierarchy. Since each Q outputs numeric values of the $c \pm 0.5$ type, it must also be able to accept inputs of the same type. This is the reason why Q is composed of ± 0.5 rectangles surrounding each elementary-rule point. Overall, Q is represented as a collection of such rectangles.

QQ has a major weakness: it depends on the notion of (numeric) weights. This seems "unnatural" for a fully qualitative method like DEX. Furthermore, weights make sense only if decision rules form a multi-dimensional pattern that is linear or close to linear. This is often not the case, and is particularly pronounced in cases involving arguments that are not preferentially ordered. Also, the assumption that the weight ratio is constant throughout the whole decision space restricts individual hyperplane partitions to adapt to the nearby contexts and better discriminate between the points.

2.2 Method QQ2

1.2 QQ2 is based on the same principles as QQ, but constructs the function $\boldsymbol{Q}$ differently. Instead of weights, it employs the concept of *preferential dominance* of decision rules. Consider decision table CAR in

1.3 Figure *1*. There, rules 1, 2, 3, 4, 5 and 9 all map to the same class CAR="unacc". However, by comparing conditional rule parts, we can see that rule 1 represents a worse situation than rule 2, because TECH.CHAR. are "bad" and "acc", respectively, and "bad" is worse than or equal to "acc". Therefore, we can say that $\boldsymbol{r_1 \preccurlyeq r_2}$, where '$\preccurlyeq$' is a weak preference relation. In this way, we can establish two rankings for CAR="unacc": $\boldsymbol{r_1 \preccurlyeq r_2 \preccurlyeq r_3 \preccurlyeq r_4}$ and $\boldsymbol{r_1 \preccurlyeq r_5 \preccurlyeq r_9}$, and represent them with the lattice in Figure *2*. Each rule has an associated value $\boldsymbol{q_i \in \mathbb{R}, i = 1, \ldots, n}$. Two elements are added, $\boldsymbol{q_L, q_U} \in \mathbb{R}$: the lower and upper bound of $\boldsymbol{Q}$, respectively. Similar lattices can be constructed for any other class.

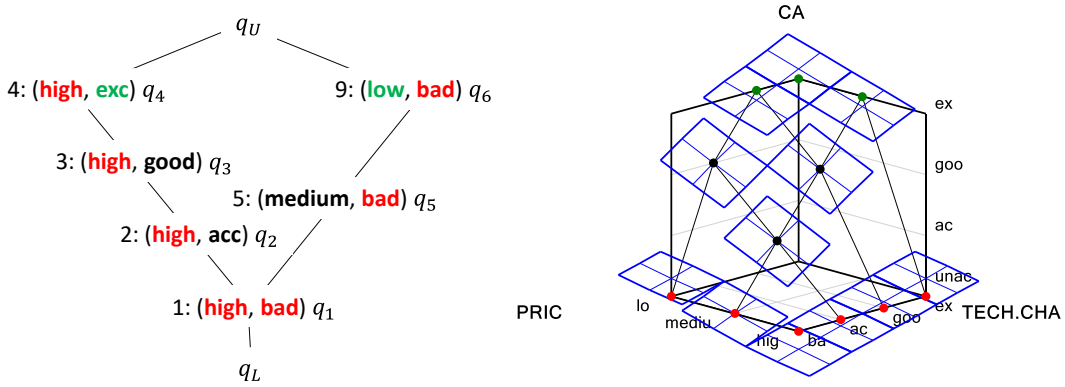


Figure 2: Lattice of CAR decision rules that map to C="unacc" (left) and Q constructed by QQ2 (right).

Having set the lattice, it is now clear what to do: place $q_1, \ldots, q_n$ evenly between q_L and q_U, considering all lattice branches. We formulate this task in terms of a quadratic optimization problem, so as to minimize the sum of squared distances between adjacent points in the lattice:

Variables: $q_0 = q_L, q_1, q_2, \ldots, q_n, q_{n+1} = q_U$
Minimize $\sum_{(i,j)\in L}(q_j - q_i)^2 = \sum_{i=0}^{n+1}(|N_i| {q_i}^2 - 2\sum_{j=0}^{n+1} q_i q_j)$
with respect to constraints: $q_0 = 0$, and $q_j - q_i \geq 1$ for $\forall (i,j) \in L$.

Here, L denotes the set of indices of adjacent rule pairs in the lattice, and N_i is the set of neighbors to rule r_i. The value $q_U = q_{n+1}$ is determined as part of the solution and need not be specified in advance. After solutions $q_1, q_2, \ldots, q_n, q_{n+1}$ have been obtained, they have to be scaled so that the ± 0.5 rectangles around each rule stay within the $c \pm 0.5$ interval.

Figure *2* (right) shows the result for the CAR decision table.

The results of QQ (

Figure *1*) and QQ2 (

Figure *2*) appear very similar. However, a closer look reveals that QQ's hyperplanes are generally narrower than QQ2's in the vertical direction. This is because QQ's are constrained with the 60:40 weight ratio, while QQ2's successfully adapt to the absence of adjacent rules that may have constrained their positions.

3 EXPERIMENTAL EVALUATION AND IMPLEMENTATION

We implemented QQ and QQ2 in R and experimentally evaluated them on a set of 3322 DEX decision tables, which were extracted from the database of real-life DEX models [3]. For each decision table, we observed: (1) S*eparation*: average distance between dominated adjacent points, defined as $\frac{1}{|L|}\sum_{(i,j)\in L}|q_j - q_i|$; (2) *Time*: average execution time per decision table. Results (Table 1) indicate that QQ2 clearly outperforms QQ with respect to both measures; the difference is statistically significant. The difference in execution time is likely due to the fact that QQ was implemented fully in high-level R, while QQ2 used a machine-level optimization package "quadprog". Anyway, both algorithms turned out fast and suitable for practice.

Table 1: Results of experimental evaluation of QQ and QQ2.

Measure	**QQ**	**QQ2**
Separation	0.1737442	0.2122702
Time [ms]	197.95	8.34

We also implemented QQ2 as part of the software DEXiWin [5]. The supported functionalities include: (1) calculating and presenting q-values for each decision table, (2) drawing Q for both QQ and QQ2, and (3) evaluating alternatives using QQ2.

4 CONCLUSIONS

The main contribution of this study is an improvement of the old, but still state-of-the-art DEX ranking algorithm QQ. Instead of using the concept of attribute weights, which is somewhat alien to the qualitative modeling approach of DEX, the new algorithm QQ2 employs the principle of dominance, which seems more "natural" and appropriate. QQ2 constructs the numeric aggregation function Q using quadratic optimization with linear constraints. Experimental evaluation confirmed that QQ2 statistically significantly outperforms QQ in terms of separation, i.e., coverage of the output dimension. Consequently, QQ2 is more sensitive and better separates decision alternatives being ranked. QQ2's rectangles, drawn around each rule, are not constrained by QQ's constant weight ratio and better adapt to the positions of adjacent decision rules.

Acknowledgement
The author acknowledges the financial support from the Slovenian Research and Innovation Agency under the programme Knowledge Technologies (No. P2-0103).

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PROCEDURE FOR COMPARING GROUP VALUES OF DIFFERENT AGGREGATION METHODS

Vesna Čančer
University of Maribor, Faculty of Economics and Business
Razlagova 14, 2000 Maribor, Slovenia
E-mail: vesna.cancerm.si

Abstract: This paper develops a procedure for comparing group values obtained by selected methods for aggregating individual decision-makers' values in group decision-making. It also aims to illustrate the simulation-based procedure using samples of individual values obtained from a uniform distribution, for both small and large-scale groups. The results show that, except for the pair arithmetic mean–weighted arithmetic mean, there are statistically significant differences between the group values obtained by different aggregation methods for all considered groups.

Keywords: aggregation method, group value, multi-criteria decision-making, related samples, simulation, uniform distribution.

1 INTRODUCTION

In group multi-criteria decision-making (MCDM), various group aggregation methods [1, 3, 4, 5, 6] can be used in different group decision situations to calculate group values from the individual values of group members. To address problems with conflicting criteria and particularly the differing interests of the involved decision-makers (DMs), Aggregation of Individual Priorities (AIP) procedures are most commonly used [5]. When group members – individual DMs are capable of independently executing all phases of the frame procedure for MCDM [2], aggregate individual values are obtained. The question arises whether the choice of group aggregation method significantly affects the obtained group values.

This paper therefore explores the impact of aggregation methods, used for calculating group values from individual values, on group values. For this purpose, it develops a procedure for comparing group values obtained by selected methods for aggregating individual values of participating DMs in group MCDM. It also aims to illustrate the simulation-based procedure, on samples of 10 000 individual values obtained from a uniform distribution, for 2, 3, 10, and 33 DMs. It determines whether there are statistically significant differences between the group values obtained using different methods for aggregating individual values into group values.

2 METHODS

The methodology used in the procedure for comparing group values of different aggregation methods includes simulation, the adjusted selected measures of central tendency, and paired-samples tests.

2.1 Selected methods for aggregating individual values in group values

In this research is assumed that the aggregated individual values of DMs are already known – they are obtained by simulation, so we focus only on the aggregation of the aggregated individual values of alternatives into group values of alternatives. The research addresses methods for calculating group values from individual values that are easy to use and are frequently applied in group MCDM, namely:

- Arithmetic mean (AM):

$$AM_i = \frac{1}{K}\sum_{k=1}^{K} v_{ik}, \quad (1)$$

where AM_i is the arithmetic mean of the ith case, K is the number of DMs, and v_{ik} is the individual aggregate value of the kth DM at the ith case.

- Geometric mean (GM):

$$GM_i = \sqrt[K]{\prod_{k=1}^{K} v_{ik}}, \tag{2}$$

where GM_i is the geometric mean of the ith case.

- Weighted arithmetic mean (WAM):

$$WAM_i = \sum_{k=1}^{K} w_k v_{ik}, \tag{3}$$

where WAM_i is the weighted arithmetic mean of the ith case and w_k is the weight of the kth DM.

- Weighted geometric mean (WGM):

$$WGM_i = \prod_{k=1}^{K} v_{ik}{}^{w_k}, \tag{4}$$

where WGM_i is the weighted arithmetic mean of the ith case. The DMs' weights in (3) and (4) are normalized: $\sum_{k=1}^{K} w_k = 1$.

2.2 Procedure for comparing group values obtained by different aggregation methods

The procedure for comparing group values obtained by different aggregation methods for calculating group values from individual values in MCDM is based on simulation and paired-samples tests. First, the number of DMs in the group is determined. Then, simulation is used for generating individual aggregate values: the number of cases is determined, and for each DM, random values are generated using uniform distribution U(min, max), where min is the minimal value and max is the maximal value. In the next step, group values are calculated from simulated individual values using selected aggregation methods. The Kolmogorov-Smirnov test can be used to determine whether the group values obtained by each aggregation method conform to a normal distribution. If the answer is positive for the group values obtained by the considered aggregation methods, the paired-samples t-test can be used to compare means of group values of various aggregation methods; otherwise, the Wilcoxon signed rank test can be used to compare medians of group values of various aggregation methods. Together with statistical significance (p), the value of the effect size:

$$r = \frac{Z}{\sqrt{N}}, \tag{5}$$

where Z is the Z-score and N is the total number of non-zero differences, should be calculated, and its absolute value should be interpreted because of the large sample size.

3 EXAMPLES WITH RESULTS

The presented procedure for comparing group values obtained by various aggregation methods was supported by IBM SPSS Statistics (Version 29). We created four examples, specifically simulations for small groups with 2 and 3 DMs, and large-scale groups with 10 and 33 DMs. For each DM, we created 10 000 random values based on the uniform distribution U(0.001, 0.999), which are simulated aggregated values of the individual DM. Then, 10 000 group values were calculated by each aggregate method, namely by AM (1), WAM (3), GM (2), and WGM (4), in each example. Following the results of the Kolmogorov-Smirnov test for the obtained group values by each aggregation method ($p < 0.001$), the nonparametric Wilcoxon signed rank test was used to determine whether there are statistically significant differences between the group values obtained by several aggregation methods. The exception is a pair of

AM–WAM in the case of 33 DMs, where the Kolmogorov-Smirnov test showed that the values of AM and WAM comply with a normal distribution ($p > 0.05$), so we used a paired-samples t-test.

In example 1, we simulated random individual aggregation values for 2 DMs. For the calculation of group values using the WAM (3) and WGM (4), weights of 0.6 and 0.4 are considered, which are often used for weighting, for example, quantitative and qualitative criteria. In example 2, we simulated random individual aggregation values for 3 DMs. For the calculation of group values using (3) and (4), weights of 0.5, 0.3, and 0.2 are considered, which are often used for weighting when dealing with three elements [5]. The results of the Wilcoxon signed rank test are, together with the calculated effect size, for the considered small groups presented in Tab. 1. Example 3 included the simulation of random individual aggregation values for 10 DMs. The weights of DMs were determined by using the SWING method. Under the assumption that the first DM is the most important, which is why she was assigned 100 points, and each subsequent one is 10 points less important, the obtained weights used in (3) and (4) are as follows: 0.182, 0.164, 0.145, 0.127, 0.109, 0.091, 0.073, 0.055, 0.036, and 0.018. Example 4 included the simulation of random individual aggregation values for 33 DMs. The weights of DMs were determined by using the SWING method. Under the assumption that the first DM is the most important, which is why she was assigned 100 points, and each subsequent one is 3 points less important, the obtained weights used in (3) and (4) are as follows: 0.0583, 0.0565, 0.0548, 0.053, 0.0513, 0.0495, 0.0478, 0.046, 0.0443, 0.0425, 0.0408, 0.039, 0.0373, 0.0355, 0.0338, 0.0321, 0.0303, 0.0286, 0.0268, 0.0251, 0.0233, 0.0216, 0.0198, 0.0181, 0.0163, 0.0146, 0.0128, 0.0111, 0.0093, 0.0076, 0.0058, 0.0041, and 0.0023. Tab. 2 shows the results of the Wilcoxon signed rank test and the values of effect size (5) for the considered large-scale groups.

Table 1: Results of the Wilcoxon signed-rank test and effect size value for small groups

	Two decision-makers			***Three decision-makers***		
	GM	***WAM***	***WGM***	***GM***	***WAM***	***WGM***
AM	$p < 0.001$ $\|r\| = 0.866$	p = 0.993 $\|r\| = 0$	$p < 0.001$ $\|r\| = 0.636$	$p < 0.001$ $\|r\| = 0.866$	p = 0.461 $\|r\| = 0.007$	$p < 0.001$ $\|r\| = 0.686$
GM		$p < 0.001$ $\|r\| = 0.646$	$p < 0.001$ $\|r\| = 0.04$		$p < 0.001$ $\|r\| = 0.712$	$p < 0.001$ $\|r\| = 0.097$
WAM			$p < 0.001$ $\|r\| = 0.866$			$p < 0.001$ $\|r\| = 0.866$

Table 2: Results of the Wilcoxon signed-rank test and effect size value for large-scale groups

	Ten decision-makers			***Thirty-three decision-makers***		
	GM	***WAM***	***WGM***	***GM***	***WAM***	***WGM***
AM	$p < 0.001$ $\|r\| = 0.866$	p = 0.275 $\|r\| = 0.011$	$p < 0.001$ $\|r\| = 0.851$	$p < 0.001$ $\|r\| = 0.866$	p = 0.333* $\|r\| = 0.010$*	$p < 0.001$ $\|r\| = 0.866$
GM		$p < 0.001$ $\|r\| = 0.861$	$p < 0.001$ $\|r\| = 0.034$		$p < 0.001$ $\|r\| = 0.866$	$p < 0.001$ $\|r\| = 0.046$
WAM			$p < 0.001$ $\|r\| = 0.866$			$p < 0.001$ $\|r\| = 0.866$

*As AM and WAM conform to a normal distribution, the results of the paired samples t-test should be taken into consideration: $p = 0.301$, Cohen's $d = 0.01$.

The results in Tab. 1 and Tab. 2 show that the medians of the group values obtained by AM (1) and WAM (3) are not statistically significantly different ($p > 0.05$); this is also supported by the effect size: $|r|$ is 0 in the case of two group members and tiny ($|r| < 0.1$) for larger group sizes. In case of 33 DMs, the results of the paired-samples t-test show that the null hypothesis

that the mean values of the group values obtained by AM and WAM are equal cannot be denied ($p > 0.05$). Furthermore, although the results of the Wilcoxon signed-rank test indicate statistically significant differences between the medians of group values obtained by GM (2) and WGM (4), the effect size ($|r| < 0.1$) shows that these results are practically not meaningful. The differences between medians of the pairs of group values obtained by other aggregation methods are statistically significant ($p < 0.001$).

4 CONCLUSION

The obtained results can let us respond to the research question posed in the introduction, that the choice of group aggregation method significantly affects the obtained group values. Except for the group values obtained by AM and WAM, for which we did not find statistically significant differences, and with GM and WGM, where there were statistically significant differences but are not practically meaningful, for other pairs of group values, statistically significant differences between the medians apply. Furthermore, it can be concluded that the number of DMs does not significantly affect the differentiation between group values, regardless of the group size.

The paper contributes to the theoretical implications of comparing the aggregate group values. Conclusions apply to a wide range of group MCDM methods where group values are aggregated from individual alternatives' values and are not limited to a particular multi-criteria method. Regarding practical implications, it illustrates how to apply the developed procedure. In a large number of cases and based on statistically significant differences, it suggests that the choice of method for aggregation matters. Based on other analytical tools and results [3, 4, 5], similar conclusions can be drawn for other aggregation methods. The research presented in this paper, however, indicates that in selected cases, it may not be necessary to pay too much attention to assigning weights to DMs.

The simulations are limited to two small and two large-scale groups, selected AIP methods, and selected normalized weights of DMs. Further research can include various weight ratios of DMs and other aggregate group methods.

Acknowledgement
This research is a part of the research program P5-0023: Entrepreneurship for Innovative Society, supported by the Slovenian Research Agency.

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AN INTERVAL-BASED GROUP AGGREGATION APPROACH TO IMPROVE SATISFACTION IN DEMATEL

Petra Grošelj
University of Ljubljana, Biotechnical Faculty
Jamnikarjeva 101, 1000 Ljubljana, Slovenia
petra.groselj@bf.uni-lj.si

Gregor Dolinar
University of Ljubljana, Biotechnical Faculty
Jamnikarjeva 101, 1000 Ljubljana, Slovenia
gregor.dolinar@bf.uni-lj.si

Abstract: Group decision-making often requires a balance between diverse expert opinions that cannot be captured by traditional aggregation methods such as the arithmetic mean, resulting in lower expert satisfaction. This study introduces an interval-based aggregation approach within the DEMATEL method to improve the representation of expert opinions and their satisfaction. Expert judgments are aggregated into intervals and satisfaction is measured using a distance-based function. A trade-off between satisfaction and reliability is investigated using different interval construction methods. A case study on supplier selection in the automotive industry shows that interval-based aggregation, especially using standard deviation-based widths, increases satisfaction. A combined objective function is proposed that balances satisfaction and uncertainty and provides a flexible tool for improving decision quality in group MCDM environment.

Keywords: Multi-Criteria Decision-Making, DEMATEL, Group Decision Making, Interval Weights, Uncertainty, Satisfaction of Decision Makers

1 INTRODUCTION

Group decision making is an important part of multi-criteria decision-making as it involves multiple decision makers with diverse knowledge, preferences and objectives. One of the biggest challenges in group decision making is to reach a decision that is widely accepted by the decision makers. Several satisfaction indices have been proposed in the literature to assess the satisfaction of decision makers [2; 3; 5].

Group decisions are usually determined by aggregating individual opinions using mean value operators [6]. However, representing group opinion with a single aggregated value often does not capture the diversity of opinions [4] and may result in lower satisfaction. The novelty of this study is to introduce interval-based aggregation approach that leads to interval group judgments, obtained by aggregating individual judgments into intervals rather than single values. To measure satisfaction in this framework, we introduce a linear function based on the distance between expert judgment and the resulting interval group judgment. Since satisfaction is closely related to the quality of the decision, the width of interval judgments should also be taken into account: wider intervals may reduce the reliability of the outcome. In this study, we focus on Decision making trial and evaluation laboratory (DEMATEL), an effective method for analyzing the interdependencies among the elements of the system. Our main contributions include a novel distance-based function for measuring expert satisfaction and an investigation of the trade-off between satisfaction and uncertainty. Our aim is to improve decision quality in group MCDM by balancing satisfaction and reliability through interval-based aggregation.

2 METHODS

DEMATEL [7] is a method for evaluating the influences between the elements of the system. Experts use a scale from 0 (no influence) to 4 (very high influence) to express the influence of

one element on another. Usually, the arithmetic mean is used to aggregate expert judgments. In this study, we propose the use of interval group judgments as they can increase satisfaction of the experts $k=1,\ldots,m$ with the final decision:

$$\bar{Z} = \left(\left[z_{ijl}, z_{iju} \right] \right)_{nxn}. \tag{1}$$

To measure the satisfaction of an expert k, we define the satisfaction function with value 1 if expert judgment falls within the interval, 0 if the distance of expert judgment is more than 1 from the interval, and linear function in between:

$$c_{ij}^{(k)} = \begin{cases} 1, & \text{if } z_{ij}^{(k)} \in \left[z_{ijl}, z_{iju} \right] \\ 1 - d_{ij}^{(k)}, & \text{if } z_{ij}^{(k)} \in \left[z_{ijl} - 1, z_{ijl} \right] \bigcup \left[z_{iju}, z_{iju} + 1 \right], \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

with $d_{ij}^{(k)} = \min \left\{ \left| z_{ij}^{(k)} - z_{ijl} \right|, \left| z_{ij}^{(k)} - z_{iju} \right| \right\}$. Group satisfaction is the average of experts' satisfaction across all their judgments:

$$S = \frac{1}{k\left(n^2 - n\right)} \sum_{k=1}^{m} \sum_{i,j=1}^{n} c_{ij}^{(k)}. \tag{3}$$

According to our definition of satisfaction, experts would be most satisfied with very wide group intervals. However, wide interval judgments reduce the reliability of decisions. Therefore, we should seek a trade-off between group satisfaction and the uncertainty of interval judgments. We propose four different ways to obtain interval group judgments.

The first way is the max-min method, in which minimum and maximum individual judgments are used as lower and upper bounds of group intervals [1]:

$$\bar{Z} = \left(\left[\min_{k=1,\ldots,m} z_{ij}^{(k)}, \max_{k=1,\ldots,m} z_{ij}^{(k)} \right] \right)_{nxn}. \tag{4}$$

This method leads to very wide intervals, while group satisfaction is equal to 1. The second option is to use arithmetic mean centred interval group judgments. In this case, the interval judgments have the same width $2d$ for all judgments:

$$\bar{Z} = \left(\left[\bar{z}_{ij} - d, \bar{z}_{ij} + d \right] \right)_{nxn}, \tag{5}$$

or the width of the interval judgments may differ from judgment to judgment and depend on the standard deviation of individual judgments σ_{ij} and parameter λ [3]:

$$\bar{Z} = \left(\left[\bar{z}_{ij} - \lambda\sigma_{ij}, \bar{z}_{ij} + \lambda\sigma_{ij} \right] \right)_{nxn}. \tag{6}$$

For the 0-4 scale, we should make sure that in all cases the lower bound is greater than or equal to 0 and the upper bound is less than or equal to 4. We will compare the presented options using data form the case study [8].

2.1 Case study

We selected a case study from the literature in which the authors presented direct influence matrices of all experts [8] for seven main criteria that are important for auto parts manufacturers to be qualified suppliers: price, delivery, production, quality, service, technological capability

and organization. Seven managers of auto parts manufacturers evaluated the causal relationships among the criteria.

$$Z^{(1),\ldots,(8)} = \begin{pmatrix} 0 & 2,0,0,2,3,1,1 & 2,0,1,1,3,2,0 & 3,1,2,3,3,3,1 & 2,0,0,1,1,1,1 & 2,0,0,1,3,2,0 & 2,0,0,0,1,1,0 \\ 2,0,2,1,3,2,2 & 0 & 2,2,3,0,3,2,2 & 1,1,3,1,3,1,1 & 1,0,2,1,1,3,0 & 1,0,1,0,3,1,0 & 1,0,1,0,1,1,0 \\ 3,0,3,1,3,2,3 & 2,2,3,2,3,2,2 & 0 & 3,1,3,0,3,3,3 & 2,0,2,0,2,1,0 & 3,0,2,1,2,1,2 & 3,0,2,0,1,1,0 \\ 3,0,3,3,3,2,3 & 2,1,3,1,3,1,2 & 2,1,3,0,2,1,3 & 0 & 3,1,0,1,2,0,0 & 3,1,0,1,2,0,1 & 2,1,0,0,1,1,0 \\ 2,1,1,2,3,3,0 & 2,1,1,1,3,2,2 & 3,1,1,0,3,1,0 & 3,1,0,1,3,1,2 & 0 & 3,1,0,1,1,0,1 & 2,1,0,0,1,0,1 \\ 3,0,3,0,3,2,2 & 2,1,3,0,2,1,1 & 3,1,3,1,3,1,3 & 3,2,3,1,3,1,3 & 2,1,0,1,1,1,0 & 0 & 3,1,0,0,1,1,1 \\ 2,1,1,0,3,1,1 & 2,1,1,0,3,2,2 & 3,2,1,0,3,2,2 & 3,2,0,0,3,2,1 & 2,1,0,1,1,0,0 & 3,1,3,0,1,1,1 & 0 \end{pmatrix} \quad (7)$$

3 RESULTS AND DISCUSSION

We aggregated individual judgments using different techniques: arithmetic mean, max-min method, intervals of equal width with *d=1* and intervals derived by standard deviation with λ=0.5 and λ=1.

Table 1: The width of group interval judgments and expert satisfaction

		Arithmetic mean	Interval max-min	Interval d=1	Interval λ=0.5	Interval λ=1
Width of group interval judgments	Mean	0	2.595	1	0.923	1.836
	Min	0	1	1	0.452	0.904
	Max	0	3	1	1.245	2.491
Group satisfaction	Mean	0.357	1	0.664	0.642	0.860
	Min	0.082	1	0.439	0.479	0.797
	Max	0.592	1	0.969	0.824	0.967
Individual satisfaction	Min	0.238	1	0.554	0.531	0.746
	Max	0.510	1	0.832	0.815	0.953

The results (Table 1) show that the use of arithmetic mean leads to low group satisfaction. Using the minimum and maximum of the individual judgments for interval bounds leads to group satisfaction with a value of 1, but the group intervals are very wide, their widths varying from 1 to 3, which leads to unreliable final results. By setting the same width of 1 for all group interval judgments, we have absolute control over the width of the intervals, but the group satisfaction with specific interval judgment varies greatly from 0.439 to 0.969. If the width of group intervals depends on the standard deviation of expert judgments, specific interval judgments would vary in width from 0.452 to 1.245 in the case of λ=0.5 with an interval width equal to σ_{ij} and from 0.904 to 2.491 in the case of λ=1 with an interval width equal to $2\sigma_{ij}$.

We believe that a lower variability in group satisfaction with certain judgments is beneficial and leads to an overall higher group satisfaction. Therefore, we propose aggregation based on standard deviations as the most appropriate method to derive aggregated interval group judgments. To determine the optimal value of parameter λ, which influences the width of group intervals, the uncertainty of the interval judgments should also be taken into account. A balance should be sought between the group satisfaction and the uncertainty of interval group judgments.

We propose to define the function $f(\lambda)$, to be maximized as a linear combination of group satisfaction $S(\lambda)$ and uncertainty $U(\lambda)$:

$$f(\lambda) = \beta S(\lambda) + (1-\beta)\mathrm{U}(\lambda), \quad (8)$$

with $\beta \in [0,1]$ determining the importance of group satisfaction over uncertainty. In a future study, the shape of uncertainty function *U(λ)* should be determined as a function of the width of interval. Another issue that should be studied in more detail is the shape of the satisfaction function $c_{ij}^{(k)}$ that measures the satisfaction of expert *k* with group interval *ij*. In this study, a linear function was used, but we believe that the satisfaction might decrease faster than a linear function if the expert judgment is not included in the group interval, therefore convex function with $q \geq 1$ should be considered:

$$c_{ij}^{(k)}(\lambda) = \begin{cases} 1, & \text{if } z_{ij}^{(k)} \in \left[z_{ijl}, z_{iju} \right] \\ \left(1 - d_{ij}^{(k)}(\lambda)\right)^q, & \text{if } z_{ij}^{(k)} \in \left[z_{ijl} - 1, z_{ijl} \right] \cup \left[z_{iju}, z_{iju} + 1 \right]. \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

4 CONCLUSIONS

This study addresses the inadequate representation of individual opinion diversity in the aggregation of expert judgments using single value methods. By introducing interval group judgments within the DEMATEL framework, we preserve the diversity of individual judgments rather than averaging them, providing a richer and more transparent view of collective judgment. Our results show that the use of intervals, especially intervals based on standard deviation, can significantly improve decision maker satisfaction while maintaining an acceptable level of uncertainty. Future research should focus on optimizing the shape and parameters of the satisfaction and uncertainty functions to further refine the method.

Acknowledgement
This study was supported by the Slovenian Research Agency, Research Program P4-0059.

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BEHAVIOURAL GROUP DECISION SUPPORT FOR POST-INDUSTRIAL TOURISM: INTEGRATING PREFERENCES, SYNERGIES, AND DECISION-MAKERS' COGNITIVE LIMITATIONS

Tomasz Wachowicz
University of Economics in Katowice,
ul. 1 Maja 50, 40-287 Katowice, Poland
E-mail: tomasz.wachowicz@uekat.pl

Eduarda Asfora Frej
Center of Decision Systems and Information Development – CDSID, Universidade Federal de Pernambuco
Av. Da Arquitetura – Cidade Universitária, Recife, PE, Brazil
E-mail: eafrej@cdsid.org.br

Marek Czekajski
University of Economics in Katowice,
ul. 1 Maja 50, 40-287 Katowice, Poland
E-mail: marek.czekajski@edu.uekat.pl

Abstract: Cultural heritage is a key foundation for cultural tourism products (CTPs), but their design is complex due to diverse stakeholders, heritage assets, and evaluation criteria. Stakeholders differ in how they perceive the attractiveness and synergy of CTP components, as well as in their cognitive capacities to use decision-aiding tools. This study demonstrates how the Multiple Criteria Behavioural Group Decision Aiding (MCBGDA) protocol can support decision-makers in formulating a consensual recommendation for designing complex CTPs from available mono-products. Particular attention is given to handling differing stakeholder synergy declarations. The protocol is applied to post-industrial CTP selection in Czeladź, Poland.

Keywords: cultural tourism product, group decisions, multiple criteria decision making, cognitive capabilities, synergy effect, consensus.

1 INTRODUCTION

Cultural tourism plays a key role in local development, providing local government units with branding opportunities while attracting tourists seeking authentic experiences. As post-industrial areas repurpose heritage assets, integrated cultural tourism products (CTPs) have gained prominence. Local government units are therefore often interested in developing such products, seeking transparency in decision-making, and balance among the needs of various cultural institutions (stakeholders) in their territories. Thus, designing complex CTPs requires balancing diverse stakeholder perspectives, financial constraints, and multi-criteria evaluations [1, 2].

To address these challenges, the Multiple Criteria Behavioural Group Decision Aiding (MCBGDA) protocol was developed to formalize the decision-making process. The initial model focused on optimizing stakeholder decisions for predefined CTP packages [3, 4], later evolving into a modular optimization framework [5]. Given the stakeholders' limited cognitive capabilities, a procedure was also proposed to identify them and assign best-fit decision-aiding tools, drawing on findings from earlier experimental research [6, 7]. A key challenge lies in aggregating stakeholder input on synergy effects among CTP components. Stakeholders differ not only in how strongly they perceive synergies within particular combinations of mono-products, but also in which mono-products they believe generate synergies in the first place. This study aims to develop a methodology for systematically integrating such synergy effects into the MCBGDA model, ensuring that stakeholder insights on interdependencies are effectively captured and incorporated.

2 METHODOLOGY

The MCBGDA protocol follows a six-stage approach, incorporating various analytical techniques: (1) Problem structuring for identification of CTP categories, component products, stakeholders, and evaluation criteria [8]; (2) Analyzing stakeholder's cognitive styles to identify limitations in the effective use of decision-support techniques [9]; (3) Eliciting individual preferences through the multi-criteria decision-aiding (MCDA) technique best-fitting the stakeholder's style to ensure a reliable representation of their preference structure [7]; (4) Aggregating stakeholders' preferences to a consensus scoring system [10]; (5) Elicitating synergy-related information from stakeholders regarding the attractiveness and cost-reduction effects of combining specific mono-products, and constructing a model for group-level synergy integration; and (6) Selecting the optimal mix of components to build a complex CTP through the mathematical programming model based on the knapsack problem approach [11].

The summary of the elements involved in the CTP design problem, as required by the MCBGDA protocol to generate a final recommendation for the optimal systemic cultural tourism product (CCTP), is formalized by the following structure:

$$\mathcal{P} = \{C, A,\ I, \mathbb{F}, P,\ \mathbb{S}, S^{\mathrm{G}}, E, R^{S}, R^{E}, \mathrm{CCTP}\} \tag{1}$$

where where:

C – set of CTP component types, $C = \{C_j\}_{j=1,\dots,J}$,

A – set of alternatives defined within subsets for each CTP component seperately, $A = \{a_i\}_{i=1,\dots \sum_{j=1}^{J}|A^j|} = \bigcup_{j=1}^{J} A^j$, $A^j = \{a_c^j\}_{c=1,\dots,|A^j|}$,

I – set of stakeholders, $I = \{I^k\}_{k=1,\dots,K}$,

$\mathbb{F}$ – set of collections of criteria used by the stakeholders to evaluate alternatives, $\mathbb{F} = \{F^k\}_{k=1,\dots,K}$, $F^k = \{f_p^k\}_{p=1,\dots,P^k}$,

P – cognitive profiles characterizing given stakeholders, $P = \{P^k\}_{k=1,\dots,K}$,

$\mathbb{S}$ – set of collections of stakeholders' scoring systems with cardinal scores for all alternatives from A, $\mathbb{S} = \{S^k\}_{k=1,\dots,K}$, $S^k = \{v_i^k\}_{i=1,\dots,I}$,

S^{G} – set of consensual scores which represent group preferences, $S^G = \{\ddot{v}_i\}_{i=1,\dots,I}$,

E – set of costs of producing alternatives from A, $E = \{e_i\}_{i=1,\dots,I}$,

R^S – set of synergistic relations describing an increase of the attractiveness for given mixes of alternatives from A,

R^E – sets of synergistic relations describing cost reductions for given mixes of alternatives from A,

CCTP – optimization model (based on the knapsack problem) for selecting the optimal complex CTP.

3 POSTINDUSTRIAL CTP IN CZELADŹ

The MCBGDA protocol was successfully applied to design an optimal post-industrial cultural tourism product in Czeladź, Poland. A total of 21 stakeholders were identified, and 22 CTP component alternatives were analyzed, including maps, multimedia experiences, events, museum exhibitions, guided tours, and heritage trails. Stakeholder interviews provided evaluation criteria and preferences, processed using the analytical engine from Step 4. The final 12-component CTP was selected to maximize tourism potential within a PLN 100,000 budget. A sensitivity analysis addressed potential biases in stakeholder-defined preferences.

4 SYNERGY EFFECTS IN MCBGDA

Incorporating synergy information in Step 5 of the MCBGDA model is straightforward under the assumption of consistent, consensus-based data. They are modeled then in Step 6 utilizing dummy variables through a set of linear constraints, enabling precise solutions using classical analytical methods [12]. In the MCBGDA model for the Czeladź study, synergy was assessed by a subject-matter expert (for attractiveness) and a financial specialist (for costs). However, during interviews, synergy assessments were also suggested by other stakeholders, leading to the challenge of constructing consensus-based synergy information.

In our approach, we draw inspiration from robust portfolio modelling [13, 14] and introduce additional mechanisms for enumerating all mixes of synerges defined and representing their strengths through lower and upper bounds, deriving them from varying estimates provided by the stakeholders. o address the challenges associated with individual synergy declarations, we propose the following:

- Designing mechanisms to combine synergy strength for overlapping synergies (i.e., those that share common subsets of alternatives) by defining synergy clusters.
- Introducing mechanisms to prevent double-counting of synergy strengths, ensuring that overlapping synergies within clusters cannot both contribute simultaneously, by defining supersets of synergies within clusters.
- Modelling synergies strengths within clusters as interval numbers, with max-max rules used for selection of lower and upper bounds derived from diverse declarations of individual stockholders.
- Analyzing the CCTP problem through simulation, incorporating random sampling from the defined synergy intervals during the optimization process. This allows identification of optimal CTP configurations that are robust to variations in perceived synergy strength.

The final step of the procedure can be extended with an additional sensitivity analysis, aimed at examining the stability of the solution set under changes to selected model parameters. It is particularly recommended for attractiveness scores elicited from stakeholders in Step 3. Despite the deliberate selection of decision-aiding methods best aligned with individual cognitive capacities, stakeholders may still commit minor errors or rely on heuristics [15]. These tendencies may introduce imprecision into the scoring systems they generate, which in turn can affect the aggregated group preference model used in the CCTP optimization process.

5 RESULTS

We applied our approach, including the synergy-handling module, to the CTP design problem in Czeladź Commune—a small town in southern Poland with post-industrial heritage from two coal mines: Ernest-Michał (1876) and Czeladź (1900), and their associated facilities. In total, 21 stakeholders evaluated 22 mono-products. To derive a general CTP recommendation reflecting their preferences and synergy declarations, we used Monte Carlo simulation with 5,000 replications. Each replication involved random sampling of lower and upper bounds to construct interval-based synergy representations.

The same solution was consistently recommended across all replications, including the following mono-products: a1 – Map, a2 – Guidebook, a4 – Educational Tour, a6 – Guidebook on the web, a7 – Multimedia guidebook-gadget, a15 – Post-industrial concert and music workshops, a17 – Post-industrial lights and lasers, a18 – Mix trail, a19 – Questing of ”Saturn” mine heritage, a20 – Questing of ”Czeladź” mine heritage, a22 – Real existing post-industrial heritage trail.

The robustness of this solution was primarily driven by synergy constraints introduced by our model. Notably, pairwise synergy effects were defined for the combinations a15 + a17 and a19 + a20, while a superset synergy constraint was applied to the combination a1 + a2 + a4 + a18 to prevent overcounting and enforce interdependency consistency.

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The 18th International Symposium on
Operational Research in Slovenia

SOR ’25

Bled, SLOVENIA
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Session 7:

Mathematical Optimization and Production Planning

AN EFFICIENT PARALLEL COMPUTATION IMPLEMENTATION OF THE PATH BRIDGING ALGORITHM FOR THE HIGHLY-CONSTRAINED ASYMMETRIC TRAVELLING SALESMAN PROBLEM

Segundo Álvarez-García
Sr. Scientist [a], PhD Student [b]
E-mail: uo53882@uniovi.es, segundo.alvarez-garcia@arcelormittal.com

Nicolás Álvarez-Gil
Sr. Researcher [a]
E-mail: nicolas.alvarez@arcelormittal.com

Rafael Rosillo
Professor [b]
E-mail: rosillo@uniovi.es

[a] ArcelorMittal Global R&D Asturias, Mathematical Optimization Department
P.O. Box 90 (33400), Avilés (Asturias), Spain

[b] University of Oviedo, Business Management Department
Edificio Departamental Este, 1a planta, Campus de Gijón (33204), Gijón (Asturias), Spain

Abstract: This study addresses the scalability of the Path Bridging (PB) algorithm for the Constrained Asymmetric Travelling Salesman Problem (CATSP). The PB is robust in finding feasible solutions in hard-to-solve benchmark instances of size up to 114 nodes, achieving best known solutions. In this work we challenge the algorithm generating new hard-to-solve instances of up to 1000 nodes and we design a parallel PB (PaPB) algorithm able to exploit multiple cores efficiently, after having satisfactorily overcome arisen parallelization issues. We test exhaustively and report results that show that the PaPB efficiently outperforms the PB when sizes start to get big.

Keywords: Combinational optimization, sequencing, metaheuristics, local search algorithms, parallel computation, travelling salesman problem.

1 INTRODUCTION

The renowned Travelling Salesman Problem (TSP) has a long history in the domain of Operations Research and numerous applications, an important one in the field of manufacturing scheduling. This is the case of the Continuous Galvanizing Line Scheduling Problem in the steel making industry, an NP-hard Constrained Asymmetric TSP (CATSP) for which the PB algorithm [2], specifically designed for challenging highly-constrained scenarios, proves to be robust and efficient. A typical batch of steel coils to be scheduled does not usually exceed 120 coils. Yet even for small instances, the previous algorithm in use, the well-known Ant Colony Optimization (ACO), as well as exact solvers struggle to find feasible solutions. The PB is a Local Search (LS) algorithm designed to handle constraints; it iteratively improves an initial sequence by chaining 2- and 3-opt moves until it gets a better sequence. The PB runs two distinct phases, feasibility (< 0.1 s) and cost, running for the bulk of the budget time (180 s).
The motivation of this work is to study the scalability of the PB, by (i) generating bigger hard-to-solve instances and analyzing the impact of instance size in the algorithm, and (ii) eventually evolving the PB to solve these challenging problem sizes with good quality in reasonable time, by means of parallel computation. In our steel making company, often the scheduling software is run not in supercomputers, but in standard machines that nowadays often embed 2 processors with 4 cores each. Parallel algorithms try to exploit this full computation capacity of the machine and can be regarded as a new class of algorithms [1]. Parallel computation has been

widely applied to most optimization algorithms including Genetic Algorithms [3], ACO and LS methods like the Chained Lin-Kernighan and the Lin-Kernighan Heldsgaun-2 [4].

2 METHOD

Building hard-to-solve instances is itself a difficult task. Our methodology has been to analyze the structure of the hard-to-solve small real-world instances, not solved by commercial exact solvers in reasonable time, and mimic their structure: in their underlying graphs we observe that they generally show variable-sized clusters poorly interconnected by a few nodes. We then run these generated instances in the commercial solver Hexaly and hand-pick the worst-performance ones for our benchmark set. In Fig. 1 we can see an example graph for one of the instances picked. We generate a set of 101 selected instances of sizes ranging from 17 to 1000 nodes, all with a known optimum fitness of value 0 by design, for studying purposes, yet they are hard-to-solve instances.

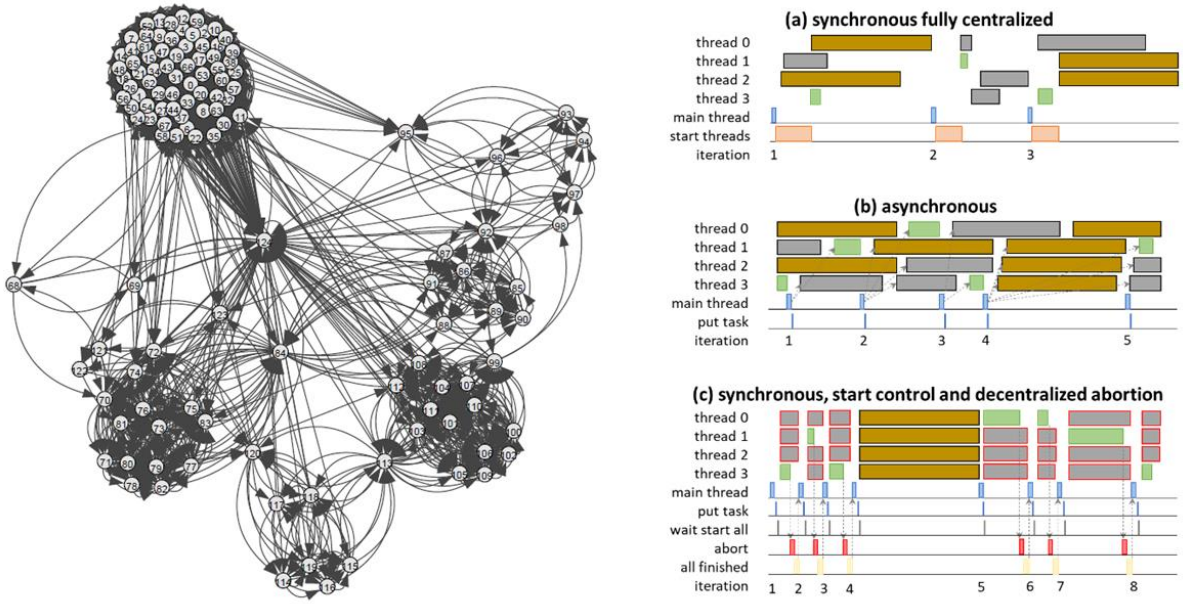


Figure 1: To the left, underlying graph for instance zero_set_C_instance_250_blocks_14z_out_fbh_rcols, of size 125 and 11 clusters. To the right, different parallelization strategies for the PB.

For the design of the PaPB, we neglect the parallelization of the PB's feasibility phase after verifying its performance is still robust, taking below 0.3 s to reach feasibility in all the new instances. For the PB's cost phase, unlike in [4], we opt for a fine-grain parallelization that aims at running several parallel executions of each iteration on multiple cores. We ponder 3 strategies (a), (b) and (c), of which we finally choose (c). A basic method (a) would simply run synchronously several instances of the iteration, i.e. in 4 cores, and consolidate the best result –see Fig 1. to the right, where brown tasks reach maximum length without improvement. Yet this would not capture the full potential of parallelization, leaving cores unloaded while waiting for the rest to end. To fix this, a second strategy (b) can be to run asynchronously. But in this setting, chains that might improve their reference sequence could actually be improving an older best cost, thus not being effective –that is, performing waste computation. See strategy (b) in Fig. 1, tasks in grey color improve an older sequence. A third strategy (c) is to perform synchronous runs of all threads *with earliest successful thread abortion*. We target to minimize those slow iterations –the unsuccessful longest chains of 200 moves, depicted in brown color–, which consume by far the bulk of the time after the initial easy gains. See an example run in Fig. 3 to the left, iterations with k=200 moves (maximum length), blue dots. In our approach, the shorter the chain of LS moves the faster the algorithm, capturing most room for

streamlining. For this implementation, each thread is a stream worker that receives a queue of tasks (the iterations' sequences) and communicates with the other threads via events, aborting the work of all if it finds an improving sequence –see strategy (c) depicted in Fig. 1, where red-contour tasks are those aborted by green tasks. This method promotes shorter chains of moves. An important problem encountered during the implementation of strategy (c) was the way Windows manages threads. The Operative System would not start all threads at once but *optimize* their run attending to its own criteria. To solve this issue, we do a false-start of all the tasks, delaying any k-opt move until all the threads have effectively (false-)started their task. This trick works fine: once all tasks are started by Windows, the threads work in parallel as expected without idle times. See the scheme of this mechanism depicted in Fig. 1, strategy (c).

3 EXPERIMENTAL RESULTS

We run 30 times each algorithm PB and PaPB for each instance, for a tight budget time of 0.33 times the size of the instance (in seconds), in a machine 11th Gen Intel(R) Core(TM) i7-1185G7 @ 3.00GHz with a 64 bits Windows 10 Enterprise. The PaPB is set to run 4 cores.

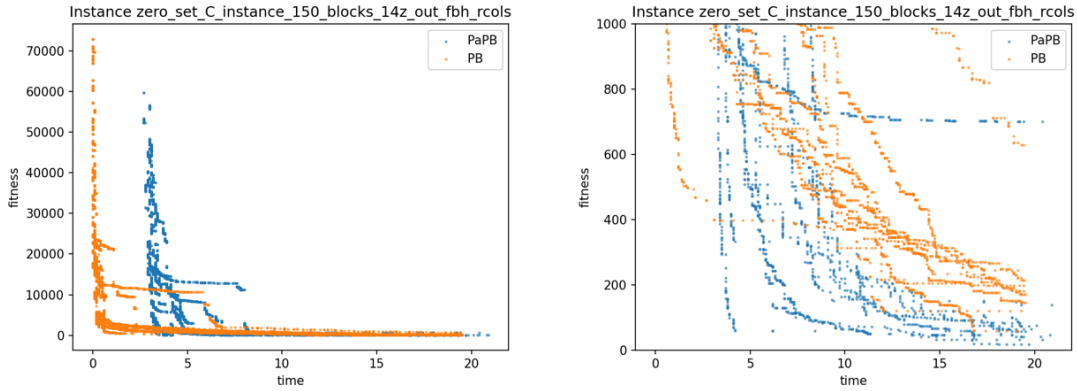


Figure 2: A representative performance comparison for 10 runs of the PB and the PaPB for a big instance, right image zooming-in below fitness 1000. To the left we observe the important delay it takes to start the threads.

One first outcome of the results is the long time it takes to start the 4 threads, typically around 3.8 s. This penalizes the performance of the PaPB in the smallest sizes. But once the 4 threads are at work, the boost in performance is conspicuous, achieving a steeper decreasing cost/time ramp due to the shortening of the length of the chains –see Fig. 2 to the right.

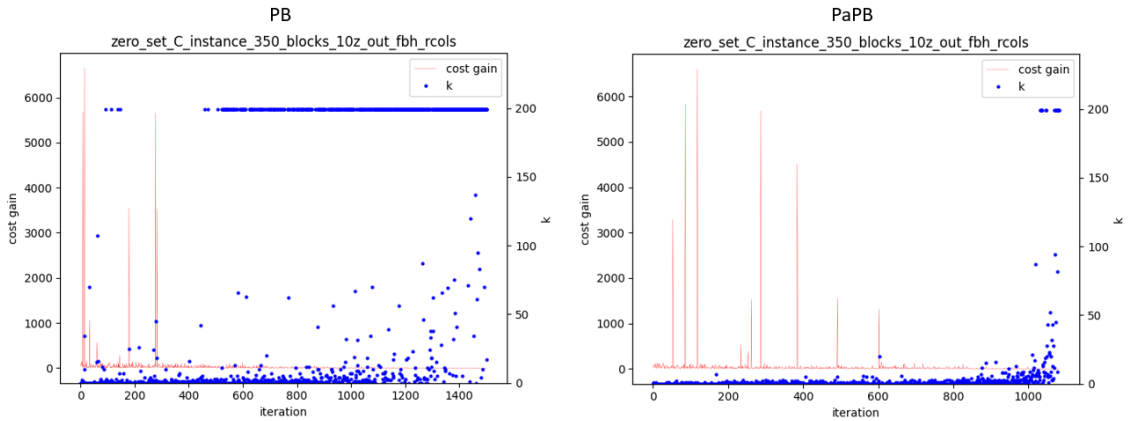


Figure 3: A visual comparison of the PB and the PaPB for one run of a big instance (350 nodes). We can see how the PaPB conspicuously reduces both the number of unsuccessful iterations (those that reach the limit length of the 2-, 3-opt chain, set to k=200 moves, without improving the sequence) and the length of the chains.

It is not trivial to compare the performance of algorithms [1]. As comparing running times would only be fair for runs reaching same fitness quality, and being scalability the object of

our study, we choose to compare fitness quality, averaged per instance size. Note that gathering problems by size only makes sense because they all have same optimum cost 0. Figure 4 displays the results of the experiment, showing the statistics for the PaPB superiority in best cost found in a boxplot chart, in logarithmic scale (chart to the left), and detailing the minimum best cost comparison per size (to the right). The PaPB outperforms the PB in all instances except sizes 17 to 28, the only sizes where the PB still finds more times the optimum value 0.

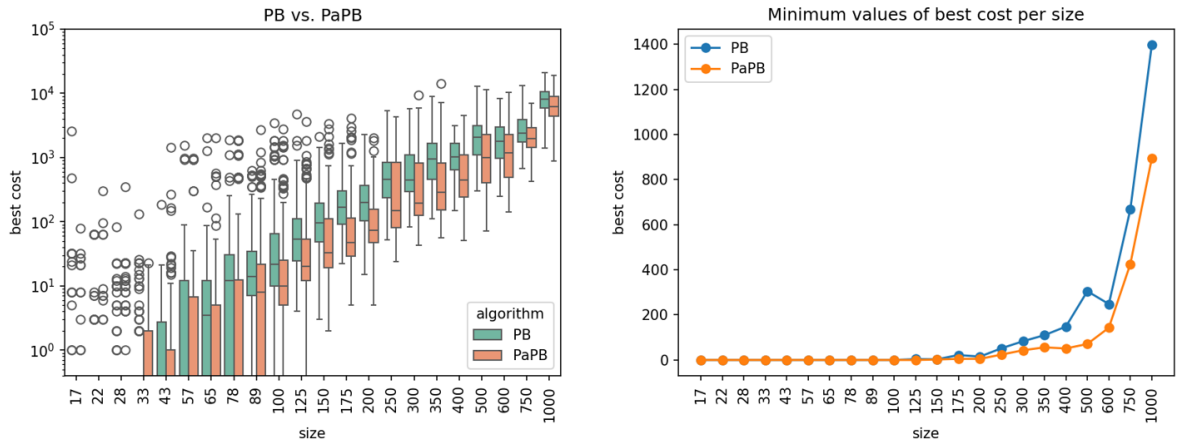


Figure 4: Best costs per instance size, comparison between the PB and the PaPB. To the left, boxplot in logarithmic scale, 30 runs per instance. To the right, minimum of best costs per size.

4 CONCLUSIONS

This work studies the scalability of the Path Bridging (PB) algorithm for the Constrained Asymmetric Travelling Salesman Problem (CATSP). First, we build a set of challenging instances of up to 1000 nodes by analyzing smaller hard-to-solve real-world instances and replicating their graph and cost structure, hand-picking the difficult ones. Then we design a parallel computation algorithm PaPB that improves the performance of the sequential PB. We weigh several parallelization strategies, adopting a fine-grain synchronous parallelization at iteration level, with cores competing for a successful chain of LS moves that improves the reference sequence. Threads communicate with each other and abort the computation as soon as one improves the previous best sequence. The abort signal cuts short the long chains that penalize performance –a key identified inefficiency. The results show that the PaPB algorithm is efficient, outperforming the PB as instance size grows. We observe a scalability problem that rends less efficient the PaPB with 8 cores for the sizes studied. As future work, studying longer running times and even bigger instances should allow to better assess the performance. New parallelization strategies might also be devised. Results suggest that a hybrid multi-start approach with parallel computation might be a setting to be explored for higher reliability.

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AN OPEN-SHOP SCHEDULING PROBLEM WITH OPERATIONS BATCHING

Daniil Baldouski[1]
University of Primorska, IAM, 6000 Koper, Slovenia
Balázs Dávid
InnoRenew CoE, 6310 Izola, Slovenia
University of Primorska, FAMNIT, 6000 Koper, Slovenia
Miklós Krész
InnoRenew CoE, 6310 Izola, Slovenia
University of Primorska, IAM, 6000 Koper, Slovenia
University of Szeged, 6725 Szeged, Hungary
Martin Milanič
University of Primorska, FAMNIT and IAM, 6000 Koper, Slovenia

Abstract: This work generalizes the open-shop and batch scheduling problems by considering merging of operations. We propose an efficient two-phase heuristic, which combines a graph-based optimal merging algorithm with a mixed-integer linear programming (MILP) model for the open-shop scheduling part of the problem. We introduce a benchmark instance set to validate our methodology.

Keywords: job scheduling, batch scheduling, interval graphs, mixed-integer linear programming

1 INTRODUCTION

In this work, we consider a new scheduling problem that combines the theory of job and batch scheduling and is inspired by real-world laboratory experiment scenarios. For each job, there is a set of tasks (operations) to complete, each task following a protocol (operation category) to be performed on one of the suitable machines for a given period of time. Each job is considered completed when all its operations are finished. Operations can be performed in any order, which relates to the theory of *open-shop scheduling* (OSSP) [5]. Moreover, several operations can be performed at the same time on the same machine, so we are interested in the efficient groupings of operations to be analyzed in parallel. We call such a grouping process *merging*, which is similar to the concept of *batching*. In the field of *batch scheduling* [6], it is standard for the jobs to be batched and the jobs in the same batch can then be executed on the same machine. In our case, the operations are ones to be batched and executed together, distinguishing our approach from traditional batch scheduling.

Due to space limitations, in this work we omit examples of the problem with discussions of additional potential parameters and the proof of Lemma 3.1, and only the key parts of the mixed-integer linear programming (MILP) model for scheduling of merged operations (Section 3.2) are mentioned. All the details can be found in the appendix [1].

2 PROBLEM DESCRIPTION

The environment of the open-shop scheduling problem with operations batching (OSSP-OB) is given by the sets of jobs, operations, and machines, with various parameters for each of the sets. Formally, it consists of:

- The time horizon of the problem, represented by a discrete set T of consecutive time units.
- A finite set J of **jobs**. For each job $j \in J$ there is a **time window** $[a_j, b_j]$ within the time horizon of the problem for which the operations of this job should be analyzed and approved.

[1]Corresponding author. E-mail: daniil.baldouski@iam.upr.si.

- A finite set O of **operations** associated with jobs. For each job $j \in J$, a set $O_j \subseteq O$ of operations associated with the job.
- A finite set C of operation **categories** associated with operations. Each operation $o \in O$ has a category $C(o)$ and there are no two operations of the same category that belong to the same job. The time it takes to perform the operation is the same within each operation category and is then defined for each category $c \in C$ as the *length* of the category and denoted by $\ell(c)$. The set of all operations of the same category is denoted by $O(c)$ for each category $c \in C$.
- A finite set M of **machines**. For each machine $m \in M$, a set $C_m \subseteq C$ of operation categories that can be carried out on the machine.
- Operations can be performed in batches if they belong to the same category. This will be referred to as **merging** of the batched operations.

In this work, we consider two additional parameters that are of high importance in real-world cases (e.g. laboratory experiments): *internal deadlines* and *activity periods*. The *internal deadline* (d_j) can be defined as k time units before b_j, which is the desired internal target for operations to be analyzed and approved, which then gives some time (k time units) for corrections in the case of exceptions (failures). Furthermore, *periods of inactivity* are all the gaps in the time horizon (e.g. weekends, national holidays, other special events) during which machines are unavailable and none of the tasks can be performed. Let us then consider an *activity period* to be any maximal interval of time units that does not contain periods of inactivity.

The objective of this work is to find the most efficient scheduling of operations to machines given the merging constraints, with the objectives to consider including tardiness, number of batches, flowtime, makespan, and overall costs.

3 SOLUTION METHODOLOGY

Due to the complexity of the problem and as a proof of concept, a two-phase approach is introduced that separates the merging of operations and scheduling of merged operations. Due to the specifics of the problem, merging of operations can be solved optimally with a graph-based algorithm, which is described in Section 3.1. Given the merging of operations, a MILP model is introduced in Section 3.2 to finalize the scheduling of merged operations.

3.1 Operations merging heuristic

The optimality of the merging heuristic is measured by minimizing the total amount of resources (machines) used. To each job $j \in J$ and each operation $o \in O_j$, we associate subinterval $I_o = [a_j, b_j]$ of the time window of the corresponding job.

For each category $c \in C$, our goal is to find a way to partition the set of operations $O(c)$ of category c into as few parts as possible so that any two operations in the same part can be analyzed simultaneously. This problem can be formulated in graph theoretic terms and admits a simple solution.

Consider the graph $G(c) = (V_c, E_c)$ obtained by introducing a vertex v_o for each operation $o \in O(c)$, and joining two vertices v_{o_1} and v_{o_2} by an edge if and only if there exists a corresponding interval that is large enough for the two operations to be merged, that is, if there exists an activity period A such that $|A \cap I_{o_1} \cap I_{o_2}| \geq \ell(c)$.

We want to find an optimal clique cover of $G(c)$, that is, a minimum-sized collection of cliques in $G(c)$ with union V_c. In general, this problem is NP-hard, but in our case it can be solved efficiently. This is because of the following.

Lemma 3.1 *Each vertex v_o of $G(c)$ can be assigned an interval I'_o on the real line such that two distinct vertices v_{o_1} and v_{o_2} of $G(c)$ are adjacent if and only if $|I'_{o_1} \cap I'_{o_2}| \geq \ell(c)$.*

Known results on tolerance graphs (see [3, 4]) imply that $G(c)$ is an interval graph. The exact definition of interval graphs is not required here, we only need the following property.

Remark 3.1 *Let $v_1, v_2, \ldots, v_n$, for $n = |V_c|$ be an ordering of the vertices of $G(c)$ such that, for all $o_1, o_2 \in O(c)$, if the interval I_{o_1} ends strictly before the interval I_{o_2}, then v_{o_1} precedes v_{o_2} in the ordering. Then, for all $o \in \{1, \ldots, n\}$, any two neighbors of the vertex v_o in the subgraph of $G(c)$ induced by $\{v_j : o \leq j \leq n\}$ are adjacent.*

This leads to an efficient algorithm (provided in full in the appendix [1]) that optimally solves the clique cover problem on $G(c)$ in polynomial time and, hence, an optimal merging of the operations of category c.

3.2 Scheduling of merged operations

In the second phase of our two-phase approach, merged operations can be viewed as regular operations. Thus, the following MILP model considers only the universal set of operations O and a feasible solution to the second phase together with the operations merging algorithm provides a feasible solution for the whole problem. Only the key parts of the model are presented here; full details are available in the appendix [1].

3.2.1 Problem data

For each $j \in J$, $o \in O$, $m \in M$, $c \in C$, and $\tau \in T$, we establish the following parameters.

- $aOJ(o)$ - the index of the job associated with the operation o.
- $tJ^d(j)$ - the internal deadline of the time interval for which the operations of job j should be analyzed.

3.2.2 Variables

- $x_{o,m,\tau} = 1$ if and only if operation $o \in O$ is active on machine $m \in M$ in time unit $\tau \in T$.
- $y_{o,\tau} = 1$ if and only if operation $o \in O$ starts in time unit $\tau \in T$.
- $z_{o,m} = 1$ if and only if operation $o \in O$, machine $m \in M$ are assigned to each other during the time horizon.

3.2.3 Objective function

Minimize the overall tardiness of operations with respect to internal deadline of the corresponding jobs:

$$\text{Minimize} \sum_{o \in O} \left(\sum_{m \in M} \sum_{\tau \in T, \tau \geq tJ^d(aOJ(o))} x_{o,m,\tau} \right) \tag{1}$$

4 RESULTS

There are a sufficient number of benchmarks available for job and batch scheduling problems, but according to our research, none of them satisfy all the specifics of the OSSP-OB. As a proof of concept, in our specific case of the two-phase heuristic solution, we introduce a set of benchmark instances for the OSSP part of the problem, adapting the benchmark generation ideas of E. Taillard [7].

Benchmark instances include both feasible and infeasible instances and, similarly to Oven Scheduling Problem (OSP) benchmarks [2], come in three sizes: small, medium, and large, based on the number of jobs. Small instances include configurations with 10 and 25 jobs, medium size instances have 50 and 100 jobs, and large infeasible instances have 250 and 500 jobs. Each instance is represented by the number of jobs, machines, and categories, with machines and categories ranging from 3 to 5. Other parameters, such as the size of the time horizon, processing times, number of operations per job, among others, are left constant. The benchmark generation algorithm can be easily expanded to include randomness for any other model parameter, including additional new parameters outside of the scope of this research. Moreover, the algorithm is expected and encouraged to be modified according to the specifics of the problem considered.

Computational experiments were conducted using the Gurobi 11.0.0 solver on an AMD Ryzen 7 5800H 3.2 GHz CPU and a 16 GB RAM setup. The time limit for the tests was set to 60 minutes. All feasible instances were solved optimally in less than 1 second for small-sized instances and less than 200 seconds for medium-sized instances. A portion of medium-sized instances and all the large instances were generated to be infeasible. Such instances were correctly identified as infeasible in less than 21 seconds for medium sizes and less than 150 seconds for large sizes. Based on the preliminary results, it can be assumed that infeasibility can be identified rather quickly, even for large instances.

5 CONCLUSION

In this work, we demonstrated an efficient two-phase heuristic for the specific case of OSSP-OB. Using the underlying properties of merging constraints, we were able to construct an efficient algorithm, which, combined with the MILP model for the OSSP part of the problem, can be used to find feasible solutions. The quality of the two-phase approach is yet to be determined, and further work will include a comparison between an exact solution method for the general problem (both phases at once) and the two-phase idea considered in this work. We believe that the exact solution algorithm for the entire problem will be limited to only smaller instances, so we expect the two-phase heuristic to be more practical. In this case, further work might include the development of an efficient (meta-)heuristic algorithm for the OSSP part of the problem.

6 Acknowledgements

The research was supported by the BioLOG project: Balázs Dávid and Miklós Krész are grateful for the support of National Center of Science (NCN) through grant DEC-2020/39/I/HS4/03533, the Slovenian Research and Innovation Agency (ARIS) through grant N1-0223 and the Austrian Science Fund (FWF) through grant I 5443-N. Martin Milanič is supported in part by the Slovenian Research and Innovation Agency (I0-0035, research program P1-0285 and research projects J1-3001, J1-3002, J1-3003, J1-4008, J1-4084, and N1-0370). This work is supported by the Slovenian Research Agency, research program P1-0404 and by the research program CogniCom (0013103) at the University of Primorska. Balázs Dávid is grateful for the support of ARIS through grant J1-50000.

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Parabolic target space method for monotone linear complementarity problem

Marianna E.-Nagy, **Tibor Illés**, **Yurii Nesteriv**, **Petra Renáta Rigó**
Corvinus Centre for Operations Research at Corvinus Institute for Advanced Studies,
Corvinus University of Budapest,
Közraktár utca 4-6., 1093 Budapest, Hungary

Abstract: We propose a parabolic target-space interior-point algorithm for solving monotone linear complementarity problems. This is a member of a novel family of interior-point algorithms, which operate in a lifted space and follow a new type of central path. We investigate different search directions, namely the classical Newton direction, the universal tangent direction and the auto-correcting version. In the case of latter, we prove local quadratic convergence of the algorithm. We will also present some preliminary numerical results.

Keywords: interior-point algorithm, parabolic target-space, monotone linear complementarity problems, universal tangent direction, polynomial complexity

INVENTORY PLANNING FOR 3D-PRINTED SPARE PARTS UNDER CAPACITY CONSTRAINTS

Marko Jakšič
University of Ljubljana, School of Economics and Business, Kardeljeva ploščad 17, Ljubljana, Slovenia, e-mail: marko.jaksic@ef.uni-lj.si

Abstract: We analyse a spare parts inventory problem in which spare parts are 3D printed to stock so that the stochastic demand can be fulfilled immediately depending on the availability of the stock. The complexity of the problem lies in the fact that the 3D printing capacity allocated to a given order depends on both the order size and orders for other spare parts. We assume a base-stock structure of the inventory policy, which allows us to derive the conditional probability for the allocated capacity when ordering a spare part. The numerical analysis of the optimal cost function shows how the capacity utilisation and the number of different spare parts affect the optimal base and safety stock levels.

Keywords: 3D printing, inventory, base-stock policy, proportional allocation, safety stock

1. INTRODUCTION

The motivation for our study is the problem of a company that manufactures household appliances and has reorganised its distribution system for spare parts. The company has decided to move from a decentralised network of local warehouses where spare parts were stored to a centralised facility from which all demand for spare parts is served. Each spare part ordered is then sent to the local sales office or directly to the repair technicians throughout Europe. To ensure fast delivery, the spare part is dispatched the same day if required. Our analysis focuses on spare parts that are 3D printed in-house. Most 3D printing applications in spare parts management assume that printing is done on demand [5, 6]. In our case, however, such a strategy is not responsive enough and would result in a delivery delay of a day or more, so the company's strategy is to keep spare parts in stock.

The inventory problem that arises from this practical situation is a multi-product inventory problem with stochastic demand for each spare part and limited total production capacity per period. The objective is to define an inventory policy for each spare part, which determines the target inventory level at which the inventory of a spare part should be held. This target inventory level should take into account both the stochastic demand for the spare part and the limited stochastic capacity allocated to a spare part given the overall 3D printing production schedule for all parts.

Multi-product production inventory models are inherently difficult to optimise due to endogenous capacity allocation, where the capacity allocated to an individual product also depends on the production requirements for other products. Glasserman [4] examines this problem in the context of a continuous review and focuses on asymptotic behaviour when the backorder costs become very large in order to circumvent the intractability of the problem. Most of the analytical results are derived for a simplified problem assuming a fixed capacity allocation and homogeneous products (with equal demand distribution), which is also the case we analyse below in the context of periodic review. The model that comes closest to our model is the periodic review model of [2], which shows that the base-stock inventory policy is optimal for homogeneous products, assuming a so-called symmetric resource allocation policy, where capacity is first allocated to the product with the smallest inventory and then the inventories of all products are gradually increased until all capacity is exhausted. Such an allocation policy is similar to the proportional allocation policy that we assume in our model, but our allocation policy increases the stock level proportionally to the size of the order and does not per se aim at the same inventory level for all products.

2. MODEL

We assume a periodic review inventory policy in which the daily demand $D_{i,t}$ for a selected spare part i is exponentially distributed at the rate λ_i, stationary and independent over the periods. If the daily demand is low, the exponential distribution provides a good description of the actual spare part demand. The spare part is kept in stock and replenished when needed by placing a production order to the in-house 3D printing production source. In-house 3D printing is done within the same day based on the total available 3D printing capacity C, which is divided among all ordered spare parts.

The sequence of events on a given day is as follows: At the beginning of the day, the on-hand inventory position $x_{i,t}$ is reviewed for each spare part i. The stock requirement is determined and in-house print order $q_{i,t}$ is placed up to the capacity allocated to the spare part c_i, possibly limiting the order. It should be noted that the orders are placed without knowing the demand realisation $D_{i,t}$, which accumulates during the day. During the day, the 3D printed spare parts become available and are added to the stock from which the demand is to be satisfied. At the end of the day, the daily demand $D_{i,t}$ is observed and the spare parts are delivered according to their stock availability. Depending on the end-of-day inventory position, inventory holding and possible backorder costs are incurred.

When orders $q_{i,t}$ for spare parts are known, the total capacity is allocated proportionally. It is assumed that the different spare parts ordered compete for capacity simultaneously according to their order size (and corresponding 3D printing times), without favouring any particular spare part. To derive the inventory policy for each spare part, we need to find a way to model the allocated capacity faced by each spare part, which depends on the orders for all spare parts. Due to this interdependence, the optimal inventory policy is not tractable for general capacity allocation rules, which is why we have to resort to approximations. First, we assume that the inventory policy has the structure of the base-stock policy, which corresponds to the optimal policy structure in a single-product inventory problem with fixed [3] or stochastic production capacity [1]. Second, we assume that for each spare part, the distribution of orders follows the distribution of demand, which is a reasonable assumption since the orders under the base-stock policy structure effectively cover the inventory shortages caused by the demand replenishments. Therefore, in each period the order $q_{i,t}$ is placed in such a way that the inventory level $x_{i,t}$ is increased to the base-stock level S_i, $q_{i,t} = \min((S_i - x_{i,t})^+, C_i(\mathbf{q}_t))$, whereby the order $q_{i,t}$ can be limited by the allocated capacity C_i, which depends on the order vector $\mathbf{q}_t$ across all ordered spare parts.

In the remainder of the paper, we restrict ourselves to analysing a simplified problem with n spare parts with the same demand distribution (therefore we omit the indices i and t in the further text if possible). Assuming that the orders q follow the same distribution as demand, $q_1, \ldots, q_n \sim \text{Exp}(\lambda)$ holds. The total capacity C is allocated proportionally to each spare part ordered $C_i = (q_i / \sum q_i)C$, and the PDF of $C_i = c$ is:

$$f_{C_i}(c) = \frac{n-1}{C}\left(1 - \frac{c}{C}\right)^{n-2}, \quad 0 < c < C, \tag{1}$$

this is a scaled $Beta(1, n-1)$ distribution, scaled to $[0, C]$ with a mean value at C/n and the variance $C^2(n-1)/(n^2(n+1))$. However, the allocated capacity depends on the order q_i, with the proportional allocation rule allocating more capacity when a higher q_i is placed. Therefore we need to derive the conditional PDF of $C_i = c$ given $Q_i = q_i$:

$$f_{C_i|Q_i=q_i}(c) = \frac{q_i C}{c^2} \cdot \frac{\lambda^{n-1}}{\Gamma(n-1)} \left(q_i \cdot \frac{C-c}{c}\right)^{n-2} e^{-\lambda q_i \cdot \frac{C-c}{c}}, \quad 0 < c < C \tag{2}$$

Finally, we derive the minimum discounted expected cost function v_t that captures the cost of inventory management of a spare part over a finite horizon T with the starting inventory

position x in period t:

$$v_t(x,q) = \min_{0\le q\le c(q)} \{pq + G(x+q) + \alpha \mathbb{E} v_{t+1}(x+q-D)\}, \quad \text{for} \quad 1 \le t \le T, \tag{3}$$

where q is ordered up to the allocated capacity $c(q)$ with the distribution given in (2), pq is the 3D printing cost, $G(y) = h\mathbb{E}(y-D)^+ + b\mathbb{E}(D-y)^+$ is the one-period inventory holding and backorder cost function, the expectation in the cost-to-go function is taken with respect to D and the ending condition is $v_{T+1} = 0$.

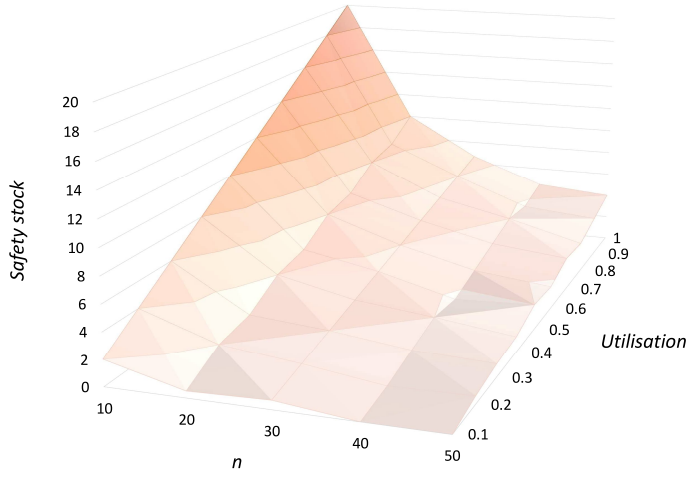


Figure 1: Safety stock levels as a function of system parameters.[1]

3. NUMERICAL ANALYSIS

In this section, we present the results of the numerical analysis we performed to determine the optimal base-stock levels, the key parameter of the inventory policy that dictates that the order q. The numerical calculations were performed by solving the dynamic programming formulation given in (3).

Figure 1 shows the optimal safety stock levels for different scenarios of capacity utilisation nD/C and the number of spare parts SKUs n. In the optimal inventory policy, the optimal safety stock represents the additional stock held over the average daily requirement to enable the direct replenishment of stochastic customer orders for spare parts.

The most important findings regarding the level of safety stocks correspond to intuition. Firstly, the optimal base-stock levels and the resulting safety stocks are higher in situations with higher capacity utilisation, in which every opportunity to build up stocks must be taken in view of the limited capacity. If the number of different spare parts SKUs increases (with the same capacity utilisation), the safety stock decreases. This is due to the risk pooling effect, as the variance of (1) decreases approximately linearly with the increase in n.

4. CONCLUSIONS

We derive the model formulation and characterise the optimal base-stock policy for inventory control of 3D-printed spare parts. We assume an endogenous 3D printing capacity that potentially limits the production of each spare part, with capacity distributed proportionally across spare parts based on order sizes.

[1]The following input parameters were used: $T = 20$, $h = 1$, $b = 20$, $\alpha = 0.99$, with fixed $C = 100$.

Due to the limited scope of the paper, we have focused on the simplified model with n homogenous spare parts. However, the model can be considered as a stepping stone for integrated analysis of inventory decisions and strategic capacity decisions, as well as for exploring other sourcing options, such as dual-sourcing models for 3D printing, comparison with on-demand 3D printing, and combination with traditional injection moulding production.

Acknowledgments: The author gratefully acknowledges the financial support of the Slovenian Research Agency (core funding no. P5-0128).

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INTERIOR-POINT ALGORITHM BASED ON STANDARD KERNEL FUNCTIONS

Petra Renáta Rigó and **Marianna Eisenberg-Nagy** and **Goran Lesaja**
Corvinus Centre for Operations Research at Corvinus Institute for Advanced Studies, Corvinus University of Budapest, Hungary
Zsolt Darvay
Faculty of Mathematics and Computer Science, Babeş-Bolyai University, Romania
Anita Varga
Department of Mathematics, North Carolina State University, Raleigh, NC, USA

Abstract:

We present an interior-point algorithm for sufficient linear complementarity problems, based on a new class of kernel functions, called standard kernel functions. We provide a comprehensive and unified complexity analysis of the algorithm and develop a general procedure to determine the iteration bounds for long-step and short-step versions of the method for the entire class of standard kernel functions. The general procedure is illustrated by determining the iteration bounds for several parametric standard kernel functions, which include all eligible kernel functions from the literature having rational or exponential barrier terms. We match the best iteration bounds obtained in the literature for these special cases of standard kernel functions.

Keywords: sufficient linear complementarity problems; interior-point algorithms; standard kernel functions.

The 18th International Symposium on
Operational Research in Slovenia

SOR '25

Bled, SLOVENIA
September 24 - 26, 2025

Session 8:

Methodologies and Case Studies

RESEARCH DOMAINS AND SOFTWARE TOOLS IN DISCRETE EVENT SIMULATION (DES) STUDIES: IMPLICATIONS FOR ORGANISATIONAL DECISION-MAKING

Mario Jadrić, Tea Mijač, Maja Ćukušić
University of Split, Faculty of Economics, Business and Tourism
Cvite Fiskovica 5, 21000 Split, Croatia
E-mail: {mario.jadric, tea.mijac, maja.cukusic}@efst.hr

Abstract: We examine the evolution of Discrete Event Simulation (DES) from 1968 to 2024. A scoping review synthesises findings from key studies published, highlighting significant advancements in DES by analysing publication trends, study types, application areas, and the types of software employed. Search strategy targeted 14 journals focused on simulations and operational research, all ranked as Q1 or Q2. After removing duplicates from Web of Science and Scopus search query results, 192 journal articles remained for further screening and analysis following the Prisma framework. A detailed content analysis was conducted using Python for text mining, and pivoting for quantitative trend analysis, enabling the identification of key research domains, and software tools in DES studies. By identifying current trends and gaps in the literature, this paper provides an overview of the latest developments in DES and suggests directions for future research to enhance its effectiveness and broaden its applicability in organisational decision-making.

Keywords: Operational Research, Discrete Event Simulation, Software, DES

1 INTRODUCTION

This paper focuses on a scoping review of the various domains and software tools used in Discrete Event Simulation (DES) studies. We find this particularly important as DES is utilised across many industries and academic fields to enhance systems and support better decision-making. Namely, DES is one of the most effective decision-support tools, not only in manufacturing but also in the knowledge economy, reinforcing its critical role in organisational decision-making [2]. Increasingly used in Industry 4.0 to help manage uncertainty and improve flexibility, DES also has clear benefits in logistics system design where human expertise is still needed in understanding and interpreting data [4]. Simulation models allow researchers to study system behaviour under various conditions without affecting the actual system, which can be costly or impractical. Real systems often have unpredictable variables, like customer service times and order arrival intervals. By incorporating these stochastic elements, DES provides valuable insights into system performance in different scenarios, helping decision-makers identify and address potential bottlenecks or delays [5]. A pervious study [7] presents a review of simulation applications from 1997 to 2006, highlighting the role of simulation techniques in manufacturing and business. The most widely used simulation approaches include DES, Agent-Based Simulation, and System Dynamics. Each approach offers unique strengths and is applied based on the complexity and nature of the system being modelled. DES has established itself as the preeminent simulation method within the realm of Operational Research largely due to its adeptness in modelling real-world processes characterised by discrete events (e.g. transit through manufacturing line, flow of patients within a healthcare facility, movement of customers within a service queue). DES enables the examination of bottlenecks, inefficiencies, and potential avenues for optimisation. For example, DES can accurately measure the time products spend in queues awaiting processing or assess the temporal availability of machinery. In the context of healthcare, DES could effectively evaluate patient wait times alongside the accessibility of medical personnel or equipment. This highlights the evolving significance of DES in the optimisation of business processes, which carries numerous implications for organisational decision-making.

In the context set in this introduction, the focus of the paper is on DES and its evolution from 1968 to 2024. A content analysis and text mining [10] were conducted following standard steps to streamline the process and enhance the efficiency of identifying patterns. This approach allows for the extraction of key terms and recurring concepts from the analysed literature, enabling a more thorough and automated understanding of the data. The rise of research repositories offering both textual and comprehensive data sources, along with the growing volume and diversity of these sources, has made screening increasingly complex [1], [14]. This is why Knowledge Discovery in Text [8] has become vital for facilitating knowledge discovery and is regularly used to uncover valuable information and patterns within text [9], explicitly aiming to extract implicit knowledge from unstructured text and articulate it clearly. Here, text mining has been used to evaluate trends in the field of simulation modelling [6].

2 METHODOLOGY

The study objective was to synthesise findings from key studies, highlighting significant advancements in DES by analysing publication trends, study types, application areas, and the types of software employed. In scoping review, the first step involves formulating a clear and structured search strategy to ensure the review is based on predefined criteria. In this case, the authors began by identifying relevant keywords aligned with the research questions. To achieve broad coverage of peer-reviewed papers, the focus is on two major academic databases: Web of Science (WoS) and Scopus. The following search queries were employed:

- Scopus: (TITLE ("Discrete-Event Simulation") OR TITLE ("Discrete Event Simulation ")) AND (LIMIT-TO (DOCTYPE , "ar")) AND (LIMIT-TO (LANGUAGE, "English"))
- WoS: (“Discrete-Event Simulation" (Title) or "Discrete Event Simulation" (Title) and Article (Document Types) and English (Languages))

A structured and transparent process for identifying and selecting relevant studies was then ensured by applying the PRISMA framework. Specific inclusion and exclusion criteria were defined to guide the selection process. The inclusion criteria (IC) for this study were as follows: IC1 – journal articles published in high-quality, peer-reviewed Q1 and Q2 journals were considered, which narrowed the selection to 14 journals; IC2 – only papers published in English were included. On the other hand, there was one exclusion criteria (EC), EC1 – papers from highly specialised journals, such as those focused on niche areas like forestry, were excluded. Based on SJR 2023, the following high-quality, peer-reviewed journals were selected: Q1 journals include the *Journal of the Operational Research Society, European Journal of Operational Research, Simulation Modelling Practice and Theory, Simulation Practice and Theory, Computers and Operations Research, Mathematics and Computers in Simulation, Annals of Operations Research, International Transactions in Operational Research,* and *Operations Research*; Q2 journals include *Simulation, Journal of Simulation, International Journal of Simulation Modelling, and International Journal of Modelling and Simulation*. Additionally, quality criteria were applied to ensure that the selected studies were robust and relevant. Papers featuring case studies were prioritised, providing practical insights into how DES is used in real-world scenarios. After defining the inclusion and exclusion criteria, the next step involved collecting and analysing the data. Full papers were downloaded to conduct a detailed content analysis to identify key themes, research approaches, and software tools used in the selected studies. This allows for synthesising findings and drawing conclusions about trends in DES research. Two tools for the analysis were used – Excel for quality analysis where pivot tables sort and categorise data, and Python for text mining, specifically for the extraction of key terms, enhancing the identification of recurring concepts

within the dataset. This enabled quantitative analyses of trends, such as the frequency of different research approaches and software tools used in the selected studies.

3 RESULTS

The analysis of results follows the PRISMA framework and is structured into three key steps: identification, screening, and inclusion, all illustrated in Figure 1. Identification phase involves gathering all potential studies related to the research topic. Comprehensive searches were conducted in major academic databases, including Scopus and WoS, as was presented. Relevant keywords were used to ensure a wide-ranging collection of articles. This initial search resulted in a total of 2,147 articles. The selection was assessed against the predefined inclusion and exclusion criteria, thus excluding 1,829 articles. Further, 126 articles were excluded due to duplication in the two databases.

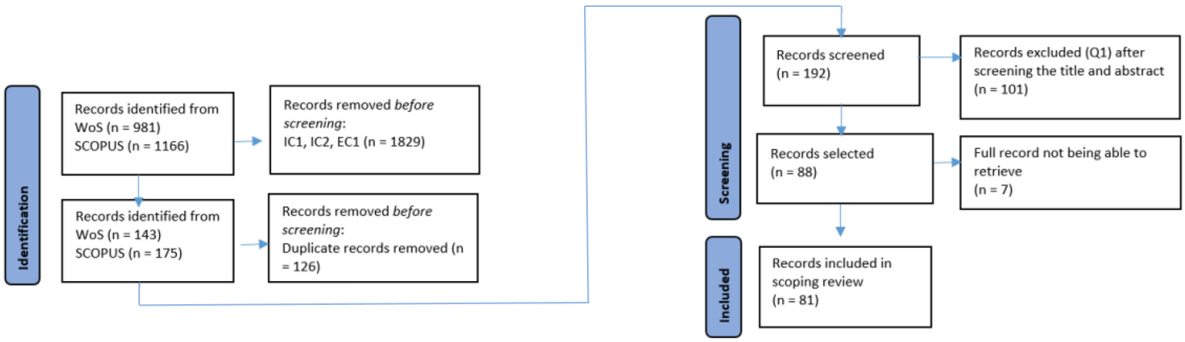


Figure 1: Identification of studies via Scopus and WoS databases - PRISMA results

More than half of the 192 screened records are exclusively methodological papers, while 45% employed the case study method. During the screening phase, titles and abstracts were reviewed. This rigorous screening process narrowed the selection down to 88 articles. Seven studies could not be retrieved, resulting in a final count of 81 articles included in the analysis. In the inclusion phase, the remaining studies underwent a full-text review to confirm their relevance and quality. Specific attention was given to the methodological rigor and the applicability of findings in discrete-event simulation. The analysis reveals that a significant concentration of the reviewed articles was published in a small number of journals. Specifically, more than 75% of the total 81 analysed articles appeared in just four journals: *Simulation* (20 articles), *Journal of the Operational Research Society* (16 articles), *Journal of Simulation* (14 articles), and *Simulation Modelling Practice and Theory* (11 articles).

Next, Python has been used for text mining to extract useful insights and patterns from titles and abstracts. A variety of tools, libraries, and frameworks specifically designed for text processing is available out-of-the-box, making Python one of the most popular programming languages for natural language processing and text analytics [13]. Pandas and Scikit-learn (sklearn) libraries were used in this study. After importing the file, the text data was cleaned, and word frequencies were calculated. The results were also arranged by decades to analyse potential trends and changes. The CountVectorizer function from sklearn.feature_extraction.text was used for feature extraction. Visualisation was performed using the Matplotlib library (Figure 2).

It can be seen that during the last three decades, the most common terms are related to „simulation“ and „model”. What has changed over the decades is the appearance of specific terms. For example, in the most recent decade, „patients“ and „time“ have emerged, likely reflecting a focus on healthcare simulations and time-related factors. In contrast, during the 2000s, the terms „discrete“ and „event“ were more frequent, focusing on the basic concepts of DES. These results indicate a shift in focus from the core principles of DES to more practical applications, such as healthcare.

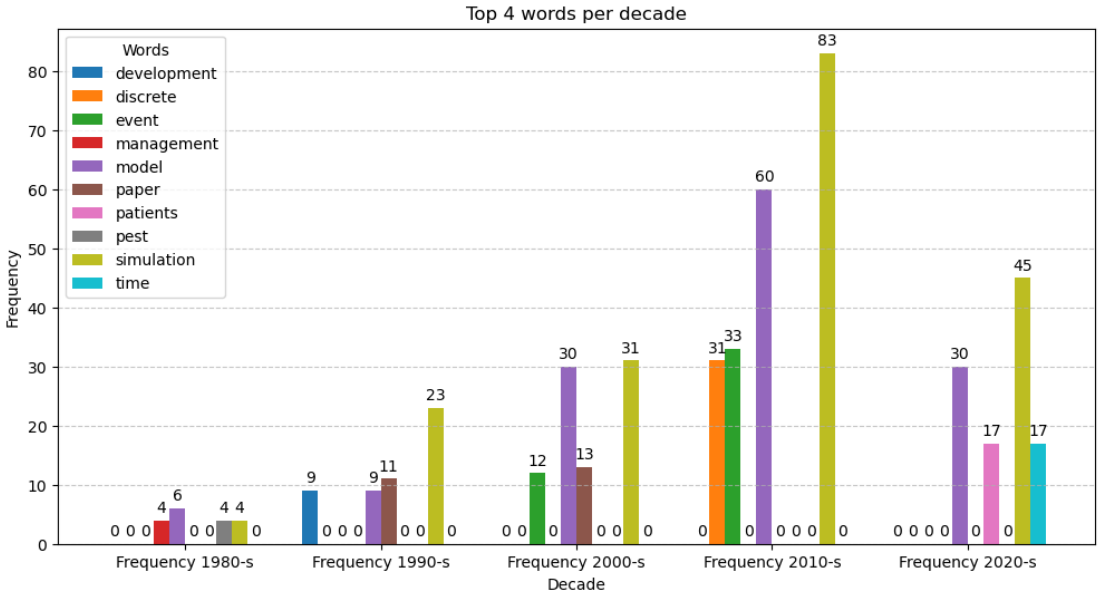


Figure 2: Visualisation of text mining results using Python and the Matplotlib library

As mentioned earlier, a quality analysis was conducted to identify trends related to application areas and software used over the past five decades. To clarify the main trends, we focused on the top 5 domains and top 5 software tools, which helped highlight the key patterns. The analysis highlights the most frequently researched domains in DES studies over the decades (Figure 3). "Healthcare" is the most commonly studied domain, with 25 occurrences. It is followed by "Production" with 16, "Transport" with 12, "IT" with 6, and "Supply Chain" with 5. However, in the past two decades, the focus has shifted significantly towards "Healthcare" and "Supply Chain", while "Production" and "Transport" have maintained more consistent representation throughout the 1990s, 2000s, and the last two decades. The analysis reveals that Arena is the most frequently used software tool in DES studies, with 21 occurrences. It is followed by Simul8 (8), C++ (3), and AutoMod with two occurrences. Arena was first used in the 2000s, but its usage increased the most starting in the 2010s. A similar trend is observed with Simul8, which has also gained more attention in the past two decades. On the other hand, C++ was most widely used in the 1990s, and it has shown a decline in usage in recent years. These results are shown in Table 1.

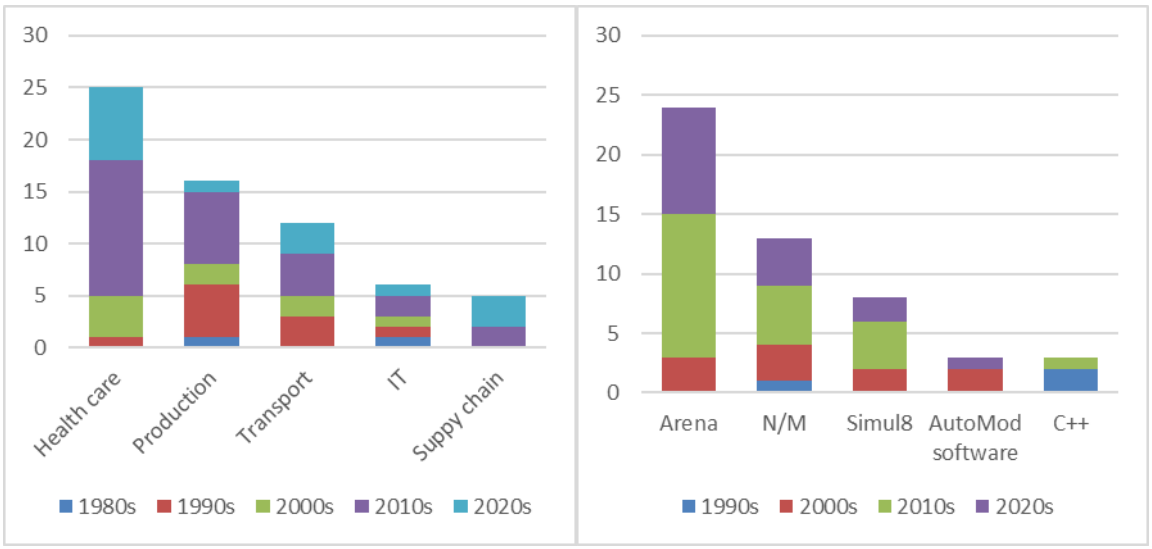


Figure 3: Visualisation of the top 5 domains and software used in DES studies by decade

Table 1: Overview of the top 5 research domains and software applications in DES studies

Software	***Health care***	***Transport***	***Supply chain***	***Production***	***IT***	***Total***
Arena	13	4	2	2		**21**
N/M	2	1	2	2	3	**10**
Simul8	7		1			**8**
C++		1		1	1	**3**
AutoMod	1	1				**2**

Arena remains the most widely used simulation software, particularly in healthcare (13 applications). It is also utilised in transport (4), supply chain (2), and production (2). Arena's versatility and robust capabilities make it a top choice across various industries. The N/M category indicates cases where the specific simulation software used is unclear. However, it is known that this software is applied across several domains: healthcare (2 applications), transport (1), supply chain (2), production (2), and IT (3). This suggests that multiple simulation software options are being used in these cases without further specificity. Simul8 is particularly used in healthcare (7 applications) and has a more minor application in supply chain management (1), which indicates that Simul8 is especially popular for healthcare simulations, but it has a more limited presence in other sectors. C++ as a programming language is often used for custom simulation development. It appears in studies related to transport (1), production (1), and IT (1), showing its application in tailored simulation solutions for specific use cases in these industries. AutoMod is used in healthcare and transport (1 application each), showing a smaller share than other tools in the top 5 domains.

4 DISCUSSION AND CONCLUSION

Our paper aimed to combine text mining and domain/software trend analysis over a long horizon (1968–2024). As the field of simulation has evolved, the preference has shifted towards using simulation software tools like Arena and Simul8, which offer more user-friendly interfaces and built-in features for modelling and analysing complex systems. Our results showed that the most popular software used is Arena, which is aligned with previous research [3], [11]. This shift in preference has led to a decline in the use of programming languages like C++ in DES studies in recent years. The results also clearly show the change in research trends, with "Healthcare" and "Supply Chain" dominating recent studies, while the focus on "Production" and "Transport" remains more balanced across the decades. It is important to emphasise that the health domain is the most represented despite the exclusion of specialised journals (EC1). The results of this work can contribute to how trends in the use of DES software and application areas influence decision-making in different organizational contexts. Simulation tools can be categorised according to their focus on operational, tactical, or strategic decision-making and whether the decision context appears centralized or decentralized. For example, the rise of simulation in healthcare with tools such as Arena and Simul8 could be linked to the general need for data-driven decision support in complex environments where the stakes are high. For example, tools like Arena and Simul8 are primarily used in operational and tactical decision-making, especially in healthcare and supply chain environments where rapid, localized optimization is required, e.g. [12]. Arena-based DES provides valuable insights that help organizations make informed decisions. In contrast, tools such as AutoMod and C++, although less commonly used in recent years, tend to be used in strategic and highly customized simulation scenarios - particularly in transportation and manufacturing systems. By relating the choice of software to the nature and scope of decision-making, our findings can help practitioners tailor the use of DES tools to the needs of the organisation.

This study is subject to several limitations. First, the exclusion of grey literature and non-English publications may have led to a partial view of the DES research landscape. Second, while the analysis offers useful trend insights, it remains largely descriptive and lacks deeper qualitative synthesis. Future research should aim to address these limitations by incorporating a broader range of literature and employing qualitative methods to enrich the understanding of DES trends and their implications. Addressing these gaps will not only enhance the robustness of future studies but also ensure that the evolving landscape of DES continues to meet the dynamic needs of decision-making in various industries.

Acknowledgement

This research is supported through the project COEXE (2023–1–MK01–KA220–HED–000161238), co-funded by the European Union under the Erasmus+ programme.

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EXPLORING DATA ENVELOPMENT ANALYSIS IN ECONOMICS AND OPERATIONS RESEARCH: A BRIEF BIBLIOMETRIC PERSPECTIVE

Jelena Jardas Antonić
University of Rijeka, Faculty of Economics and Business, Department of Quantitative Economics
Ivana Filipovića 4, 51000 Rijeka, Croatia
E-mail: jelena.jardas.antonic@efri.uniri.hr

Nenad Vretenar
University of Rijeka, Faculty of Economics and Business, Department of Organisation and Management
Ivana Filipovića 4, 51000 Rijeka, Croatia
E-mail: nenad.vretenar@efri.uniri.hr

Jana Špirková
Faculty of Economics, Matej Bel University, Department of Quantitative Methods and Information Systems
Tajovského 10, 975 90 Banská Bystrica, Slovakia
E-mail: jana.spirkova@umb.sk

Abstract: Data Envelopment Analysis as a non-parametric method for analysing technical efficiency and benchmarking decision making units has become very popular over the years and is used in all fields, especially in economics. The aim of this study is to provide a brief overview of 282 scientific publications from 1992 to 2025 based on a bibliometric analysis of publications from the Web of Science database. The analysis has shown that the number of citations has increased over the years, as has the number of publications in the field. Authors from the USA and China are particularly noteworthy for their work in the field, while from the point of view of application, the method is mainly used in the field of education and, in terms of keywords, the group of sustainable and green economy stands out.

Keywords: Data Envelopment Analysis, economics, bibliometric analysis, efficiency.

1 INTRODUCTION

The concept of efficiency has always preoccupied mankind. However, the concept of technical efficiency was first introduced to economics by Vilfredo Pareto with the optimal allocation of resources. Later, in 1957, Michael J. Farrell elaborated on the concept of technical efficiency further by introducing a radial approach to evaluating efficiency. In his work, he tried to illustrate utilised value-based data instead of quantity-based data [1]. This approach became the basis for the method of Data Envelopment Analysis (DEA), which is presented in this overview in the context of economics, as Farrell laid the foundations of technical efficiency in his work by functionally linking input and output quantities in production function. The problem with using a theoretically given function is that it is very difficult to determine a realistic theoretical function that would describe a complex problem well. On the other hand, the empirically determined efficient production function was estimated by observing the input and output of a large number of companies. For this reason, it is easier to compare performance with the best that can be achieved in the current situation (empirical production function) than with the unattainable ideal situation represented by the theoretical production function. The need to measure such technical efficiency led to the development of the method by Charnes, Cooper and Rhodes in the same year, which they labelled with the capital letters of their names as the CCR model [2]. With the development of the CCR model, which provides a measure of the overall efficiency of each unit observed in the model by aggregating the technical and proportional efficiency of inputs and outputs, DEA gained popularity, as confirmed by the

number of papers published thereafter. Later papers deal with other assumptions, and in 1984 Banker, Charnes and Cooper introduce the BCC model, a variant of the CCR model with the assumption of variable returns to scale. The flexibility of this method, which does not require strict functional dependence between input and output, and the fact that this method is non-parametric and does not require functional dependence compared to classical regression methods, were the cause of its rapid rise in popularity, which continues to this day. The applied component of this method has paved the way for its popularity and research in the field of operations research, as it enables benchmarking between units and their performance improvement by calculating projections on the efficiency frontier. With all this in mind, we will highlight two research questions that will be answered through a bibliometric analysis: RQ1: What are the current trends in the use of the DEA method to measure performance in economics? RQ2: Which authors, countries and topics are most relevant and what research gaps have been identified in the field of economics?

2 LITERATURE REVIEW

Over the years, data envelopment analysis has distinguished itself as a method in the applied sense and as such is used to calculate technical efficiency in all fields, from medicine and education [3] to public institutions [4] and agricultural sciences, but as a special category we highlight in this brief overview works from the field of bibliometrics on applications of Data Envelopment Analysis (DEA). Recent works of bibliometric analysis on the application of the DEA method are diverse and deal with general overviews [5],but also with performance measurement in specific areas such as management [6], education [7], health sector [8], the area of supply chain management [9], bank sector [10] and trendy topics such as eco efficiency [11], energy sector [12], sustainable efficiency and ESG standards [13]. If we observe the level of the region and highlight the most deserving authors in the field of operational research and the economics of the DEA methodology itself, it is certainly distinguished author Luka Neralić [14] who has written about the methodology and its theoretical possibilities, and he himself had the opportunity to write with big names such as Abraham Charnes and Rajiv Banker on the methodology improvement through the sensitivity analysis of additive and other DEA models. In the region, we can highlight the Slovenian author Aleksander Aristovnik, who validated the DEA method by measuring efficiency in education [15], and Danijela Rabar with her contribution to measuring efficiency with the DEA method in healthcare [16] .

3 BIBLIOMETRIC ANALYSIS OF THE TRENDS, KEYWORDS AND AUTHORS IN ECONOMICS

The initial selection of data started with 20,925 articles on Data Envelopment Analysis, we narrowed this selection with the field of economics and business, obtaining 8,200 records, and we decided to narrow the number of publications by using the additional word “economics”. The bibliometric analysis was continued by filtering out articles from the Web of Science database containing the keywords "data envelopment analysis" and "economics" in the title, abstract or authors' keywords. In the first filtering, 574 scientific papers were found, which were reduced to 517 by restricting them to journal articles. A further filtering via the scientific fields of Business Economics, Operations Research Management Science, Public Administration, Mathematical Methods in Social Sciences and Mathematics reduced the papers to 282 scientific publications.

The period of observed publications, as shown in the graph in Figure 1, covers the period from 1992 to 2025. It shows how the number of publications and citations grows and reaches its highest level in 2021, then falls slightly and begins to rise again in 2024.

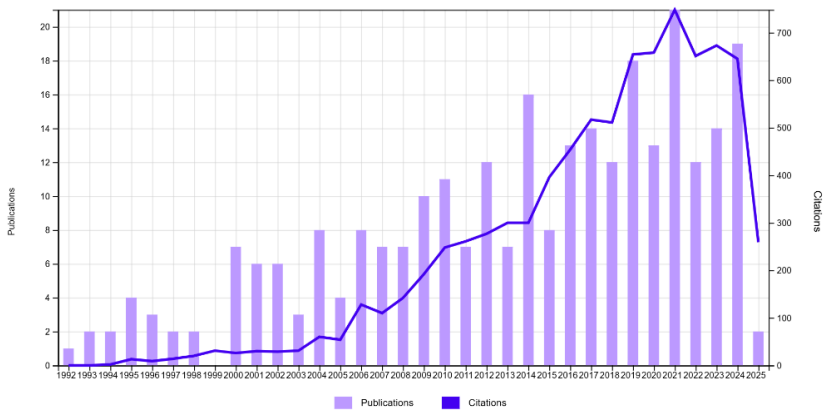


Figure 1: Graphical representation of the number of published works and citations over the years

The number of citations is stable and continues to increase over the years, which confirms the interest in the method and its relevance for publications. The next two charts in Figure 2 show the number of publications in relation to the scientific field and publisher.

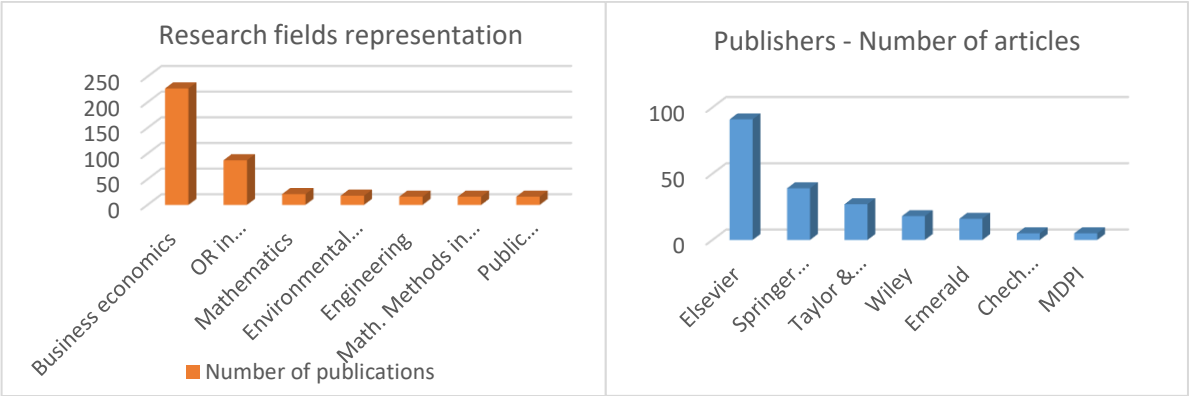


Figure 2: Graphical representation of the number of published works by the fields and Publishers

It can be seen that most publications on the topic of the application of data envelopment analysis in economics come from the field of Business and Economics with 227 articles, followed by the field of Operations Research in Management Science with 87 articles and the field of Mathematics with only 21 articles. Looking at publishers, the vast majority of papers are published by researchers at Elsevier, Springer Nature, Taylor and Frances and Wiley, but MDPI also appears in the top ten publishers. MDPI has begun to take centre stage in publishing in recent years but is not yet prominent in these specific topics.

The graphical representation of the co-occurrence of keywords shows that the keywords are grouped into five clusters. The red cluster refers terminologically exclusively to the method itself, which can be recognised by the keywords it contains, such as *return to scale, decision making units, stochastic frontier* and others.

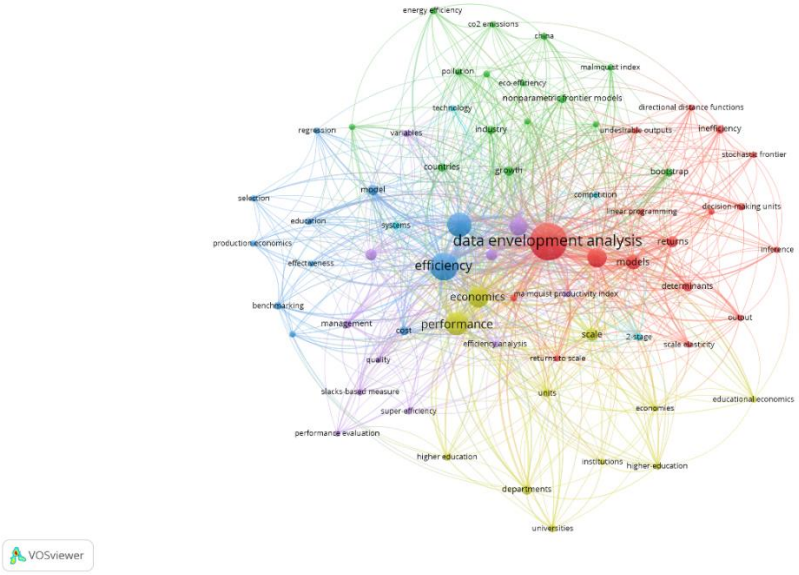


Figure 3: Graphical representation of the keyword co-occurrence

The keyword we used to filter the publications, "data envelopment analysis", belongs to this cluster, and almost all the others are associated with it, and its occurrence is the highest. The second cluster is blue and is related to efficiency, effectiveness and productivity, the keywords that occur most frequently in this cluster and are linked to other economic terms from the cluster, such as *benchmarking*, *effectiveness*, *model*, etc. The third cluster is yellow and relates to education topics, which can be identified by the keywords *higher education*, *universities*, *departments*, *institutions*, while the fourth cluster is green and relates to eco-topics in the economy, which can be identified by the keywords CO2 emissions, eco-efficiency, pollution, energy efficiency, etc. The last cluster is purple and refers to the area of management and quality, which is determined by words such as *quality*, *variables*, *efficiency analysis*, etc.

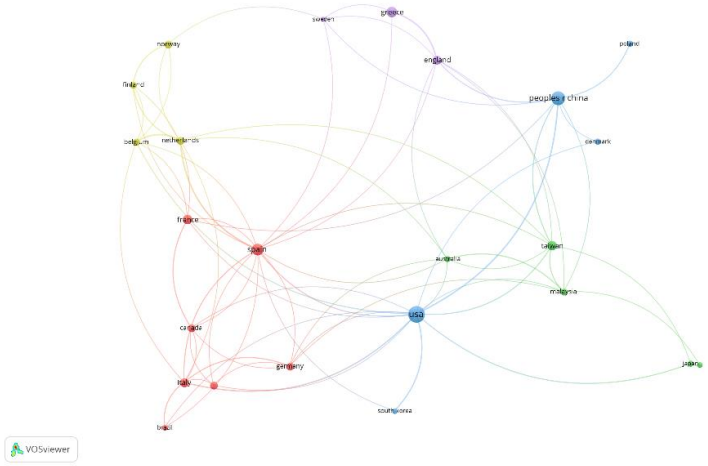


Figure 4: Graphical representation of the co-authorship between countries

If we analyse the graphical representation in Figure 4, we can see that the most cited authors are from China, the USA and Spain, and from this representation we can also see which authors collaborate with each other, if we look at the colour of the cluster, it is not surprising that there

are many authors from the USA collaborating with authors from China (blue cluster). The red cluster represents collaboration between authors from Southern Europe and Canada, while the yellow cluster represents collaboration between authors from Scandinavian and Northern European countries. The green group represents collaborations between authors from Australia, Japan, Malaysia and Taiwan.

If we look at the authors from the region, they modestly use the possibilities that DEA method offers, as of all publications in the observation period 1992 - 2025 in the field of economics and sustainable development, only 72 out of 10,390 publications were published in Croatia, 43 in Serbia and 32 in Slovenia, which is an extremely low percentage considering that this is a period of more than 30 years. Looking at the filtered volume of publications from this analysis, the number of publications from the region is even much lower.

4 DISCUSSION AND CONCLUSION

By observing the development of the DEA method, it can be said that it has become very well established in practise through work in all fields, especially in economics, as it is primarily used for benchmarking by calculating the technical efficiency of decision-making units and is therefore particularly used in problems where efficiency depends on non-quantitative indicators. For this reason, this analysis has shown that the method is most frequently used in economic sectors such as education and new trends such as the green economics, sustainability, etc., which are also among the newer keywords in the field. The countries that publish the most on the topic are the United States and China, while among the European countries that stand out in this analysis, which focuses on the field of economics and operational research, are Spain and England, as well as the European Mediterranean countries. Among the countries in the region, Croatia has the most publications, followed by Serbia and Slovenia. However, in the 33 years in which the method has been applied, the number of scientific publications is very low compared to other countries in Europe and the developed world. Given the trends shown in this bibliometric analysis, it is suggested that authors from the region apply the DEA method in the context of new topics such as sustainability, green economy, energy and eco-economy and use the current trend of low number of publications to their advantage.

Acknowledgement

This study was supported through project ZIP-UNIRI-2023-14 by the University of Rijeka.

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FORMAL ANALYSIS OF SELECTED BOND TILT PROPERTIES

Vedran Kojić and Margareta Gardijan Kedžo
University of Zagreb, Faculty of Economics & Business
Trg J. F. Kennedyja 6, HR-10000 Zagreb, Croatia
vkojic@efzg.hr, mgardijan@efzg.hr

Abstract: Duration and convexity, which are standard measures of a bond's price sensitivity to interest rate fluctuations, are derived from the first and second derivatives of an option-free bond's price, respectively. Recently, a third-derivative measure, termed 'bond tilt', was introduced to quantify changes in convexity and to assess its contribution to the accuracy of estimating the impact of small interest rate changes on bond prices. To date, certain properties of tilt, including its monotonicity with respect to the coupon rate, yield to maturity, and time to maturity (under ceteris paribus conditions), have only been demonstrated empirically. Consequently, the primary objective of this paper is to provide formal mathematical proofs for these properties, complemented by novel numerical examples.

Keywords: bonds, tilt properties, formal proofs

1 INTRODUCTION

Duration and bond convexity are well-established and widely applied risk measures in both bond theory and financial practice, as they correspond to the first and second coefficients, respectively, in the Taylor series approximation of bond price changes. Consequently, a substantial body of literature addresses the fundamental concepts and properties of these measures (see, for example, [3, 4, 5, 6, 7, 8, 9, 10]). More recently, in 2020, Angel et al. [1] introduced 'bond tilt' as a third-derivative measure and an additional coefficient within the Taylor expansion of bond prices. This measure was proposed to evaluate its contribution to the accuracy of bond price estimates, particularly under small interest rate fluctuations. Extending this line of inquiry, Crack (2025) [2] examined the relative importance of each term up to the seventh-order coefficient in the Taylor series expansion, in explaining bond price changes. However, it is noteworthy that the properties of bond tilt presented in [1] and [2] been demonstrated exclusively through empirical observations. Therefore, this paper aims to provide formal mathematical proofs for selected properties of bond tilt.
The remainder of this paper is structured as follows: Section 2 introduces the notation and fundamental formulas used throughout the analysis. The main theoretical results are presented in Section 3, followed by illustrative numerical examples in Section 4. Finally, Section 5 offers concluding remarks and suggestions for future research.

2 NOTATION AND PRELIMINARIES

We use the following notation: N — face value (par value) of the bond, i — annual coupon payment (annual interest payment), n — bond maturity (number of payments, in years), k — annual yield to maturity of the bond, $r = 1 + k$ — annual period discount factor at rate k, P — (market) bond price, D_{mod} — modified duration, D_{Mac} — Macaulay duration, Con — bond convexity, T — bond tilt, $T(k)$ — bond tilt as a function of k (ceteris paribus (CP)), $T(i)$ — bond tilt as a function of i (CP), $T(n)$ — bond tilt as a function of n (CP). Let us begin with the formula for the present value of the bond:

$$P = \sum_{t=1}^{n} \frac{iN}{r^t} + \frac{N}{r^n}. \tag{1}$$

Starting from $\Delta P = P(r+\Delta r) - P(r)$, the Taylor series yields expressions for duration, convexity, and tilt, respectively:

$$\frac{\Delta P}{P} \approx \frac{1}{P}\cdot\frac{\partial P}{\partial r}\Delta r + \frac{1}{2P}\cdot\frac{\partial^2 P}{\partial r^2}\Delta r^2 + \frac{1}{6P}\cdot\frac{\partial^3 P}{\partial r^3}\Delta r^3, \tag{2}$$

$$D_{mod} = \frac{D_{Mac}}{r} = -\frac{1}{P}\cdot\frac{\partial P}{\partial r},\ Con = \frac{1}{P}\cdot\frac{\partial^2 P}{\partial r^2}, \tag{3}$$

$$T = -\frac{1}{P}\cdot\frac{\partial^3 P}{\partial r^3} = \frac{1}{Pr^3}\left(\sum_{t=1}^{n} t(t+1)(t+2)\frac{iN}{r^t} + n(n+1)(n+2)\frac{N}{r^n}\right). \tag{4}$$

3 BOND TILT PROPERTIES

Proposition 3.1 *Let* $T_1(n) = 6r^{n+3} - (r-1)^3n^3 - 3(2r-1)(r-1)^2n^2 - (11r^3 - 18r^2 + 9r - 2)n - 6r^3$, $T_2(n) = n(n+1)(n+2)(r-1)^4$, *and* $T_3(n) = r^3(r-1)^4\big((r^n-1)\frac{i}{k}+1\big)$. *The closed form of the formula* (4) *is given by the following expression:*

$$T(n) = \frac{T_1(n)i + T_2(n)}{T_3(n)}. \tag{5}$$

The proof of formula (5), involving only finite sums, is omitted due to space limitations.

Theorem 3.2 *Properties of* T *with respect to change of its variables are as follows:*

(i) For the function $T = T(n)$ *the following inequality holds:*

$$1 \le r^3T(n) \le n(n+1)(n+2). \tag{6}$$

(ii) The function $T = T(i)$ *is decreasing (ceteris paribus), i.e., for any two coupon rates such that* $i_1 \le i_2$, *the inequality* $T(i_1) \ge T(i_2)$ *holds.*

(iii) The function $T = T(k)$ *is decreasing (ceteris paribus), i.e., for any two yields to maturity such that* $k_1 \le k_2$, *the inequality* $T(k_1) \ge T(k_2)$ *holds.*

(iv) The limit of the function $T = T(n)$ *is equal to* $L_T = \lim_{n\to+\infty} T(n) = \frac{6}{k^3}$. *Furthermore, if* $i \ge k$, *then the function* $T(n)$ *is increasing with a maximal asymptotic value of* L_T. *In the case where* $0 < i < k$, *the additional analysis is required.*

Proof.

(i) Since $1 \le t(t+1)(t+2) \le n(n+1)(n+2)$ for all $t = 1, 2, \ldots, n$, inequality (6) follows directly from (1) and (4).

(ii) Let $A = \sum_{t=1}^{n}\frac{t(t+1)(t+2)N}{r^t}$, $B = \sum_{t=1}^{n}\frac{iN}{r^t}$, $C = \frac{n(n+1)(n+2)N}{r^n}$, $D = \frac{N}{r^n}$, and $\delta > 0$. Then

$$\begin{aligned} T(i+\delta) - T(i) &= \frac{((i+\delta)A + C)(iB+D) - (iA+C)((i+\delta)B+D)}{r^3P(i+\delta)P(i)} = \frac{\delta(AD-BC)}{r^3P(i+\delta)P(i)} \\ &= \frac{\delta N^2}{r^{n+3}P(i+\delta)P(i)}\sum_{t=1}^{n}\frac{t(t+1)(t+2) - n(n+1)(n+2)}{r^t}. \end{aligned} \tag{7}$$

Since $\frac{\delta N^2}{r^{n+3}P(i+\delta)P(i)} > 0$ and $t(t+1)(t+2) \le n(n+1)(n+2)$ for all $t = 1, 2, \ldots, n$, it follows from (7) that $T(i+\delta) - T(i) \le 0$, which proves that $T = T(i)$ is decreasing (ceteris paribus).

(iii) Let us show that inequality $T(k+\delta) \le T(k)$ holds for any $\delta > 0$, i.e.,

$$\frac{r^3}{(r+\delta)^3} P(k) \left(\sum_{t=1}^{n} \frac{t(t+1)(t+2)iN}{(r+\delta)^t} + \frac{n(n+1)(n+2)N}{(r+\delta)^n} \right) \le$$
$$\le P(k+\delta) \left(\sum_{t=1}^{n} \frac{t(t+1)(t+2)iN}{r^t} + \frac{n(n+1)(n+2)N}{r^n} \right). \tag{8}$$

Since $\frac{r^3}{(r+\delta)^3} < 1$, the proof of inequality (8) reduces to proving the following inequality:

$$P(k) \left(\sum_{t=1}^{n} \frac{t(t+1)(t+2)iN}{(r+\delta)^t} + \frac{n(n+1)(n+2)N}{(r+\delta)^n} \right) \le$$
$$\le P(k+\delta) \left(\sum_{t=1}^{n} \frac{t(t+1)(t+2)iN}{r^t} + \frac{n(n+1)(n+2)N}{r^n} \right). \tag{9}$$

Let $\alpha = \frac{r}{r+\delta}$, $K_t = \frac{iN}{r^t}$, $M = \frac{N}{r^n}$. Note that $0 < \alpha < 1$. The inequality (9) is equivalent to the following inequalities

$$\Big(\sum_{t=1}^{n} K_t + M \Big) \Big(\sum_{t=1}^{n} t(t+1)(t+2) K_t \alpha^t + n(n+1)(n+2) M \alpha^n \Big) \le$$
$$\le \Big(\sum_{t=1}^{n} K_t \alpha^t + M \alpha^n \Big) \Big(\sum_{t=1}^{n} t(t+1)(t+2) K_t + n(n+1)(n+1) M \Big), \tag{10}$$
$$\sum_{j=1}^{n-1} \sum_{k=j+1}^{n} \big(j(j+1)(j+2) - k(k+1)(k+2) \big) \big(\alpha^j - \alpha^k \big) K_j K_k +$$
$$+ M \sum_{t=1}^{n} \big(t(t+1)(t+2) - n(n+1)(n+2) \big) \big(\alpha^t - \alpha^n \big) \le 0. \tag{11}$$

Since $0 < \alpha < 1$ and $j < k$, we have $\alpha^j - \alpha^k > 0$. Furthermore, since $t \le n$, it follows that $\alpha^t - \alpha^n \ge 0$. Thus, $\big(j^2 + j - k^2 - k \big) \big(\alpha^j - \alpha^k \big) \le 0$ and $\big(t(t+1) - n(n+1) \big) \big(\alpha^t - \alpha^n \big) \le 0$. This implies that inequality (11) holds, and consequently inequality (8) also holds, which proves that $T = T(k)$ is decreasing (ceteris paribus).

(iv) Using the well-known result for the limit $\lim_{n \to +\infty} \frac{n^p}{a^n} = 0$, where $n, p \in \mathbb{N}$, and $a > 1$, it follows from (5) that $L_T = \lim_{n \to \infty} T(n) = \lim_{n \to +\infty} \left(\frac{T_1(n) i}{r^{n+3}} + \frac{T_2(n)}{r^{n+3}} \right) \left(\frac{T_3(n)}{r^{n+3}} \right)^{-1} = \frac{6}{k^3}$. Let us now determine the sign of the difference $T(n+1) - T(n)$ using (1) and (4). On one hand, we have

$$P(n+1)T(n+1) - P(n)T(n) = \frac{1}{r^3} \left(\frac{(n+1)(n+2)(n+3)iN}{r^{n+1}} \right.$$
$$\left. + \frac{(n+1)(n+2)(n+3)N}{r^{n+1}} - \frac{n(n+1)(n+2)N}{r^n} \right). \tag{12}$$

On the other hand, we have

$$P(n+1)T(n+1) - P(n)T(n) = \left(P(n) + \frac{iN}{r^{n+1}} + \frac{N}{r^{n+1}} - \frac{N}{r^n} \right) T(n+1) - P(n)T(n)$$
$$= (T(n+1) - T(n))P(n) + \left(\frac{iN}{r^{n+1}} + \frac{N}{r^{n+1}} + \frac{N}{r^n} \right) T(n+1). \tag{13}$$

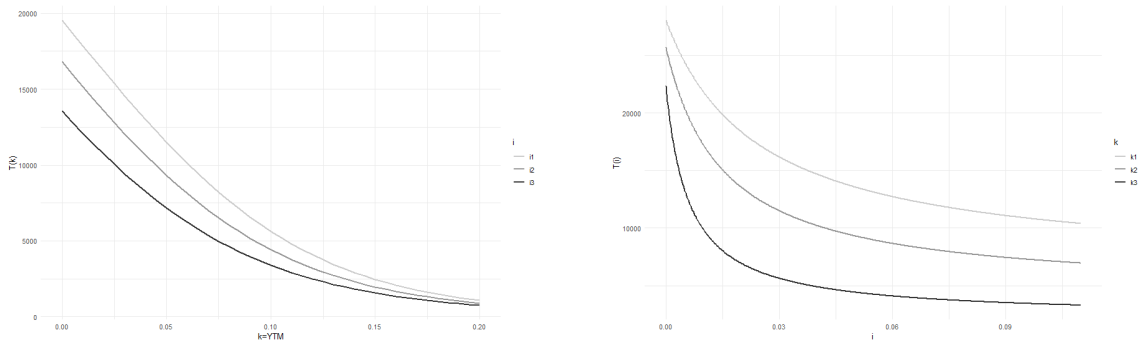
Equating (12) and (13) implies

$$(T(n+1)-T(n))P(n) =$$
$$= \frac{N}{r^{n+4}}\left(\Big((n+1)(n+2)(n+3)-r^3T(n+1)\Big)(i-k)+3r(n+1)(n+2)\right). \qquad (14)$$

Note: In case $i \geq k > 0$, (6) and (14) imply $T(n+1)-T(n) > 0$, i.e., the function $T(n)$ is increasing with maximal asymptotic value L_T. This completes the proof of the theorem. Q.E.D.

4 EMPIRICAL TILT

This section aims to empirically illustrate the properties of bond tilt, derived and proved in the preceding section, using selected values for the variables k (yield to maturity), i (coupon rate), and n (time to maturity).
Table 1 presents the calculated values of bond tilt for a fixed coupon rate of $i = 0.05$, while varying the interest rate $k \in \{0, 0.01, 0.03, 0.05, 0.1, 0.2\}$, and time to maturity $n = \{1, 5, 10, 20, 30, 50, 100\}$ in years. Consistent with the theoretical derivations presented in equations (7) - (11), the results demonstrate that bond tilt decreases as the yield to maturity k increases, ceteris paribus. Conversely, the values of bond tilt are observed to increase with an increase in time to maturity (n), ceteris paribus. Figure 1a provides a graphical representation of the function $T(k)$, illustrating its decreasing nature with respect to k for specific coupon rates $i_1 = 3\%, i_2 = 5\%, i_3 = 10\%$, and a fixed time to maturity $n = 30$ years. Similarly, Figure 1b depicts $T(i)$ as a decreasing function of the coupon rate i for elected interest rates $k_1 = 2\%, k_2 = 5\%, k_3 = 10\%$ and constant time to maturity $n = 30$.



(a) (b)

Figure 1: (a) $T(k)$ at 3 different values of $i > 0$ (b) $T(i)$ at 3 different values of $k > 0$

The relationship between bond tilt and time to maturity n is graphically represented in Figure 2, also illustrating property (iv) of the bond tilt for a fixed value of yield to maturity ($k = 0.05$). Figure 2 examines two different scenarios: where $k > i_1 = 0.02$ and where $k < i_2 = 0.08$. Consistent with equation (5), the horizontal line shown represents the theoretical limit $L_T = 6/0.05^3$. The 'hump' in the tilt function is observed in the case where the coupon rate i is smaller than the yield to maturity k, particularly at greater values of n. While our numerical analyses did not reveal alternative functional forms or shapes for $T(n)$, we acknowledge that different scenarios cannot be excluded.

Figures 3a and 3b illustrate the explanatory power of approximations utilizing duration, convexity, and tilt (equation (1)) in predicting the expected change in the price of the bond due to changes in YTM. This part of the analysis varies changes in yield to maturity (YTM),

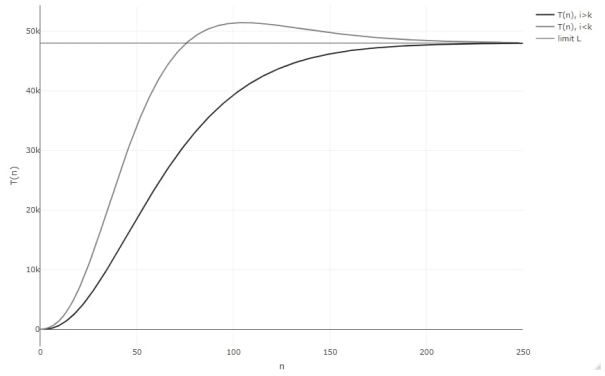


Figure 2: Tilt(n) in cases where $k < i$ and $k > i$

denoted as dr (where $dr = k_1 - k_0$) at different levels of YTM and a fixed level of coupon rate $i = 0.05$ and a time to maturity of $n = 30$ years. Values approaching 1 in these figures indicate that the approximation accounts for nearly 100% of the actual price change.

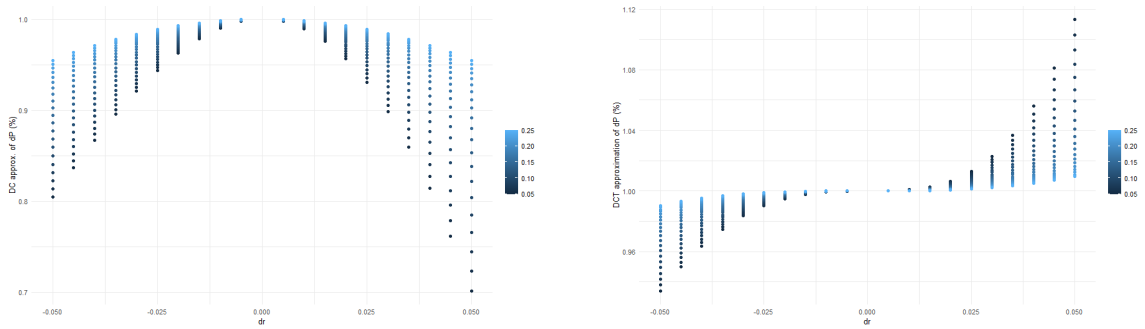
Specifically, Figure 3a depicts the percentage of bond price change explained solely by modified duration and convexity in Taylor series expansion (the DC-approximation). As anticipated, the accuracy of this approximation is highest for small changes in dr, and notably, all approximations consistently underestimate the actual price change. Conversely, Figure 3b presents the percentage of bond price change explained by incorporating modified duration, convexity, and tilt (the DCT-approximation) in the Taylor series expansion. This demonstrates that including the tilt term substantially enhances the accuracy of approximations, particularly for larger magnitudes of YTM change. An interesting observation is that the DCT approximation tends to underestimate the actual change for negative changes of YTM ($dr < 0$) and overestimate it for positive changes ($dr > 0$).

	Time to maturity						
YTM	1	5	10	20	30	50	100
0	6.00	184.80	1023.00	5948.25	16814.40	62985.00	392763.75
0.01	5.82	178.77	980.34	5543.98	15159.04	52812.62	271979.50
0.03	5.49	167.42	899.65	4775.30	12055.38	35004.20	110114.39
0.05	5.18	156.93	824.73	4065.45	9317.47	21746.34	41716.93
0.1	4.51	133.98	660.14	2586.20	4424.75	5975.75	6033.16
0.2	3.47	99.00	412.29	899.18	896.56	771.35	750.02

Table 1: Bond tilt for $i = 0.05$ and chosen values of YTM and time to maturity

5 CONCLUSION AND FUTURE WORK

This paper presents novel formal mathematical proofs for selected properties of bond tilt, which, to date, have predominantly been demonstrated through empirical observations in the existing literature. By formally proving the monotonicity of bond tilt with respect to the coupon rate, yield to maturity, and time to maturity (under ceteris paribus conditions), this work addresses a significant gap in the understanding of tilt's behavior. While acknowledging that the inclusion of tilt may offer only marginal improvements in pricing accuracy for option-free bonds under small interest rate changes, the mathematical foundation established here is essential for guiding future research. The subsequent research should be directed towards the detailed analysis of tilt's dynamic behavior over time to maturity, especially in scenarios where the annual coupon payment is smaller than the yield to maturity.



(a) (b)

Figure 3: (a) DC approximation w.r.t. real price change (b) DCT approximation w.r.t. real price change

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The 18th International Symposium on
Operational Research in Slovenia

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Bled, SLOVENIA
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Session 9:

Different Problems Approaching OR

WHAT IS "PRIOR"?

Ziv Hellman
Department of Economics, Bar-Ilan University, Ramat-Gan, Izrael,
E-mail: ziv.hellman@biu.ac.il

Miklos Pinter
Corvinus Center for Operations Research, Corvinus University of Budapest, Budapest, Hungary,
E-mail: pmiklos@protonmail.com

Abstract: We consider finite information structures, and quest for the answer of the question: What is the proper definition of prior? In the single player setting we conclude that a probability distribution is a prior if it is disintegrable, because this definition excludes money pump. In the multiplayer setting our analysis does not boil down to one proper notion of common prior (the multiplayer version of prior). The appropriate notion is a choice of the modeller in this setting. We consider three variants of money pump, each "defines" a notion of common prior. Furthermore, we also consider three variants of trade, each correspond to one of the money pump variants, hence to one of the common prior variants.

Keywords: Information structure, Common prior, Trade, Money pump, Disintegrability, Conglomerability.

PREDICTING CUSTOMER CHURN IN THE CONSUMER GOODS INDUSTRY USING MACHINE LEARNING

Tin Horvat
Samsung Electronics Austria GmbH Zagreb branch office
Radnička cesta 37/B, 10000 Zagreb, Croatia
E-mail: tin.horvat44@gmail.com

Silvija Vlah Jerić
Faculty of Economics & Business, University of Zagreb
Trg J. F. Kennedya 6, 10000 Zagreb, Croatia
E-mail: svlah@net.efzg.hr

Abstract: In today's competitive business landscape, predicting customer churn has become a crucial element for companies seeking to maintain profitability. This study focuses on predicting churn within the consumer goods sector in Croatia, utilizing machine learning techniques: Decision Tree and Random Forest. The data spans from March 2022 to May 2024 and consists of customer purchase records from an online retailer. Key variables such as average delivery time, monetary value, and product engagement metrics were used as predictors. The models were evaluated using 5-fold cross-validation and then assessed on the entire dataset, indicating high predictive power for both models measured by metrics such as accuracy and F1 score. The RF model slightly outperformed the DT in terms of F1 score, suggesting better generalization capability. Feature importance analysis highlighted delivery time and purchase value as the most significant factors influencing churn. The findings suggest that companies can improve retention by focusing on logistical efficiency and customer engagement. This study contributes to the understanding of churn prediction in the consumer goods industry and offers practical insights for enhancing customer retention strategies.

Keywords: churn prediction, e-commerce, machine learning applications, decision trees, random forest.

1 INTRODUCTION

Customer churn represents a direct loss of revenue and can negatively impact a company's market position. The manner in which customer relationships are terminated varies across industries. In the telecommunications industry, for instance, the relationship typically ends when the customer decides not to renew their contract. In contrast, in non-contractual industries like consumer goods, the relationship begins with the first transaction. Today, a growing number of these transactions occur through online sales, driven by consumer preferences for convenience, broader product choices, and often lower prices [12].

Following the initial transaction, customers may choose to purchase again or stop buying from the company, leading to customer churn, defined as the shift of a customer to a competitor [17]. As noted in [13], customer churn refers to the number of customers who have stopped using a product due to various factors, such as changing preferences, inadequate customer relationship strategies, relocation, price, product quality, or delivery times. Therefore, implementing churn analysis has become essential for businesses to identify and anticipate customers most likely not to repurchase.

With the rise of e-commerce, understanding and predicting customer churn has become a strategic necessity for companies in the consumer goods sector. Unlike industries such as telecommunications, where customer relationships are often contractual, the consumer goods sector is more volatile because customers can freely switch between competitors without any formal obligation. This makes it crucial for companies to use sophisticated data analysis techniques to predict which customers are likely to churn. One such technique is machine learning, a branch of artificial intelligence that leverages historical data to predict future

outcomes. As mentioned in [16], advanced analytics allow businesses to anticipate customer loss and implement targeted retention strategies. Research in [15] suggests that ML models continuously improve by learning from past customer data, making them increasingly accurate over time. Research by [1] demonstrates the effectiveness of joint models for addressing both usage and churn, especially in settings with contractual agreements. Machine learning has long been applied in the telecommunications industry for churn management; for instance, data mining techniques to improve retention strategies are used in [5], highlighting the early adoption of these methods. Recent research has also focused on early churn prediction in non-contractual settings, such as mobile gaming, where personalized targeting is crucial [8].

Recent research on customer churn in Croatia has predominantly focused on telecommunications and retail banking sectors. Research in [9] applied extended RFM logit models in the mobile gaming and telecommunications industries, highlighting the integration of marketing metrics with predictive analytics. Neural networks for churn prediction in banking were explored in [3], while authors of [11] proposed a novel methodology for churn definition based on user engagement in mobile gaming. Another study utilized player behavior clustering to improve churn prediction models, providing further insights into the gaming industry [11].

In this work, the aim is to explore how machine learning can be used to predict customer churn in the consumer goods sector given the data from an e-commerce site. It contributes to the growing body of knowledge on customer churn by developing predictive models using decision trees (DT) and random forest (RF), some of the most widely used machine learning algorithms. Both methods classify customer behavior into categories, identifying key variables that indicate whether a customer is likely to stop purchasing. Understanding these factors enables businesses to tailor their efforts in order to retain high-value customers and optimize overall marketing investment.

The structure of the paper is as follows: First, data and methodology section describes the steps involved in building the decision tree model and a random forest model, including data collection, cleaning, and preparation. The results section evaluates the performance of the models, providing an analysis of how well the models predict churn. Finally, the conclusion provides the practical implications of the results and suggests directions for further research.

2 DATA AND METHODOLOGY

The data used to create the model spans the period from March 8, 2022, to May 2, 2024, and includes records of customer purchases through an online store operated by a company in Croatia's consumer goods sector. Data was extracted from the company's SAP system and a platform that tracks product ratings and customer comments generated on the e-commerce site. The literature also discusses the selection of optimal time windows for churn analysis, for example by [2] who discuss the impact of historical data length on churn prediction. Paper [7] illustrates how a backtesting framework can be applied to assess churn models.

Once extracted, the data was loaded into Python [14] for further preparation and analysis. The analysis was conducted using the libraries pandas, NumPy, scikit-learn, matplotlib, and seaborn for data processing, modeling and visualization. The original data consisted of two data sets with columns including transaction details, customer information, product reviews, etc. Some columns were excluded from further analysis because they provided redundant or irrelevant information. After merging and filtering, essential columns such as order number, customer name, quantity, net value, currency, order date, delivery date, product group, and product reviews were retained for analysis.

To prepare the data for modeling, missing values were addressed, and some data was transformed. For example, missing values in delivery dates were identified as errors, and other missing values in columns like product reviews and comments were replaced with zeroes, as

these represented products that had not been reviewed or commented on. Currency values were standardized by converting Croatian kuna (HRK) to euros (EUR) using a fixed conversion rate of 7.53450, applicable to transactions before January 1, 2023. The data was then adjusted for tax, converting net amounts to gross values using a 25% VAT rate. A unique customer ID was generated for each transaction using customer names, as the original dataset did not assign unique identifiers to each customer. This transformation resulted in a dataset containing 22,728 customers and 28,684 transactions, totaling 35,246 rows of data. A key target variable, churn, was created to classify customers who made fewer than two purchases as churned. After cleaning and creating the necessary features, the dataset contained 83.78% churned customers and 16.22% retained customers. This final dataset contained total of 22,728 unique customer records (samples). The final set of variables used in the model is included the following:

- Monetary value: The total value of all purchases.
- Average delivery time: The average number of days between order creation and delivery.
- Average Number of Purchased Products
- Average Number of Free Products (TV)
- Average Number of Free Products (Monitors)
- Average Number of Free Products (Soundbar)
- Average Number of Free Products (Smart Devices)
- Average Number of Free Products (Home Appliances)
- Average Rating of Purchased Products
- Average Number of Comments on Purchased Products

With the data prepared, two machine learning models were applied: Decision Trees and Random Forests. Both Decision Tree and Random Forest algorithms, as well as many others are thoroughly described in [4]. These models were chosen due to their effectiveness in handling classification tasks, particularly churn prediction, where the goal is to classify customers as churners or non-churners [6]. A decision tree classifier as well as a random forest classifier were implemented with default hyperparameter settings. The data was standardized ensuring the models are more consistent and less sensitive to the scale of variables.

In order to evaluate and compare the models effectively, 5-fold cross-validation was used, where the data was split into five subsets for training and testing. Finally, after the cross-validation, the model was trained on the entire dataset to assess its overall performance. Additionally, a feature importance analysis was conducted for both final models (DT and RF) to assess the influence of each variable on the model's performance.

3 RESULTS AND DISCUSSION

The performance of the algorithms is assessed, followed by observing the most important features, interpretation of results and discussion of practical implications. The metrics used to assess the models include accuracy and F1 score. Table 1 presents cross-validation scores for both algorithms. It can be observed that the Random Forest outperformed the Decision Tree slightly, achieving a higher average F1 score (0.95 vs. 0.94). This suggests that Random Forest has better generalization capability, likely due to its ensemble nature, which reduces the likelihood of overfitting compared to a single Decision Tree.

Table 1: Cross-validation F1 scores for DT and RF

	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5
Decision Tree	0.943	0.944	0.940	0.951	0.948
Random Forest	0.950	0.954	0.951	0.958	0.958

Figure 1 shows the confusion matrix obtained by the models on the full set, for DT and RF respectively. The results show that both models perform well, with high accuracy rates.

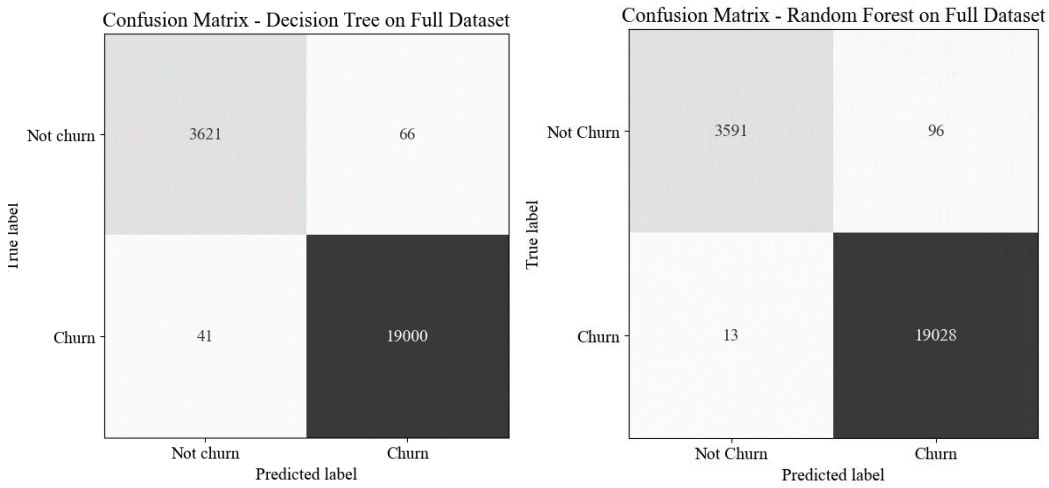


Figure 1: Confusion matrix on full dataset for DT (left panel) vs RF (right panel)

Figures 2 and 3 present the variable importance derived from DT and RF model respectively.

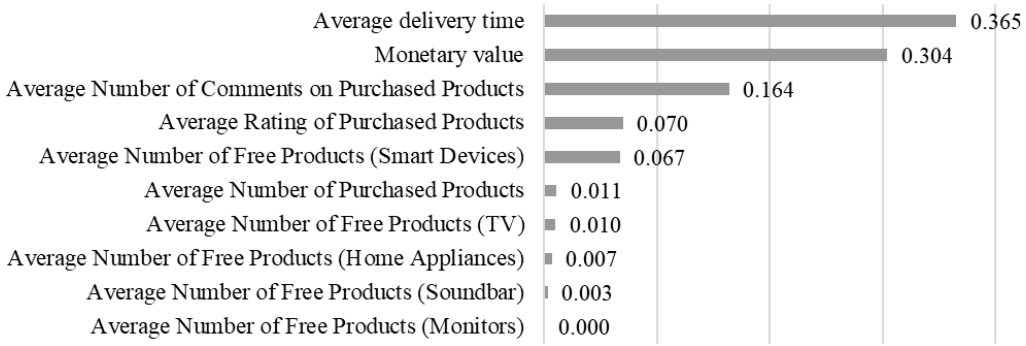


Figure 2: Feature importance obtained by DT

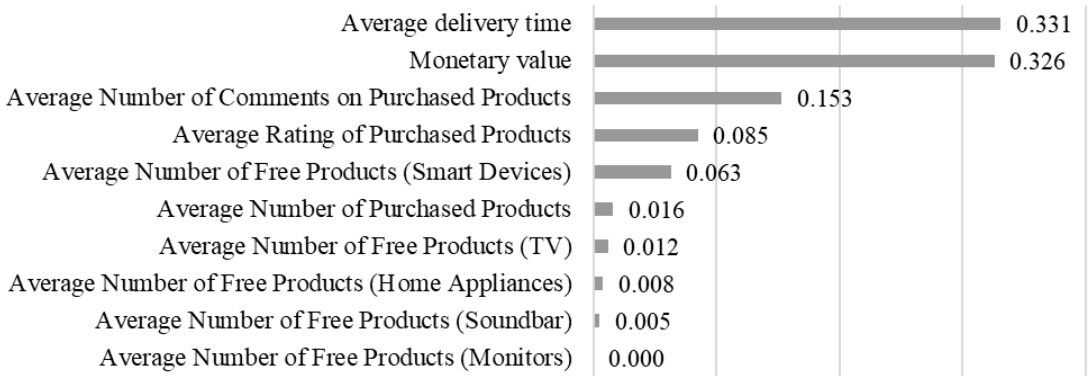


Figure 3: Feature importance obtained by RF

For both models, "Average delivery time" and "Monetary value" are the most important features for predicting churn, followed by "Average number of comments on purchased products." With the DT model, there is a larger gap between the importance between the two most important features, while the two features are of more similar importance in the RF model.

These findings suggest that how quickly a product is delivered and the monetary value associated with purchases significantly influence customer retention. Other features, such as free products and the average number of purchased products, had a smaller impact on the predictions.

4 CONCLUSIONS

This study aimed to predict customer churn in the consumer goods sector using machine learning algorithms Decision Trees and Random Forests. The findings demonstrate that both models performed exceptionally well, with high accuracy and F1 scores, indicating their effectiveness in predicting customer churn. The Random Forest model slightly outperformed the Decision Tree, which aligns with its known strength in reducing variance and handling noisy data.

The analysis revealed that the most significant predictors of churn were "Average delivery time" and "Monetary value," highlighting the importance of efficient logistics and spending patterns in retaining customers. These insights suggest that companies can enhance customer retention strategies by focusing on timely delivery and encouraging higher-value purchases. Additionally, features such as "Average number of comments on purchased products" and "Average rating of purchased products" provided valuable insights, indicating that customer engagement and satisfaction play important roles in predicting churn.

Overall, this research contributes to the understanding of customer churn in the consumer goods sector, showing the usability of machine learning for developing proactive customer retention strategies. Future work could explore integrating additional data sources, such as social media activity or customer service interactions, to further improve predictive accuracy. Additionally, experimenting with other algorithms or ensemble techniques may provide further gains in model performance.

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SELECTED WORK SITUATIONS OF FACULTY PERSONNEL AND THE IMPACT ON WORK MOTIVATION IN THE REPUBLIC OF CROATIA

Marija Vuković
University of Split, Faculty of Economics, Business and Tourism, Department of Quantitative Methods
Cvite Fiskovića 5, Split, Croatia
E-mail: marija.vukovic@efst.hr

Snježana Pivac
University of Split, Faculty of Economics, Business and Tourism, Department of Quantitative Methods
Cvite Fiskovića 5, Split, Croatia
E-mail: snjezana.pivac@efst.hr

Karmen Vrhar Šušnjara
University of Split, Faculty of Economics, Business and Tourism, Department of Quantitative Methods
Cvite Fiskovića 5, Split, Croatia
E-mail: karmen.vrhar@efst.hr

Abstract: This study examines the effects of selected negative work-related situations encompassing psychological factors, working conditions, unprofessional behaviour, and physical abuse on work motivation among faculty personnel. A survey of 113 academic and administrative staff at five Croatian universities was analyzed using PLS-SEM. Results show that psychological factors such as exclusion, verbal abuse, and task manipulation negatively affect interpersonal relationships by reducing collegiality and socialization. Unprofessional behavior, including work delays and resource misuse, positively affects the unmanageable workload by raising stress and hindering task completion. Poor working conditions, including unreasonable deadlines and denied entitlements, positively affect workload challenges, further impairing performance.

Keywords: bullying, faculty personnel, PLS-SEM, work motivation.

1 INTRODUCTION

In the higher education system, academic and administrative staff perform complex tasks requiring a high level of expertise, cooperation, and professional responsibility. Employees' work motivation plays an essential role in maintaining the quality of teaching, academic research, and management processes. Nevertheless, they are facing numerous psychosocial challenges and universities' working environments are not immune to negative work situations that can seriously damage the quality of interpersonal relationships, organizational climate, commitment and personal work motivation [4, 9]. Over the years, researchers have put more focus on exploring the impact of negative work factors such as physical abuse, unprofessional behavior, poor working conditions, and verbal or physical violence, on the employees' welfare, including the efficiency of educational institutions as a whole [3, 4]. According to the Job Demands-Resources (JD-R) model, such factors represent work demands that exceed the individual resources of employees, resulting in higher stress, drained energy, reduced motivation and burnout [2]. A lack of support and positive interpersonal relationships undermine work engagement and organizational commitment [9]. In universities, we often deal with "sophisticated, psychologically emphasised, inappropriate behavior that is difficult to label as bullying" in a traditional way [7]. Those situations are more challenging to detect and address, and are often manifested through subtle forms of mobbing. Psychological factors, including verbal insult, abuse, ignoring, manipulation of work tasks, and denial of rights such as paid leave or sick days, are perceived as professionally discouraging and emotionally draining by employees [11]. They also lead to deterioration of interpersonal relationships,

reduction of team spirit and social integration. Unprofessional behavior, such as deliberate delay of work and misuse of resources, can additionally increase the feeling of workload and make it more difficult to complete work tasks [14]. This also includes deliberate delay, concealment of information or sabotage of work processes, which can result in an unbearable workload and an increased risk of burnout [6]. Also, unfavorable and poor working conditions, such as unreasonable and inadequate deadlines and denials of legal labor rights, can increase the feeling of workload, job dissatisfaction and cause stress [2]. Additionally, they are connected with a negative perception of personal effectiveness and professional contribution [3]. In the Croatian scientific context, there has been a lack of research focusing on analyzing negative work situations and their effects on employees' motivation at universities. Moreover, the application of PLS-SEM is still relatively limited, particularly in analyzing negative work factors affecting faculty personnel. Nevertheless, there is an example of its successful use. The impact of interpersonal conflicts on employees was analyzed, with conflicts being recognized as both stressors and incentives for resource mobilization needed for facing the work demands [15]. The research topic is relevant to our field of interest, but it focuses on elementary school employees. There is still a shortage of research in Croatian higher education, especially using the PLS-SEM method to analyze the impact of negative work situations on employees' motivation, and this study aims to fill this void. The research is based on the data collected through a questionnaire among teaching and administrative staff of five public universities in Croatia. The PLS-SEM method is used to investigate the effects of psychological factors, unprofessional behavior, working conditions, and physical abuse on three key indicators of work motivation: interpersonal relationships, commitment, and workload.

2 DATA AND METHODOLOGY

Data were collected via an online survey questionnaire. The target population included teaching/research and administrative staff of the five largest universities in Croatia. The questionnaire was distributed via email to relevant university officials at selected institutions, who subsequently forwarded it to their employees. The final sample consists of 113 respondents. The questionnaire content was designed according to previous research [1, 8]. Along the basic demographics, the survey included questions regarding manifestations of experienced bullying at work, and the interpersonal relationships, commitment, and workload. These questions were ranked on a 5-point Likert scale, where 1 indicated strong disagreement and 5 strong agreement with a certain statement. The average age of the respondents is 42 years with an average of 14.89 years of work experience. The majority of the respondents are female (67.26%), and most were employed on a permanent contract (73.45%). In terms of job position, 56.64% are research and teaching staff, 17.70% teaching assistants, 22.12% administrative or professional staff, and 3.54% teaching-only staff. The respondents were predominantly from University of Split (26.55%) and Zagreb (25.66%), followed by University of Rijeka (23.89%), Osijek (19.47%) and Dubrovnik (4.42%). Exploratory factor analysis was used to extract factors related to negative work situations. Subsequently, these factors were used in the PLS-SEM analysis, due to the nature of the variables that represent psychological and behavioral concepts, captured indirectly through sets of items. This method also enables the simultaneous estimation of the interrelationships. Additionally, it is suitable for novel exploratory research models and focusing on prediction, rather than confirmation of factor interrelationships [5, 13].

3 EMPIRICAL RESULTS

The research model included seven latent constructs: psychological factors (PSY), working conditions (WC), unprofessional behavior (UB), physical abuse (PHYS), interpersonal

relationships (INT), commitment (COM), and workload (WL). Exploratory factor analysis was conducted to group the items into meaningful factors representing negative work situations. The KMO (0.915) and significant Bartlett's Test ($p<0.001$) confirmed sampling adequacy. Four factors that explained 72.26% of the total variance were extracted. Factor loadings (FL) were calculated to assess the strength of association between each item and the identified components. The convergent validity and reliability of the constructs are evaluated by examining the outer loadings (OL), average variance extracted (AVE), composite reliability (CR) and Cronbach's alpha coefficient (CA). The results are shown in Table 1. All outer loadings are relatively high and statistically significant. This is connected to AVE, which is above 0.5 for all constructs, indicating that all constructs explain a substantial part of each indicator's variance [5, 15]. CR values for all latent constructs exceed the recommended threshold of 0.70, indicating strong internal consistency and reliability of the scales [5, 12]. CA, a more traditional and conservative measure of internal consistency, also yielded satisfactory results for all constructs, although slightly lower in the case of COM ($\alpha = 0.739$) and WL ($\alpha = 0.752$), but still above the 0.7 threshold. This discrepancy is expected, as alpha typically produces lower estimates than CR.

Table 1: Factor analysis, convergent validity and reliability of the constructs

Code	***Item***	***FL***	***OL****	***CA***	***CR***	***AVE***
PSY1	You have been bullied at the workplace.	0.695	0.829	0.974	0.976	0.708
PSY2	Someone withholding information which affects your performance.	0.719	0.850			
PSY3	Being humiliated or ridiculed in connection with your work.	0.770	0.849			
PSY4	Having key areas of responsibility removed or replaced with more trivial or unpleasant tasks.	0.615	0.812			
PSY5	Spearing of gossip and rumours about you.	0.729	0.863			
PSY6	Being ignored or excluded.	0.707	0.856			
PSY7	Having insulting or offensive remarks made about your person, attitudes or your private life.	0.739	0.829			
PSY8	Being shouted at or being the target of a spontaneous anger.	0.625	0.795			
PSY9	Hints or signals from others that you should quit your job.	0.703	0.767			
PSY10	Repeated reminders of your errors or mistakes.	0.765	0.848			
PSY11	Being ignored or facing a hostile reaction when you approach.	0.746	0.883			
PSY12	Persistent criticism of your errors or mistakes.	0.754	0.871			
PSY13	Having your opinions ignored.	0.671	0.859			
PSY14	Practical jokes carried out by people you don't get along with.	0.559	0.780			
PSY15	Having allegations made against you.	0.773	0.855			
PSY16	Excessive monitoring of your work.	0.751	0.904			
PSY17	Being the subject of excessive teasing and sarcasm.	0.711	0.838			
WC1	Being ordered to do work below your level of competence.	0.656	0.831	0.839	0.891	0.672
WC2	Being given tasks with unreasonable deadlines.	0.841	0.861			
WC3	Pressure not to claim something to which by right you are entitled (e.g. sick leave).	0.648	0.751			
WC4	Being exposed to an unmanageable workload.	0.743	0.831			
UB1	You have recognised that somebody is intentionally delayed his/her working.	0.742	0.899	0.900	0.927	0.720
UB2	You have found out that somebody is purposely wasting resources (for example working time) doing something other than what belongs in his/her own work.	0.729	0.904			

UB3	Somebody has intentionally lied to you about work she/he has done.	0.643	0.920			
UB4	Somebody has been delaying or dawdling with her/his working so that you have been forced to wait for her/his contribution.	0.642	0.827			
UB5	You have found out that somebody has intentionally stolen employer´s belongings for his/her own use.	0.563	0.669			
PHYS1	Intimidating behaviours such as finger-pointing, invasion of personal space, shoving, blocking your way.	0.505	0.798	0.797	0.866	0.619
PHYS2	Threats of violence or physical abuse or actual abuse.	0.681	0.745			
PHYS3	Being the subject of sexual harassment.	0.733	0.774			
PHYS4	Being the subject of cyberbullying.	0.856	0.827			
INT1	I have a sense of friendship and team spirit with colleagues.	-	0.906	0.917	0.939	0.757
INT2	Work relations are satisfactory.		0.927			
INT3	There is a good interpersonal communication and cooperation.		0.925			
INT4	There are coordinated and integrated activities.		0.876			
INT5	There are chances for socialization with colleagues during work.		0.693			
COM1	I am ready to put extra efforts to accomplish my work.	-	0.580	0.739	0.819	0.536
COM2	I am aware of quality concepts while performing my duties.		0.664			
COM3	I have a high degree of loyalty to this University.		0.833			
COM4	I have clear achievable goals and standards for my position.		0.820			
WL1	My required workload reduces the quality of performance.	-	0.664	0.752	0.839	0.567
WL2	My work makes me stressed.		0.738			
WL3	I can not accomplish my assigned workload.		0.749			
WL4	I do not get the necessary information to accomplish my work.		0.850			

** Significant at 0.01 level*

Table 2 shows the analysis of the discriminant validity of the latent constructs through the heterotrait-monotrait ratio (HTMT) of the correlations. HTMT should be less than 0.85 in order to support the existence of the discriminant validity [5, 12, 15]. In this case, all of the HTMT values are relatively low, indicating that there is no problem with the discriminant validity and that each construct is truly distinct from other constructs in the model.The answers to the research questions were detected by observing the structural (inner) model relationships and testing their significance through the bootstrapping procedure (Table 3).

Table 2: Discriminant validity of the constructs according to the HTMT ratio

	COM	INT	PHYS	PSY	UB	WC	WL
COM							
INT	0.539						
PHYS	0.219	0.467					
PSY	0.203	0.634	0.710				
UB	0.129	0.542	0.650	0.843			
WC	0.198	0.439	0.519	0.793	0.736		
WL	0.226	0.422	0.444	0.575	0.614	0.678	

Table 3: Structural model coefficients with significance testing

	path coefficient	***t***	***p***
PHYS → COM	-0.139	0.834	0.404

PHYS → INT	-0.027	0.242	0.809
PHYS → WL	0.058	0.575	0.565
PSY → COM	-0.296	1.231	0.218
PSY → INT	-0.599	3.079	0.002
PSY → WL	0.048	0.276	0.782
UB → COM	0.374	2.160	0.031
UB → INT	-0.080	0.524	0.601
UB → WL	0.253	1.937	0.053
WC → COM	-0.137	0.988	0.323
WC → INT	0.096	0.828	0.408
WC → WL	0.351	2.455	0.014

Results of the PLS-SEM analysis indicate that PSY, involving social exclusion, verbal abuse, persistent criticism, rumours and manipulation of work tasks significantly negatively affect INT ($\beta=-0.599$, $p=0.002$), and contribute to a work environment with reduced collegiality, diminished team spirit, impaired communication and cooperation, and fewer opportunities for workplace socialization. UB, manifested through intentional work delays, resource misuse, dishonesty, stealing, and avoiding responsibilities, has a significant positive effect on COM ($\beta=0.374$, $p=0.031$), suggesting that in dysfunctional environments, commitment might increase due to compensatory mechanisms or internalised responsibility. Its effect on WL is marginally significant at 5% ($\beta=0.253$, $p=0.053$). WC (e.g. unreasonable deadlines, excessive workload) positively and significantly affect WL ($\beta=0.351$, $p=0.014$), by exacerbating stress and creating obstacles to task completion. Lastly, PHYS does not significantly influence any of the observed outcomes ($p>0.5$ in all paths), suggesting it may be less prevalent or less directly linked to constructs in this sample. Figure 1 shows the structural model with standardized path coefficients, outer loadings for each indicator, and R^2 values (the value of the coefficient of determination) for the three endogenous constructs: INT, COM and WL. The model demonstrates acceptable in-sample predictive power, with 38.0% of the variance explained in INT ($R^2=0.380$), 9.3% in COM ($R^2=0.093$), and 38.8% in WL ($R^2=0.388$). These values indicate that the model has moderate predictive power [5, 10] for INT and WL, and limited predictive ability for COM. Additionally, the model's out-of-sample predictive power was assessed with PLS predict procedure, as well as the cross-validated predictive ability test (CVPAT) [13]. PLS predict shows that all prediction errors, including both root mean squared error (RMSE) and mean absolute error (MAE) values are lower in the PLS model compared to the LM model, implying a high predictive power. CVPAT analysis demonstrates that PLS predictions significantly outperform the benchmark models ($p<0.001$) [13].

4 CONCLUSION

This research explores the impact of selected negative work situations on faculty personnel's work motivation in Croatian higher education. While some effects have been examined in previous studies, their influence on university staff remains underexplored, particularly through the PLS-SEM methodology.

Based on the sample of 113 academic and administrative staff from five Croatian universities in Split, Zagreb, Rijeka, Osijek and Dubrovnik, the PLS-SEM analysis reveals that psychological factors such as exclusion, verbal abuse, and persistent criticism have a negative effect on interpersonal relationships, reducing collegiality, social interaction, and workplace cohesion. Unprofessional behavior showed a positive effect on commitment, which may suggest the presence of compensatory motivation mechanisms among employees facing dysfunctional conditions. Poor working conditions significantly contribute to perceived workload, potentially impairing employee performance and increasing stress. The model

demonstrated acceptable reliability, validity, and predictive power. Further research is encouraged to deepen these findings by assessing the effects of negative workplace experiences that persist over time or evolve with institutional changes. Multi-group comparisons may also provide valuable insights into how demographic or positional differences influence the experience and impact of such stressors. Future research may benefit from expanding the scope to include additional psychological and organizational variables and broadening the sample to enhance the generalizability of the findings and capture a broader range of experiences.

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PRIVACY-PRESERVED POLYNOMIAL-TIME INTERIOR-POINT METHODS FOR COMPUTING MARKET EQUILIBRIA

Chuwen Zhang[1], **Chang He**[2], **Bo Jiang**[2] and **Yinyu Ye**[3,4]
[1] The University of Chicago Booth School of Business, Chicago, USA
[2] Shanghai University of Finance & Economics, Research Institute for Interdisciplinary Sciences, Shanghai, China
[3] Stanford University, Emeritus Faculty, Department of Management Science and Engineering, Stanford, California, USA
[4] Corvinus University of Budapest, Corvinus Institute for Advance Studies, Corvinus Centre for Operations Research, Budapest, Hungary
czhang12@chicagobooth.edu, chang.he@mail.shufe.edu.cn, jiang.bo@mail.shufe.edu.cn, yyye@stanford.edu

Abstract: We study the computation of equilibria in exchange markets with divisible goods and players endowed with heterogeneous utilities. In this paper, we revisit the polynomial-time interior-point methods and develop primal-dual based strategies that update only the prices with agents' responding demands, mirroring the tâtonnement process. The key ingredient is the logarithmic barrier structure inherent in the utility maximization: the utility turns unbounded when the goods are almost free of charge. Focusing on a ubiquitous class of utilities, we formalize this structure into agent utility maximization and market social optimization perspectives. The result suggests that no additional effort is required for computing high-order derivatives and all the necessary information is readily available when collecting the best responses to the current prices from agents. Building on these tools, we design two inexact interior-point methods: one such method has a linear convergence rate and, under mild conditions, the other method achieves a non-asymptotic super-linear convergence rate, which is in contrast with all previous tâtonnement methods that have only sublinear convergence rates.

Keywords: General Equilibrium, Convex Optimization, Interior-Point Methods, Privacy-Preserving

The 18th International Symposium on
Operational Research in Slovenia

SOR '25

Bled, SLOVENIA
September 24 - 26, 2025

Appendix

Authors' addresses

Addresses of SOR'25 Authors

(The 18th International Symposium on OR in Slovenia, September 24 – 26, 2025)

	First name	Surname	Institution	Street and Number	Post code	Town	Country	E-mail
1.	Kolos Csaba	Ágoston	Corvinus University of Budapest	Fővám tér 8	1093	Budapest	Hungary	kolos.agoston@ uni-corvinus.hu
2.	Zdravka	Aljinović	University of Split, Faculty of Economics, Business and Tourism	Cvite Fiskovića 5	21000	Split	Croatia	zdravka.aljinovic@efst.hr
3.	Joaquim	Alonso	proMetheus, Instituto Politécnico de Viana do Castelo	Rua Escola Industrial e Comercial Nun'Álvares	4900-347	Viana do Castelo	Portugal	malonso@esa.ipvc.pt
4.	Segundo	Álvarez-García	ArcelorMittal Global R&D Asturias, Mathematical Optimization Department University of Oviedo, Business Management Department	P.O. Box 90 Edificio Departamental Este, 1a planta	33400 Campus de Gijón (33204)	Avilés (Asturias) Gijón (Asturias)	Spain	uo53882@uniovi.es segundo.alvarez-garcia@arcelormittal.com
5.	Nicolás	Álvarez-Gil	University of Oviedo, Business Management Department, Edificio Departamental Este, 1a planta	Campus de Gijón	33204	Gijón (Asturias)	Spain	nicolas.alvarez@ arcelormittal.com
6.	Helder	Amorim	Agrarian School of Polytechnic Institute of Viana do Castelo	Rua D. Mendo Afonso, 147 Refóios do Lima	4990-706	Ponte de Lima	Portugal	h.amorim@ipvc.pt
7.	Josip	Arnerić	University of Zagreb, Faculty of Economics and Business, Department of statistics	J. F. Kennedy Sq. 6	10000	Zagreb	Croatia	jarneric@net.efzg.hr

	First name	Surname	Institution	Street and Number	Post code	Town	Country	E-mail
8.	Giuseppe	Attanasi	Sapienza University of Rome & Corvinus University of Budapest				Italy	giuseppe.attanasi@uniroma1.it
9.	Verica	Bakeva	Ss. Cyril and Methodius University, Faculty of Computer Science and Engineering		1000	Skopje	North Macedonia	verica.bakeva@finki.ukim.mk
10.	Daniil	Baldouski	University of Primorska, IAM		6000	Koper	Slovenia	
11.	Frane	Banić	Croatian National Bank, Economic Analysis Department	Trg hrvatskih velikana 3	10000	Zagreb	Croatia	frane.banic@hnb.hr
12.	Diana	Barro	Ca' Foscari University of Venice, Department of Economics San Giobbe	Cannaregio 873	30121	Venice	Italy	d.barro@unive.it
13.	Boštjan	Batagelj	University of Ljubljana, Faculty of Electrical Engineering	Tržaška cesta 25	1000	Ljubljana	Slovenia	bostjan.batagelj@fe.uni-lj.si
14.	János	Baumgartner	University of Pannonia, Department of Computer Science and Systems Technology	Egyetem str. 10	8200	Veszprém	Hungary	baumgartner.janos@mik.uni-pannon.hu
15.	Nina	Begičević Ređep	University of Zagreb, Faculty of organization and informatics	Pavlinska 2	42000	Varaždin	Croatia	nina.begicevic@foi.unizg.hr
16.	Omkar	Bihani	Rudolfovo - Science and Technology Centre Novo Mesto.	Podbreznik 15	8000	Novo mesto	Slovenia	omkar.bihani@rudolfovo.eu
17.	Aljaž	Blatnik	University of Ljubljana, Faculty of Electrical Engineering	Tržaška cesta 25	1000	Ljubljana	Slovenia	aljaz.blatnik@fe.uni-lj.si
18.	Bojan	Blažica	Jožef Stefan Institute,Department of Knowledge Technologies Proventus d.o.o.	Jamova cesta 39 Gradišče 20	1000 5270	Ljubljana Ajdovščina	Slovenia	bojan.blazica@ijs.si
19.	Marija	Bogataj	Alma Mater Europaea University Institute INRISK	Slovenska ulica 17 Kidričeva ulica 1	2000 8210	Maribor Trebnje	Slovenia	marija.bogataj@guest.arnes.si

	First name	Surname	Institution	Street and Number	Post code	Town	Country	E-mail
20.	Marko	Bohanec	Jožef Stefan Institute, Department of Knowledge Technologies	Jamova cesta 39	1000	Ljubljana	Slovenia	marko.bohanec@ijs.si
21.	Drago	Bokal	University of Maribor, Faculty of Natural Sciences and Mathematics, Laboratory for Natural Science and Mathematical Applications and Research Institute of mathematics, physics, and mechanics DataBitLab, d.o.o.	Koroška cesta 160 Jadranska ulica 19 Perhavčeva ul. 13	2000 1000 2000	Maribor Ljubljana Maribor	Slovenia	drago.bokal@um.si
22.	Maja	Borlinič Gačnik	University of Maribor, Faculty of Tourism	Cesta prvih borcev 36	8250	Brežice	Slovenia	maja.borlinic@um.si
23.	Mile	Bošnjak	University of Zagreb, Faculty of Economics and Business, Department of International Economics	Trg J. F. Kennedyja 6	10000	Zagreb	Croatia	mbosnjak1@net.efzg.hr
24.	Jure	Brečko	Agricultural institute of Slovenia, Department of Agricultural economics	Hacquetova 17	1000	Ljubljana	Slovenia	jure.brecko@kis.si
25.	Andrej	Bregar	Informatika d.d.	Vetrinjska ulica 2	2000	Maribor	Slovenia	andrej.bregar@ informatika.si
26.	Alenka	Brezavšček	University of Maribor, Faculty of Organizational Sciences	Kidričeva cesta 55a	4000	Kranj	Slovenia	alenka.brezavscek@um.si
27.	Maja	Breznik	The Peace Institute – Institute for Contemporary Social and Political Studies	Metelkova 6	1000	Ljubljana	Slovenia	
28.	Paško	Burnać	Faculty of Economics, Business and Tourism, Department of Finance	Cvite Fisković 5	21000	Split	Croatia	pburnac@efst.hr
29.	Tonči	Carić	University of Zagreb, Faculty of Transport and Traffic Sciences	Vukelićeva 4	10 000	Zagreb	Croatia	tcaric@fpz.unizg.hr

	First name	Surname	Institution	Street and Number	Post code	Town	Country	E-mail
30.	Xiaoni	Chi	School of Mathematics and Computing Science, Guangxi Colleges and Universities Key Laboratory of Data Analysis and Computation, Guilin University of Electronic Technology	Guilin	541004	Guangxi	P.R. China	chixiaoni@126.com
31.	Boris	Cota	University of Zagreb, Faculty of Economics and Business	J. F. Kennedy Square 6	10000	Zagreb	Croatia	bcota@efzg.hr
32.	László	Csató	Institute for Computer Science and Control (SZTAKI), Hungarian Research Network (HUN_REN) Corvinus University of Budapest	Kende street 13-17 Fővám tér 8	1111 1093	Budapest	Hungary	
33.	Ivana	Cunjak Mataković	University of Applied Science Baltazar	Vladimira Novaka 23	10290	Zaprešić	Croatia	icunjakmatakovci@bak.hr
34.	Marek	Czekajski	University of Economics in Katowice	ul. 1 Maja 50	40-287	Katowice	Poland	marek.czekajski@ edu.uekat.pl
35.	Peter	Czimmermann	University of Zilina, Faculty of Management Science and Informatics	Univerzitna 8215/1	010 26	Zilina	Slovak Republic	peter.czimmermann@ fri.uniza.sk
36.	Bruna	Čalija	Global Markets Investment Products at OTP banka d.d.	Domovinskog rata 61	21000	Split	Croatia	calija.bruna@gmail.com
37.	Vesna	Čančer	University of Maribor, Faculty of Economics and Business	Razlagova 14	2000	Maribor	Slovenia	vesna.cancer@um.si
38.	Anita	Čeh Časni	University of Zagreb, Faculty of Economics and Business, Department of Statistics		10 000	Zagreb	Croatia	aceh@efzg.hr
39.	Nejc	Čelik	University of Maribor, Faculty of Organizational Sciences	Kidričeva cesta 55a	4000	Kranj	Slovenia	nejc.celik1@um.si

	First name	Surname	Institution	Street and Number	Post code	Town	Country	E-mail
40.	Michal	Černý	Prague University of Economics and Business, Faculty of Informatics and Statistics, Department of Econometrics	nám. W. Churchilla 4	13067	Prague	Czech Republic	cernym@vse.cz
41.	Andraž	Čevka	EXOTERM-IT d.o.o.	Struževo 66	4000	Kranj	Slovenia	andraz.cevka@exoterm.si
42.	Elena	Čokor	DataBitLab d.o.o. University of Maribor, Faculty of Natural Sciences and Mathematics	Perhavčeva ulica 13 Koroška cesta 160	2000	Maribor	Slovenia	elena.cokor@student.um.si
43.	Nikola	Čokor	DataBitLab d.o.o. University of Maribor, Faculty of Electrical Engineering and Computer Science	Perhavčeva ulica 13 Koroška cesta 46	2000	Maribor	Slovenia	nikola.cokor@student.um.si
44.	Marko	Čular	University of Split, Faculty of Economics, Business and Tourism, Department of Accounting and Auditing	Cvite Fiskovića 5	21000	Split	Croatia	marko.cular@efst.hr
45.	Ana Marija	Ćoko	University of Split, Faculty of Economics, Business and Tourism	Cvite Fiskovića 5	21000	Split	Croatia	amcoko@efst.hr
46.	Maja	Ćukušić	University of Split, Faculty of Economics, Business and Tourism	Cvite Fiskovica 5	21000	Split	Croatia	maja.cukusic@efst.hr
47.	Zsolt	Darvay	Babeş-Bolyai University, Faculty of Mathematics and Computer Science				Romania	
48.	Balázs	Dávid	InnoRenew CoE University of Primorska, FAMNIT	Livade 6 Glagoljaška ul. 8	6310 6000	Izola Koper	Slovenia	balazs.david@ famnit.upr.si
49.	Marko	Debeljak	Institute Jožef Stefan, Department of Knowledge Technologies	Jamova Cesta 39	1000	Ljubljana	Slovenia	marko.debeljak@ijs.si
50.	Goran	Dedic	University of Split, Faculty of Economics, Business and Tourism	Cvite Fiskovica 5	21000	Split	Croatia	mpepur@efst.hr

	First name	Surname	Institution	Street and Number	Post code	Town	Country	E-mail
51.	Tanja	Dergan	Institute Jožef Stefan, Department of Knowledge Technologies	Jamova Cesta 39	1000	Ljubljana	Slovenia	tanja.dergan@ijs.si
52.	Gregor	Dolinar	University of Ljubljana, Biotechnical faculty	Jamnikarjeva 101	1000	Ljubljana	Slovenia	gregor.dolinar@bf.uni-lj.si
53.	Staš	Dolinšek	University of Ljubljana, Faculty of Electrical Engineering	Tržaška cesta 25	1000	Ljubljana	Slovenia	sd4973@student.uni-lj.si
54.	Nikola	Drinjak	University of Belgrade, Faculty of Organizational Sciences, Department of Operational Research and Statistics	Jove Ilica 154	11000	Belgrade	Serbia	nikola.drinjak@fon.bg.ac.rs
55.	Samo	Drobne	University of Ljubljana, Faculty of Civil and Geodetic Engineering	Jamova cesta 2	1000	Ljubljana	Slovenia	samo.drobne@fgg.uni-lj.si
56.	Tibor	Dulai	University of Pannonia, Department of Computer Science and Systems Technology	Egyetem St. 10	8200	Veszprém	Hungary	
57.	Ksenija	Dumičić	DOBA Business School	Prešernova 1	2000	Maribor	Slovenia	kdumicic.prof@gmail.com
58.	Marianna	Eisenberg-Nagy	Corvinus University of Budapest, Corvinus Centre for Operations Research at Corvinus Institute for Advanced Studies	Közraktár utca 4-6.	1093	Budapest	Hungary	marianna.eisenberg-nagy @uni-corvinus.hu
59.	Tomislav	Erdelić	University of Zagreb, Faculty of Transport and Traffic Sciences	Vukelićeva 4	10 000	Zagreb	Croatia	terdelic@fpz.unizg.hr
60.	Andrej	Fašánok	University of Zilina, Faculty of Management Science and Informatics	Univerzitna 8215/1	010 26	Zilina	Slovak Republic	fasanok@stud.uniza.sk
61.	Liljana	Ferbar Tratar	University of Ljubljana, Faculty of Economics	Kardeljeva pl 17	1000	Ljubljana	Slovenia	liljana.ferbar.tratar@ ef.uni-lj.si

	First name	Surname	Institution	Street and Number	Post code	Town	Country	E-mail
62.	Sarah	Fores	Manager of EURO					manager@euro-online.org
63.	Pavlína	Forstová Kuráňová	VSB – Technical University of Ostrava, Faculty of Economics, Department of Management	17. listopadu 2172/15	708 00	Ostrava-Poruba	Czech Republic	pavlina.forstova.kuranova@vsb.cz
64.	Piotr	Frąckiewicz	Institute of Exact and Technical Sciences, Pomeranian University in Słupsk	ul. Arciszewskiego 22A	76-200	Słupsk	Poland	
65.	Eduarda Asfora	Frej	Center of Decision Systems and Information Development – CDSID, Universidade Federal de, ,	Pernambuco Av. Da Arquitetura – Cidade Universitária		Recife, PE	Brazil	eafrej@cdsid.org.br
66.	Lýdia	Gábrišová	University of Zilina, Faculty of Management Science and Informatics	Univerzitna 8215/1	010 26	Zilina	Slovak Republic	lydia.gabrisova@fri.uniza.sk
67.	Qian	Gao	VSB – Technical University of Ostrava, Department of Finance	17. listopadu 2172/15	708 00	Ostrava-Poruba	Czech Republic	qian.gao@vsb.cz
68.	Elif	Garajová	Charles University, Faculty of Mathematics and Physics, Dpt. of Applied Mathematics Prague University of Economics and Business, Faculty of Inform atics and Statistics, Dpt. of Econometrics	Malostranské nám. 25 nám. W. Churchilla 4	11800 13067	Prague	Czech Republic	elif@kam.mff.cuni.cz
69.	Margareta	Gardijan Kedžo	University of Zagreb, Faculty of Economics & Business	Trg J. F. Kennedyja 6	10000	Zagreb	Croatia	mgardijan@efzg.hr
70.	Alberto	Garre	Universidad Politécnica de Cartagena, Department of Agronomical Engineering & Institute of Plant Biotechnology	Paseo Alfonso XIII 48		Murcia	Spain	alberto.garre@upct.es

	First name	Surname	Institution	Street and Number	Post code	Town	Country	E-mail
71.	Helena	Gaspars-Wieloch	Poznan Universtity of Economics and Business, Department of Operations Research and Mathematical Economics	Al. Niepodległości 10	61-875	Poznań	Poland	helena.gaspars@ue.poznan.pl
72.	Anna	Gorczyca-Goraj	University of Economics in Katowice, Department of Operations Research	ul. Bogucicka 3	40-287	Katowice	Poland	
73.	Petra	Grošelj	University of Ljubljana, Biotechnical Faculty	Jamnikarjeva 101	1000	Ljubljana	Slovenia	petra.groselj@bf.uni-lj.si
74.	Krzysztof	Grzanka	University of Economics in Katowice, Department of Operations Research	ul. Bogucicka 3	40-287	Katowice	Poland	
75.	Jelena	Gusić Munđar	University of Zagreb, Faculty of organization and informatics	Pavlinska 2	42000	Varaždin	Croatia	jelena.gusic@foi.unizg.hr
76.	Chang	He	Shanghai University of Finance & Economics, Research Institute for Interdisciplinary Sciences			Shanghai	China	chang.he@mail.shufe.edu.cn
77.	Ziv	Hellman	Bar-Ilan University, Department of Economics				Israel	ziv.hellman@biu.ac.il
78.	Milan	Hladík	Charles University, Faculty of Mathematics and Physics, Department of Applied Mathematics Prague University of Economics and Business, Faculty of Informatics and Statistics, Department of Econometrics	Malostranské nám. 25 nám. W. Churchilla 4	11800 13067	Prague	Czech Republic	hladik@kam.mff.cuni.cz
79.	Robert	Hlavatý	Charles University, Faculty of Mathematics and Physics	Ke Karlovu 2027/3	121 16	Praha 2	Czech Republic	
80.	Eloy	Hontoria	Technical University of Cartagena, Department of Business Economics	Calle Real 3		Cartagena	Spain	eloy.hontoria@upct.es

	First name	Surname	Institution	Street and Number	Post code	Town	Country	E-mail
81.	Tin	Horvat	Samsung Electronics Austria GmbH Zagreb branch office	Radnička cesta 37/B	10000	Zagreb	Croatia	tin.horvat44@gmail.com
82.	Tibor	Illés	Corvinus University of Budapest, Corvinus Centre for Operations Research at Corvinus Institute for Advanced Studies	Közraktár utca 4-6.	1093	Budapest	Hungary	
83.	Ria	Ivandić	University of Zagreb, Faculty of Economics and Business	J. F. Kennedy Square 6	10000	Zagreb	Croatia	rivandic@efzg.hr
84.	Mario	Jadrić	University of Split, Faculty of Economics, Business and Tourism	Cvite Fiskovica 5	21000	Split	Croatia	mario.jadric@efst.hr
85.	Tena	Jagačić	University of Zagreb, Faculty of organization and informatics	Pavlinska 2	42000	Varaždin	Croatia	tjagacic@foi.hr
86.	Dominik	Jagodic	EXOTERM-IT d.o.o.	Struževo 66	4000	Kranj	Slovenia	dominik.jagodic@ exoterm.si
87.	Marko	Jakšič	University of Ljubljana, School of Economics and Business	Kardeljeva pl. 17	1000	Ljubljana	Slovenia	marko.jaksic@ef.uni-lj.si
88.	Nemanja	Jalić	The University of Banja Luka, Faculty of Agriculture	Bulevar Vojvode Petra Bojovića 1A	78000	Banja Luka	Bosnia and Herzegovina	nemanja.jalic@ agro.unibl.org
89.	Jaroslav	Janáček	University of Žilina, Faculty of Management and Informatics	Univerzitná 8215/1	010 26	Žilina	Slovak Republic	jaroslav.janacek@ fri.uniza.sk
90.	Jelena	Jardas Antonić	University of Rijeka, Faculty of Economics and Business, Department of Quantitative Economics	Ivana Filipovića 4	51000	Rijeka	Croatia	jelena.jardas.antonic@ efri.uniri.hr
91.	Janja	Jerebic	University of Maribor, Faculty of Organizational Sciences	Kidričeva cesta 55a	4000	Kranj	Slovenia	janja.jerebic@um.si

	First name	Surname	Institution	Street and Number	Post code	Town	Country	E-mail
92.	Veljko	Jeremic	University of Belgrade, Faculty of Organizational Sciences, Department of Operational Research and Statistics	Jove Ilica 154	11000	Belgrade	Serbia	veljko.jeremic@fon.bg.ac.rs
93.	Bo	Jiang	Shanghai University of Finance & Economics, Research Institute for Interdisciplinary Sciences			Shanghai	China	jiang.bo@mail.shufe.edu.cn
94.	Jelena	Joksimović	Rudolfovo Science and Technology Center	Podbreznik 15	8000	Novo mesto	Slovenia	jelena.joksimovic@ rudolfovo.eu
95.	Nikola	Kadoić	University of Zagreb, Faculty of organization and informatics	Pavlinska 2	42000	Varaždin	Croatia	nkadoic@foi.hr
96.	Špela	Kajzer	University of Maribor, Faculty of Organizational sciences University of Maribor, Faculty of Natural Sciences and Mathematics	Kidričeva cesta 55a Koroška cesta 160	4000 2000	Kranj Maribor	Slovenia	spela.kajzer1@um.si
97.	Ema Luna	Karara Geršak	University of Ljubljana, Biotechnical Faculty	Jamnikarjeva 101	1000	Ljubljana	Slovenia	emaluna.kararagersak@ bf.uni-lj.si
98.	Ema	Kelin	University of Rijeka, Faculty of Economics and Business	Ivana Filipovića 4	51000	Rijeka	Croatia	ema.kelin@efri.uniri.hr
99.	Mirjana	Kljajić Borštnar	University of Maribor, Faculty of Organizational Sciences	Kidričeva 55a	4000	Kranj	Slovenia	mirjana.kljajic@um.si
100.	Vedran	Kojić	University of Zagreb, Faculty of Economics & Business	Trg J. F. Kennedyja 6	10000	Zagreb	Croatia	vkojic@efzg.hr
101.	Martin	Konestabo	Proventus d.o.o.	Gradišče 20	5270	Ajdovščina	Slovenia	martin.konestabo@ tomappo.com
102.	Špela	Korpič	Panvita d.o.o.	Lendavska ulica 5, Rakičan, p.p. 106	9000	Murska Sobota	Slovenia	spela.korpic@panvita.si

	First name	Surname	Institution	Street and Number	Post code	Town	Country	E-mail
103.	Aleš	Kresta	VSB – Technical University of Ostrava, Department of Finance	17. listopadu 2172/15	708 00	Ostrava-Poruba	Czech Republic	ales.kresta@vsb.cz
104.	Terezie	Krestová	VSB – Technical University of Ostrava, Faculty of Economics, Department of Management	17. listopadu 2172/15	708 00	Ostrava-Poruba	Czech Republic	terezie.krestova@vsb.cz
105.	Miklós	Krész	InnoRenew CoE University of Primorska, IAM University of Szeged	Livade 6a Muzejski trg 2 Boldogasszony sgt 6	6310 6000 6725	Izola Koper Szeged	Slovenia Hungary	miklos.kresz@ innorenew.eu
106.	Jože	Kropivšek	University of Ljubljana, Biotechnical Faculty	Jamnikarjeva 101	1000	Ljubljana	Slovenia	joze.kropivsek@bf.uni-lj.si
107.	Aljaž	Krpan	University of Ljubljana, Faculty of Mathematics and Physics Rudolfovo Institute	Jadranska ulica 19 Podbreznik 15	1000 8000	Ljubljana Novo mesto	Slovenia	krpan.aljaz@gmail.com
108.	Nicola	Kubacká	VSB – Technical University of Ostrava, Faculty of Economics, Department of Management	17. listopadu 2172/15	708 00	Ostrava-Poruba	Czech Republic	n.kubacka.n@gmail.com
109.	Aleš	Kuhar	University of Ljubljana, Biotechnical Faculty	Jamnikarjeva 101	1000	Ljubljana	Slovenia	ales.kuhar@bf.uni-lj.si
110.	Ana	Kundid Novokmet	University of Split, Faculty of Economics, Business and Tourism	Cvite Fiskovića 5	21000	Split	Croatia	ana.kundid.novokmet@ efst.hr
111.	Marek	Kvet	University of Žilina, Faculty of Management Science and Informatics	Univerzitná 8215/1	010 26	Žilina	Slovak Republic	marek.kvet@fri.uniza.sk
112.	Goran	Lesaja	Corvinus Centre for Operations Research, Corvinus University of Budapest at Corvinus Institute for Advanced Studies	Fővám tér 8	H-1093	Budapest	Hungary	goranlesaja@gmail.com

	First name	Surname	Institution	Street and Number	Post code	Town	Country	E-mail
113.	Klara	Ljubi	University of Ljubljana, School of Economics and Business	Kardeljeva pl. 17	1000	Ljubljana	Slovenia	klara.ljubi@ef.uni-lj.si
114.	Martin	Lukáčik	Bratislava University of Economics and Business, Department of Operations Research and Econometrics	Dolnozemská cesta 1	852 35	Bratislava	Slovak Republic	martin.lukacik@euba.sk
115.	Adriana	Lukáčiková	Bratislava University of Economics and Business, Department of Operations Research and Econometrics	Dolnozemská cesta 1	852 35	Bratislava	Slovak Republic	adriana.lukacikova@ euba.sk
116.	Velibor	Mačkić	University of Zagreb, Faculty of Economics and Business	J. F. Kennedy Square 6	10000	Zagreb	Croatia	vmackic@efzg.hr
117.	Bahate	Maidiya	VSB – Technical University of Ostrava, Department of Finance	17. listopadu 2172/15	708 00	Ostrava-Poruba	Czech Republic	bahate.maidiya@vsb.cz
118.	Nikola	Mardešić	University of Zagreb, Faculty of Transport and Traffic Sciences	Vukelićeva 4	10000	Zagreb	Croatia	nmardesic@fpz.unizg.hr
119.	Milica	Maričić	University of Belgrade, Faculty of Organizational Sciences, Department of Operational Research and Statistics	Jove Ilica 154	11000	Belgrade	Serbia	milica.maricic@ fon.bg.ac.rs
120.	Teodora	Marković	University of Maribor, Faculty of agriculture and life science	Pivola 10	2311	Hoče	Slovenia	teodora.markovic@um.si
121.	Maja	Meško	University of Maribor, Faculty of Organizational Sciences	Kidričeva cesta 55a	4000	Kranj	Slovenia	maja.mesko@um.si
122.	Zoran	Mihanović	University of Split, Faculty of Economics, Business, and Tourism	Cvite Fiskovića 5	21000	Split	Croatia	zmihanov@efst.hr
123.	Tea	Mijač	University of Split, Faculty of Economics, Business and Tourism	Cvite Fiskovica 5	21000	Split	Croatia	tea.mijac@efst.hr

	First name	Surname	Institution	Street and Number	Post code	Town	Country	E-mail
124.	Martin	Milanič	University of Primorska, FAMNIT and IAM	Glagoljaška ul. 8	6000	Koper	Slovenia	
125.	Ivan	Miloloža	Faculty for Dental Medicine & Health, University of J.J. Strossmayer	Crkvena 21	31000	Osijek	Croatia	ivan.miloloza@fdmz.hr
126.	Stefan	Mirchevski	Ss. Cyril and Methodius University, Faculty of Civil Engineering		1000	Skopje	North Macedonia	stefan_mircevski@outlook.com; stefan.mircevski@eurm.edu.mk
127.	Aldo	Montesano	Bocconi University Milan & Accademia dei Lincei Rome				Italy	aldo.montesano@unibocconi.it
128.	Ivana	Mravak	University of Split, Faculty of Economics, Business and Tourism	Cvite Fiskovića 5	21 000	Split	Croatia	ivana.mravak@efst.hr
129.	Martina	Nardon	Ca' Foscari University of Venice, Department of Economics San Giobbe	Cannaregio 873	30121	Venice	Italy	mnardon@unive.it
130.	Yurii	Nesteriv	Corvinus University of Budapest, Corvinus Centre for Operations Research at Corvinus Institute for Advanced Studies	Közraktár utca 4-6.	1093	Budapest	Hungary	
131.	Helena	Nikolić	University of Zagreb, Faculty of Economics and Business, Department of Trade	Trg J.F. Kennedyja 6	10000	Zagreb	Croatia	hnikolic@net.efzg.hr
132.	Leonel	Nunes	proMetheus, Instituto Politécnico de Viana do Castelo	Rua Escola Industrial e Comercial Nun'Álvares	4900-347	Viana do Castelo	Portugal	leonelnunes@esa.ipvc.pt
133.	Kosovka	Ognjenović	Institute of Economic Sciences, Welfare Economics Department	Zmaj Jovina Str. 12	11000	Belgrade	Serbia	kosovka.ognjenovic@ien.bg.ac.rs

	First name	Surname	Institution	Street and Number	Post code	Town	Country	E-mail
134.	Ákos	Orosz	University of Pannonia, Department of Computer Science and Systems Technology	Egyetem str. 10	8200	Veszprém	Hungary	
135.	Aleksandar	Ostojić	The University of Banja Luka, Faculty of Agriculture	Bulevar Vojvode Petra Bojovića 1A	78000	Banja Luka	Bosnia and Herzegovina	aleksandar.ostojic@ agro.unibl.org
136.	Željka	Ostović	Alma Mater Europaea University	Slovenska ulica 17	2000	Maribor	Slovenia	zeljka.ostovic@ almamater.si
137.	Irena	Palić	University of Zagreb, Faculty of Economics and Business Zagreb	Trg J.F. Kennedya 6	10000	Zagreb	Croatia	ipalic@efzg.hr
138.	Polona	Pavlovčič Prešeren	University of Ljubljana, Faculty of Civil and Geodetic Engineering	Jamova cesta 2	1000	Ljubljana	Slovenia	polona.pavlovcic-preseren@ fgg.uni-lj.si
139.	Martin	Pavlovič	University of Maribor, Faculty of Agriculture and Life sciences	Pivola 10	2311	Hoče	Slovenia	martin.pavlovic@um.si
140.	Karmen	Pažek	University of Maribor, Faculty of agriculture and life science	Pivola 10	2311	Hoče	Slovenia	karmen.pazek@um.si
141.	Mirjana	Pejić Bach	University of Zagreb, Faculty of Economics and Business, Department of Informatics Faculty of Commercial and Business Sciences IEDC-Bled School of Management	Trg J.F. Kennedyja 6 Lava 5 Prešernova cesta 33	10000 3000 4260	Zagreb Celje Bled	Croatia Slovenia	mpejic@net.efzg.hr
142.	Petra	Penga	University of Split, Faculty of Economics, Business and Tourism	Cvite Fiskovića 5	21 000	Split	Croatia	ppenga00@live.efst.hr
143.	Aljoša	Peperko	University of Ljubljana, Faculty of Mechanical Engineering Institute of Mathematics, Physics and Mechanics	Aškerčeva 6 Jadranska 19	1000	Ljubljana	Slovenia	aljosa.peperko@fs.uni-lj.si

	First name	Surname	Institution	Street and Number	Post code	Town	Country	E-mail
144.	Mario	Pepur	University of Split, Faculty of Economics, Business and Tourism	Cvite Fiskovica 5	21000	Split	Croatia	mpepur@efst.hr
145.	Ernest	Petrič	New University, European Faculty of Law		1000	Ljubljana	Slovenia	
146.	Sanja	Petrovic	Nottingham University Business School, Division of Operations Management and Information Systems	Jubilee Campus, Wollaton Road	NG8 1BB	Nottingham	UK	sanja.petrovic@ nottingham.ac.uk
147.	Miklos	Pinter	Corvinus University of Budapest, Corvinus Center for Operations Research			Budapest	Hungary	pmiklos@protonmail.com
148.	Snježana	Pivac	University of Split, Faculty of Economics, Business and Tourism, Department of Quantitative Methods	Cvite Fiskovića 5	21000	Split	Croatia	snjezana.pivac@efst.hr
149.	Martin	Pluskal	VSB – Technical University of Ostrava, Department of Finance	17. listopadu 2172/15	708 00	Ostrava-Poruba	Czech Republic	martin.pluskal.st@vsb.cz
150.	Primož	Podržaj	University of Ljubljana, Faculty of Mechanical Engineering	Aškerčeva 6	1000	Ljubljana	Slovenia	primoz.podrzaj@fs.uni-lj.si
151.	Aleksandra	Popovska-Mitrovikj	European University, Faculty of Informatics		1000	Skopje	North Macedonia	aleksandra.popovska. mitrvikj@finki.ukim.mk
152.	Melani	Potrč	University of Maribor, Faculty of Natural Sciences and Mathematics	Koroška cesta 160	2000	Maribor	Slovenia	melani.potrc1@um.si
153.	Janez	Povh	University of Ljubljana, Faculty of mechanical engineering; Rudolfovo Institute	Aškerčeva cesta 6 Podbreznik 15	1000 8000	Ljubljana Novo mesto	Slovenia	janez.povh@rudolfovo.eu
154.	Boris	Prevolšek	University of Maribor, Faculty of tourism	Cesta prvih borcev 36	8250	Brežice	Slovenia	boris.prevolsek@um.si

	First name	Surname	Institution	Street and Number	Post code	Town	Country	E-mail
155.	Dunja	Pucher	University of Klagenfurt, Department of Mathematics			Klagenfurt	Austria	dunja.pucher@aau.at
156.	Miroslav	Rada	Prague University of Economics and Business, Faculty of Informatics and Statistics, Department of Econometrics, Faculty of Finance and Accounting, Department of Financial Accounting and Auditing	nám. W. Churchilla 4	13067	Prague	Czech Republic	miroslav.rada@vse.cz
157.	Luna	Radević	University of Novi Sad – Faculty of Philosophy, Department of Psychology	Dr Zorana Đinđića 2	21000	Novi Sad	Serbia	luna.radevic@ff.uns.ac.rs
158.	Maja	Ralić	The University of Banja Luka, Faculty of Agriculture	Bulevar Vojvode Petra Bojovića 1A	78000	Banja Luka	Bosnia and Herzegovina	maja.ralic@ student.agro.unibl.org
159.	Simon	Ravnič	Panvita d.o.o.	Lendavska ulica 5, Rakičan, p.p. 106	9000	Murska Sobota	Slovenia	simon.ravnic@panvita.si
160.	Igor	Reznichenko	University of Ljubljana, Faculty of Mechanical Engineering	Aškerčeva 6	1000	Ljubljana	Slovenia	ir4016@student.uni-lj.si
161.	Petra Renáta	Rigó	Corvinus Centre for Operations Research, Corvinus University of Budapest at Corvinus Institute for Advanced Studies	Fővám tér 8	H-1093	Budapest	Hungary	
162.	Blaž	Rodič	University of Maribor, Faculty of Tourism; Rudolfovo - Science and Technology Centre Novo Mesto	Cesta prvih borcev 36 Pobreznik 15	8250 8000	Brežice Novomesto	Slovenia	blaz.rodic1@um.si
163.	Lucija	Rogić Dumančić	University of Zagreb, Faculty of Economics and Business	J. F. Kennedy Square 6	10000	Zagreb	Croatia	lrogic@efzg.hr
164.	Rafael	Rosillo	University of Oviedo, Business Management Department, Edificio Departamental Este, 1a planta	Campus de Gijón	33204	Gijón (Asturias)	Spain	rosillo@uniovi.es

	First name	Surname	Institution	Street and Number	Post code	Town	Country	E-mail
165.	Maryam	Roudneshin	UCD, Ireland	8 Glenard Hall, Friarland	D14W9E5	Dublin 14	Ireland	rudneshinmaryam@ yahoo.com
166.	Črtomir	Rozman	University of Maribor, Faculty of agriculture and life science	Pivola 10	2311	Hoče	Slovenia	crt.rozman@um.si
167.	Darja	Rupnik Poklukar	Faculty of Mechanical Engineering	Aškerčeva 6	1000	Ljubljana	Slovenia	darja.rupnik-poklukar@ fs.uni-lj.si
168.	Martina	Senekovič	University of Maribor, Faculty of agriculture and life science	Pivola 10	2311	Hoče	Slovenia	martina.senekovic@ student.um.si
169.	Renato	Silva	Agrarian School of Polytechnic Institute of Viana do Castelo	Rua D. Mendo Afonso, 147 Refóios do Lima	4990-706	Ponte de Lima	Portugal	renato.silva@esa.ipvc.pt
170.	Sandra	Silva	proMetheus, Instituto Politécnico de Viana do Castelo	Rua Escola Industrial e Comercial Nun'Álvares	4900-347	Viana do Castelo	Portugal	sandrasilva@esa.ipvc.pt
171.	Lucija	Skopljanac Macina					Croatia	lpleti01@live.efst.hr
172.	Amanda	Sosa	UCD, Ireland	8 Glenard Hall, Friarland	D14W9E5	Dublin 14	Ireland	
173.	Zoltán	Süle	University of Pannonia, Department of Computer Science and Systems Technology	Egyetem str. 10	8200	Veszprém	Hungary	
174.	Karol	Szomolányi	Bratislava University of Economics and Business, Department of Operations Research and Econometrics	Dolnozemská cesta 1	852 35	Bratislava	Slovak Republic	karol.szomolanyi@euba.sk
175.	Marek	Szopa	University of Economics in Katowice, Department of Operations Research	ul. Bogucicka 3	40-287	Katowice	Poland	

	First name	Surname	Institution	Street and Number	Post code	Town	Country	E-mail
176.	Jerneja	Šavrič	University of Maribor, Faculty of Tourism	Cesta prvih borcev 36	8250	Brežice	Slovenia	
177.	Tea	Šestanović	University of Split, Faculty of Economics, Business and Tourism, Department of Quantitative Methods	Cvite Fiskovića 5	21000	Split	Croatia	tea.sestanovic@efst.hr
178.	Anja	Šketa	DataBitLab d.o.o. University of Maribor, Faculty of Natural Sciences and Mathematics Institute of Mathematics, Physics and Mechanics	Perhavčeva ulica 13 Koroška cesta 160 Jadranska ulica 19	2000 1000	Maribor Ljubljana	Slovenia	anja.sketa@databitlab.eu
179.	Andrej	Škraba	University of Maribor, Faculty of Organizational Sciences	Kidričeva cesta 55a	4000	Kranj	Slovenia	andrej.skraba@um.si
180.	Blanka	Škrabić Perić	University of Split, Faculty of Economics, Business and Tourism	Cvite Fiskovica 5	21000	Split	Croatia	bskrabic@efst.hr
181.	Renee	Škulić Ažić	University of Rijeka, Faculty of Economics and Business, Department of National Economy and Development University of Maribor,Faculty of Economics and Business	Ivana Filipovića 4 Razlagova ulica 14	51000 2000	Rijeka Maribor	Croatia Slovenia	renee.skulic.azic@efri.uniri.hr renee.skulic@student.um.si
182.	Slavko	Šodan	Faculty of Economics, Business and Tourism, Department of Accounting and Auditing	Cvite Fisković 5	21000	Split	Croatia	ssodan@efst.hr
183.	Jana	Špirková	University of Rijeka, Faculty of Economics and Business, Department of Quantitative Economics	Ivana Filipovića 4	51000	Rijeka	Croatia	jana.spirkova@efri.uniri.hr

	First name	Surname	Institution	Street and Number	Post code	Town	Country	E-mail
184.	Tamás	Terlaky	Lehigh University, Department of Industrial and Systems Engineering, Quantum Computing Optimization Laboratory	200 W Packer Ave	PA 18015	Bethlehem	USA	terlaky@lehigh.edu
185.	Darija	Trpin Švikart	University of Maribor, Faculty of Agriculture and Life sciences	Pivola 10	2311	Hoče	Slovenia	darija.svikart@telemach.net
186.	Klemen	Tršinar	University of Maribor, Faculty of Natural Sciences and Mathematics	Koroška cesta 160	2000	Maribor	Slovenia	klemen.trsinar@ student.um.si
187.	Maja	Turnšek	University of Maribor, Faculty of Tourism				Slovenia	
188.	Tatyana	Unuchak	University of Maribor, Faculty of Organizational Sciences	Kidričeva 55a	4000	Kranj	Slovenia	tatyana.unuchak@ student.um.si
189.	Yauhen	Unuchak	University of Maribor, Faculty of Organizational Sciences	Kidričeva 55a	4000	Kranj	Slovenia	yauhen.unuchak@ student.um.si
190.	Veljko	Uskokovic	University of Belgrade, Faculty of Organizational Sciences, Department of Operational Research and Statistics	Jove Ilica 154	11000	Belgrade	Serbia	veljko.uskokovic@ fon.bg.ac.rs
191.	Anita	Varga	North Carolina State University, Department of Mathematics			Raleigh, NC	USA	
192.	Željko	Vaško	The University of Banja Luka, Faculty of Agriculture	Bulevar Vojvode Petra Bojovića 1A	78000	Banja Luka	Bosnia and Herzegovina	zeljko.vasko@ agro.unibl.org
193.	Ágnes	Vathy-Fogarassy	University of Pannonia, Department of Computer Science and Systems Technology	Egyetem str. 10	8200	Veszprém	Hungary	
194.	Josip	Visković	Faculty of Economics, Business and Tourism, Department of Finance	Cvite Fisković 5	21000	Split	Croatia	jviskovi@efst.hr

	First name	Surname	Institution	Street and Number	Post code	Town	Country	E-mail
195.	Josipa	Višić	University of Split, Faculty of Economics, Business and Tourism	Cvite Fiskovića 5	21 000	Split	Croatia	josipa.visic@efst.hr
196.	Silvija	Vlah Jerić	Faculty of Economics & Business, University of Zagreb	Trg J. F. Kennedya 6	10000	Zagreb	Croatia	svlah@net.efzg.hr
197.	Nenad	Vretenar	University of Rijeka, Faculty of Economics and Business, Department of Quantitative Economics	Ivana Filipovića 4	51000	Rijeka	Croatia	nenad.vretenar@efri.uniri.hr
198.	Karmen	Vrhar Šušnjara	University of Split, Faculty of Economics, Business and Tourism, Department of Quantitative Methods	Cvite Fiskovića 5	21000	Split	Croatia	karmen.vrhar@efst.hr
199.	Marija	Vuković	University of Split, Faculty of Economics, Business and Tourism, Department of Quantitative Methods	Cvite Fiskovića 5	21000	Split	Croatia	marija.vukovic@efst.hr
200.	Tomasz	Wachowicz	University of Economics in Katowice	ul. 1 Maja 50	40-287	Katowice	Poland	tomasz.wachowicz@ uekat.pl
201.	Guoqiang	Wang	School of Mathematics, Physics and Statistics, Shanghai University of Engineering Science		201620	Shanghai	P.R. China	guoq_wang@hotmail.com
202.	Jialei	Xiong	Technical University of Ostrava, Department of Finance	17. listopadu 2172/15	708 00	Ostrava-Poruba	Czech Republic	jialei.xiong@vsb.cz
203.	Masoud	Yavari	University of Ljubljana, School of Economics and Business	Kardeljeva pl. 17	1000	Ljubljana	Slovenia	yavari.masoud72@ gmail.com
204.	Yinyu	Ye	Stanford University, Emeritus Faculty, Department of Management Science and Engineering Corvinus University of Budapest, Corvinus Institute for Advance Studies, Corvinus Centre for Operations Research			Stanford, California Budapest	USA Hungary	yyye@stanford.edu

	First name	Surname	Institution	Street and Number	Post code	Town	Country	E-mail
205.	Lidija	Zadnik Stirn	University of Ljubljana, Biotechnical Faculty	Jamnikarjeva 101	1000	Ljubljana	Slovenia	lidija.zadnik@bf.uni-lj.si
206.	Martin	Zagorc	Rudolfovo Science and Technology Center	Podbreznik 15	8000	Novo mesto	Slovenia	martin.zagorc@ rudolfovo.eu
207.	Chuwen	Zhang	The University of Chicago Booth School of Business			Chicago	USA	czhang12@ chicagobooth.edu
208.	Kacper	Zielak	Poznan Universtity of Economics and Business, Department of Operations Research and Mathematical Economics	Al. Niepodległości 10	61-875	Poznań	Poland	kacper.zielak @ ue.poznan.pl
209.	Nikolina	Žajdela Hrustek	University of Zagreb, Faculty of organization and informatics	Pavlinska 2	42000	Varaždin	Croatia	nikolina.zajdela@foi.hr
210.	Janez	Žerovnik	Rudolfovo - Science and Technology Centre Novo Mesto. University of Ljubljana, Faculty of Mechanical Engineering	Podbreznik 15 Aškerčeva 6	8000 1000	Novo mesto Ljubljana	Slovenia	janez.zerovnik@ fs.uni-lj.si
211.	Jaka	Žgajnar	University of Ljubljana, Biotechnical faculty, Department of Animal Science	Groblje 3	1230	Domžale	Slovenia	jaka.zgajnar@bf.uni-lj.si
212.	Berislav	Žmuk	University of Zagreb, Faculty of Economics and Business, Department of Statistics	Trg J.F.Kennedyja 6	10000	Zagreb	Croatia	bzmuk@efzg.hr
213.	Bojan	Žugec	University of Zagreb, Faculty of organization and informatics	Pavlinska 2	42000	Varaždin	Croatia	bojan.zugec@foi.hr

The 18th International Symposium on
Operational Research in Slovenia

SOR '25

Bled, SLOVENIA
September 24 - 26, 2025

Appendix

Program of SOR'25



SOR '25 programme

Slovenian Society INFORMATIKA
Section for Operational Research
- organizer -

University of Maribor
Faculty of Organizational Sciences
- co-organizer -

Rudolfovo
The Scientific and Technological Centre in Novo Mesto
- co-organizer -

MDPI AG
Mathematics
- sponsor -

EURO
Association of European Operational Research Societies
- supporter -

EuroCC2
– National Competence Centre SLING
- sponsor -

Excellerat P2
The European Centre of Excellence for Engineering Applications
- sponsor -

Wednesday **September 24, 2025**	**Thursday** **September 25, 2025**	**Friday** **September 26, 2025**
8.15 – Registration	8.15 – Registration	8.15 – Registration
9.00 – 9.45 Opening ceremony (Hall I)	9.00 – 10.00 **Plenary lecture (Hall I):** **(PL3) Tamás Terlaky** Lehigh University, USA	9.00 – 10.00 **Plenary lecture (Hall I):** **(PL5) Drago Bokal** University of Maribor, Slovenia
9.45 – 10.45 **Plenary lecture (Hall I):** **(PL1) Sanja Petrovic** **(The EURO plenary)** Nottingham University, UK	10.00 – 11.15 Parallel sessions (Hall I, II and III): **(SS1) AiDAM&GT** **+ (S4) L&TG&tA + (S7) MO&PP**	10.00 – 11.00 Parallel sessions (Hall I and II): **(SS6) I&S5.0 +** **+ (S1) EM&S**
10.45 – 11.15 Coffee break	11.15 – 11.45 Coffee break	11.00 – 11.30 Coffee break
11.15 – 12.45 Parallel sessions (Hall I, II and III): **(SS5) HPC&QCfOR + (S6) MCDM +** **+ (SS3) AtRoPE&OiSCuOR&SM**	11.45 – 13.00 Parallel sessions (Hall I, II and II): **(S2) HR + (SS2) A&AE +** **+ (S3) F&I**	11.30 – 12.15 Parallel sessions (Hall I and II): **(SS6) I&S5.0 +** **+ (SS7) R&AoMCDM&DSiC**
12.45 – 13.45 Lunch break	13.00 – 14.00 Lunch break	12.15 – 13.00 Closing ceremony (Hall I)
13.45 – 14.45 **Plenary lecture (Hall I):** **(PL2) Josip Arnerić** University of Zagreb, Croatia	14.00 – 15.00 **Plenary lecture (Hall I):** **(PL4) Milica Maričić** University of Belgrade, Serbia	
14.45 – 15.30 Parallel sessions (Hall I and II): **(S5) MO&PP + (S8) M&CS**	15.00 – 16.15 Parallel sessions (Hall I, II and III): **(S1) EM&S + (SS2) A&AE +** **+ (S2) HR**	
15.45 – 18.30 Bled guided tour or Walk tour around the lake Bled	16.15 – 16.45 Coffee break	
19.00 – 22.00 Dinner with quiz Venue: Hotel Astoria	16.45 – 18.00 Parallel sessions (Hall I and II): **(S1) EM&S + (SS4) DC +** **+ (S9) DPAOR**	
	18.15 – 19.30 Social event with traditional Bled cream cake Venue: Hotel Astoria, Bled	

Plenary lectures: 35-40 minutes for presentation plus 5-10 minutes for discussion.
Contributed papers: 14 minutes all together for presentation and for discussion.

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Wednesday

September 24, 2025

8.15 – Registration

9.00 – 9.45 Opening ceremony (Hall I)

9.45 – 10.45 **Plenary lecture** (Hall I, Chair: Sarah Fores)
ORP1 - Sarah Fores: EURO Plenary Speaker: Professor Sanja Petrovic
PL1 - **Sanja Petrovic**: Well-being in personnel scheduling (The EURO plenary)

10.45 – 11.15 Coffee break

11.15 – 12.45 Parallel sessions:

(SS5) Chairs: Janez Povh and Roman Kužel **High Performance Computing and Quantum Computing for Operations Research** (Hall I)	**(S6)** Chairs: Tomasz Wachowicz and Vesna Čančer **Multi-Criteria Decision-Makin** (Hall II)	**(SS3)** Chairs: Veljko Jeremić and Milica Maričić **Analysing the Role of Platform Economy and Organisations in Societal Change using OR and Statistical Method** (Hall III)
SS5.1 Omkar Bihani, Janez Povh and Dunja Pucher: On QUBO Relaxations to the Sparsest k-Subgraph Problem SS5.2 Tomislav Erdelić, Tonči Carić and Nikola Mardešić: A Review of Quantum Computing for Vehicle Routing Problem SS5.3 Anna Gorczyca-Goraj, Krzysztof Grzanka, Marek Szopa and Piotr Frąckiewicz: Introduction to Quantum Extensions of Classical Games SS5.4 Krzysztof Grzanka, Anna Gorczyca-Goraj, Marek Szopa and Piotr Frąckiewicz: Quantum Extensions of Classical Games with QEGS SS5.5 Jelena Joksimović and Martin Zagorc: HPC-Enabled BiLSTM Architecture for Multi-Step Energy Consumption Forecasting SS5.6 Aljaž Krpan and Janez Povh: Kernelization-Based Iterative Refinement of Upper Bounds for Maximum Clique Problem	S6.1 Kolos Csaba Ágoston and László Csató: Inconsistency Thresholds for Incomplete Pairwise Comparisons: The Role of the Associated Graph S6.2 János Baumgartner, Ákos Orosz, Tibor Dulai, Ágnes Vathy-Fogarassy and Zoltán Süle: Multi-Criteria MILP-Based Scheduling of Outpatient Diagnostic Examinations S6.3 Vesna Čančer: Procedure for Comparing Group Values of Different Aggregation Methods S6.4 Marko Bohanec: An Improved Ranking Algorithm in Multi-Criteria Decision Modeling Method DEX S6.5 Petra Grošelj and Gregor Dolinar: An Interval-Based Group Aggregation Approach to Improve Satisfaction in DEMATEL S6.6 Tomasz Wachowicz, Eduarda Asfora Frej and Marek Czekajski: Behavioural Group Decision Support for Post-Industrial Tourism: Integrating Preferences, Synergies, and Decision-Makers' Cognitive Limitations	SS3.1 Tanja Dergan and Marko Debeljak: Typology of B2B and B2C Decision Support Tools in the Agri-Food Sector SS3.2 Nikola Drinjak and Milica Maričić: Factors Influencing the Use of Sports Equipment Sharing Platforms: A User-Centric Study SS3.3 Milica Maričić, Veljko Uskoković and Veljko Jeremić: Mapping Research Trends in the Sharing Economy: A Topic Modeling Approach SS3.4 Mirjana Pejić Bach, Helena Nikolić and Ivan Miloloža: From Coefficients to Connections: Comparing Regression and Network Analysis for Explaining Export Share in Turnover SS3.5 Luna Radević, Milica Maričić and Veljko Jeremić: Segmentation of Touristic Destinations Based on the Number of Domestic and Foreign Shared Nights: Insights from Spain SS3.6 Blaž Rodič, Maja Turnšek, Maja Breznik and Jerneja Šavrič: Using System Dynamics to Model the Impact of Platformisation of Services on Tourism Labour Market in the EU

12.45 – 13.45 Lunch break

13.45 – 14.45 **Plenary lecture** (Hall I, Chair: Tea Šestanović)
ORP2 - Tea Šestanović: Reveiling the (un)known - Josip Arnerić
PL2 - **Josip Arnerić:** Overcoming trading imperfections in multivariate asset prices to improve integrated covariance estimation

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14.45 – 15.30 Parallel sessions:

(S5) Chair: Marko Bohanec **Mathematical Programming and Optimization** (Hall I)	**(S8)** Chair: Sanja Petrovic **Methodologies and Case Studies** (Hall II)	
S5.1 Xiaoni Chi, Goran Lesaja and Guoqiang Wang: A Predictor-Corrector Interior-Point Method with New Search Directions for $P_*(\kapa) $-Weighted Linear Complementarity Problem S5.2 Elif Garajová, Milan Hladík and Miroslav Rada: Weakly Feasible and Optimal Solution Sets in 0–1 Interval Linear Programming S5.3 Miroslav Rada and Michal Černý: On the Generalization of the FGKLA Algorithm	S8.1 Mario Jadrić, Tea Mijač and Maja Ćukušić: Research Domains and Software Tools in Discrete Event Simulation (DES) Studies: Implications for Organisational Decision-Making S8.2 Jelena Jardas Antonić, Nenad Vretenar and Jana Špirková: Exploring Data Envelopment Analysis in Economics and Operations Research: A Brief Bibliometric Perspective S8.3 Vedran Kojić and Margareta Gardijan Kedžo: Formal Analysis of Selected Bond Tilt Properties	

15.45 – 22.00 **Social events**

15.45 – 18.30 Bled guided tour or Walk tour around the lake Bled

19.00 – 22.00 Dinner with quiz (Venue: Hotel Astoria)

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Thursday

September 25, 2025

8.15 – Registration

9.00 – 10.00 **Plenary lecture** (Hall I, Chair: Tibor Illés)

ORP3 - Tibor Illés: The unknown Tamás Terlaky: years from 1955 until 1989

PL3 - **Tamás Terlaky:** Quantum interior point methods (QIPMS) with iterative refinement for linear and semidefinite optimization

10.00 – 11.15 Parallel sessions:

(SS1) Chairs: Darja Rupnik Poklukar and Janez Žerovnik - **Advances in Discrete Applied Mathematics and Graph Theory** (Hall I)	**(S4)** Chair: Andrej Bregar **Location and Transport, Graphs and their Applications** (Hall II)	**(S7)** Chair: Miklos Pinter **Mathematical Optimization and Production Planning** (Hall III)
SS1.1 Omkar Bihani, Janez Povh and Janez Žerovnik: Towards a Randomized Algorithm for Estimating the Chromatic Number of a Graph	S4.1 Nejc Čelik and Andrej Škraba: Graph-Based Modeling of Warehouse Layouts Based on Data from Relation Database for Optimizing Order Picking Paths	S7.1 Segundo Álvarez-García, Nicolás Álvarez-Gil and Rafael Rosillo: An Efficient Parallel Computation Implementation of the Path Bridging Algorithm for the Highly-Constrained Asymmetric Travelling Salesman Problem
SS1.2 Janez Žerovnik: On 2-Domination and 2-Rainbow Domination of Cylindrical Graphs	S4.2 Robert Hlavatý: Designing an Efficient Network under Cost Uncertainty Subject to Constrained Budget and Multiple Objectives	S7.2 Daniil Baldouski, Balázs Dávid, Miklós Krész and Martin Milanič: An Open-Shop Scheduling Problem with Operations Batching
SS1.3 Lýdia Gábrišová, Peter Czimmermann and Andrej Fašánok: On the Application of Uniformly Deployed Sets to the Matrix Permutation Problem	S4.3 Jaroslav Janáček and Marek Kvet: Improving the Credibility of the Emergency Medical System Model	S7.3 Marianna E.-Nagy, Tibor Illés, Yurii Nesteriv and Petra Renáta Rigó: Parabolic Target Space Method for Monotone Linear Complementarity Problem
SS1.4 Igor Reznichenko, Primož Podržaj and Aljoša Peperko: Derivative: Free Optimization of a Magnetic Levitation System	S4.4 Masoud Yavari and Marko Jakšič: Transforming Supply Chains through Reverse Logistics: Scenario Planning with Hybrid Collection-Recycling Facilities	S7.4 Marko Jakšič: Inventory Planning for 3D-Printed Spare Parts under Capacity Constraints
SS1.5 Darja Rupnik Poklukar and Janez Žerovnik: Decay Stability in Graph Products and Trees: New Insights	S4.5 Samo Drobne, Alberto Garre and Eloy Hontoria: Graph-Based Modelling of High-Skilled Internal Migration Patterns in Slovenia	S7.5 Petra Renáta Rigó, Marianna Eisenberg-Nagy, Goran Lesaja, Zsolt Darvay and Anita Varga: Interior-Point Algorithm Based on Standard Kernel Functions

11.15 – 11.45 Coffee break

11.45 – 13.00 Parallel sessions:

(S2) Chair: Kosovka Ognjenović **Human Resources** (Hall III)	**(SS2)** Chair: Karmen Pažek **Agriculture and Agricultural Economics** (Hall II)	**(S3)** Chair: Marija Bogataj **Finance and Investments** (Hall I)
S2.1 Nina Begičević Ređep, Tena Jagačić, Nikola Kadoić and Jelena Gusić Munđar: Measuring the Learning Analytics Adoption: How Important are Different Capabilities?	SS2.1 Giuseppe Attanasi and Aldo Montesano: The Value of Endogenous above Exogenous Information in Irreversible Environmental Decisions	S3.1 Ana Marija Ćoko and Zdravka Aljinović: Green Bonds: Some Buy-Side Aspects
S2.2 Ema Kelin: Quantifying hhe Burden: Unpaid Work and Labour Market Decision Trade-Offs	SS2.2 Maja Borlinič Gačnik, Črtomir Rozman, Andrej Škraba, Boris Prevolšek and Martina Seneković: A Hybrid Modelling Approach to Winegrowing under Climate Stress: Integrating Machine Learning and System Dynamics	S3.2 Tea Šestanović and Marko Čular: MICE or no MICE? Machine Learning Models for Financial Distress Prediction in Croatian Manufacturing SME
S2.3 Terezie Krestova and Nicola Kubacká: Generational Differences in Well-Being Perception: A Mixed-Methods Analysis	SS2.3 Jure Brečko and Jaka Žgajnar: Optimizing Cattle Feed Rations under Farm-Level Constraints: A LP Approach using the SiTFarm Model	S3.3 Jialei Xiong, Diana Barro and Martina Nardon: The Effect of Market Attention Fluctuations on Portfolio Strategy Performance
S2.4 Klara Ljubi and Marko Jakšič: Rethinking Lean through a Social Sustainability Lens	SS2.4 Nemanja Jalić, Maja Ralić, Karmen Pažek, Črtomir Rozman, Aleksandar Ostojić and Željko Vaško: The Impact of Climatic Factors on Crop Yields in the Republic of Srpska	S3.4 Marija Vuković, Snježana Pivac and Karmen Vrhar Šušnjara: Selected Work Situations of Faculty Personnel and the Impact on Work Motivation in the Republic of Croatia
S2.5 Mirjana Pejić Bach, Mile Bošnjak and Maja Meško: Mapping AI-Enabled Innovation Research in the Danube Region: A Bibliometric and Topic Analysis using Vosviewer	SS2.5 Martin Konestabo, Jože Kropivšek and Bojan Blažica: Decision Support System for Garden Centers: A Rule-Based Approach to In-Store Customer Guidance	

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Thursday

September 25, 2025

13.00 – 14.00 Lunch break

14.00 – 15.00 **Plenary lecture** (Hall I, Chair: Veljko Jeremić)

ORP4 - Veljko Jeremić: Invited speaker Milica Maričić, PHD

PL4 - **Milica Maričić:** The art and science of composite indicators: where OR meets statistics

15.00 – 16.15 Parallel sessions:

(S1) Chair: Ksenija Dumičić **Econometric Models and Statistics** (Hall I)	**(SS2)** Chair: Črtomir Rozman **Agriculture and Agricultural Economics** (Hall II)	**(S2)** Chair: Mirjana Pejić Bach **Human Resources** (Hall III)
S1.1 Ivana Cunjak Mataković and Ksenija Dumičić: The Role of Monotonically Decreasing Distributions in Cryptocurrency Markets: Comparing Benford's Law and Stigler's Law S1.2 Anita Čeh Časni: Rethinking the Housing Wealth Effect in Europe S1.3 Liljana Ferbar Tratar: How to Make Forecasts if a Drastic Drop Occurs in a Time Series: A Case Study S1.4 Ria Ivandić and Velibor Mačkić: High Profile Corruption and Trust in Politics S1.5 Nikola Kadoić, Nikolina Žajdela Hrustek and Bojan Žugec: Is it Possible to Unite (by Music) with the Current Voting System at the Eurovision Song Contest?	SS2.6 Martin Pavlovič and Darija Trpin Švikart: Benchmarking Model Evaluation of Hop Industry Economics SS2.7 Karmen Pažek and Teodora Marković: The Impact of ESG Standards on the Competitiveness of Agri-Food Companies SS2.8 Roudneshin Maryam and Amanda Sosa: Harnessing Land and Sea: Strategic Modelling of Hybrid Biomass Supply Chains SS2.9 Martina Senekovič, Črtomir Rozman and Maja Borlinič Gačnik: Framework of a Multi-Criteria DEXi Model for Holistic Sustainability Assessment of the Wine Supply Chain: The Case of Radgonske Gorice SS2.10 Jaka Žgajnar and Lidija Zadnik Stirn: Bio-Economic Farm Modelling to Assess the Effects of Cap Measures: The Case of Suckler Cow Farms	S2.6 Kosovka Ognjenović: How Women Manage Career Change in Today's Labour Markets S2.7 Željka Ostović, Samo Drobne and Marija Bogataj: Long-Term Care Programs in Rural Regions Must Reduce the Risk of Social Isolation S2.8 Martin Lukáčik, Adriana Lukáčiková and Karol Szomolányi: Intertemporal Consumption Behaviour in Post-Communist EU Countries S2.9 Sandra Silva, Leonel Nunes, Helder Amorim, Renato Silva and Joaquim Alonso: Multicriteria Spatial Analysis for the Location-Allocation of Forest Biomass Transfer Stations

16.15 – 16.45 Coffee break

16.45 – 18.00 Parallel sessions:

(S1) Chair: Jaka Žgajnar **Econometric Models and Statistics** (Hall I)	**(SS4)** Chair: Mirjana Kljajić Borštnar **Doctoral Consortium** (Hall II)	**(S9)** Chair: Josip Arnerić **Different Problems Approaching OR** (Hall III)
S1.6 Ema Luna Karara Geršak, Lidija Zadnik Stirn and Aleš Kuhar: Insects as a Sustainable Feed Alternative: Cluster Analysis of Slovenian Consumer Perceptions S1.7 Aleš Kresta and Martin Pluskal: Application of Machine Learning Methods in Volatility Forecasting: A Comparative Study S1.8 Ana Kundid Novokmet and Bruna Čalija: The Weight of Country Risk in Credit Price – Evidence for the Eurozone Housing Loans S1.9 Velibor Mačkić, Lucija Rogić Dumančić and Boris Cota: Competitiveness, Cronyism and Regulation in EU New Member States S1.10 Bahate Maidiya: Industry-Specific Sensitivity to Investor Sentiment: An Empirical Study on Russell 1000 Constituents	SS4.1 Qian Gao and Aleš Kresta: Comparison of Hierarchical Clustering-Based Portfolio Optimization Framework in Multi-Asset ETF SS4.2 Stefan Mirchevski, Aleksandra Popovska Mitrovikj and Verica Bakeva: Mathematical Framework for Cost Function Constructions in some Multi-Phase Queueing Systems: Review and Further Ideas SS4.3 Renee Škulić Ažić: Digital Drivers of Human Development in the European Union: A Dynamic Panel Analysis SS4.4 Tatyana Unuchak, Yauhen Unuchak and Mirjana Kljajić Borštnar: Project Task Descriptions Quality Assesment using Entropy and Textual Metrics	S9.1 Ziv Hellman and Miklos Pinter: What is "Prior"? S9.2 Tin Horvat and Silvija Vlah Jerić: Predicting Customer Churn in the Consumer Goods Industry using Machine Learning S9.3 Blanka Škrabić Perić: Socio-Economic Determinants of Life Satisfaction in European Countries S9.4 Frane Banić and Irena Palić: Fiscal Policy Responses to Inflationary Shocks in Croatia: An Analysis of the Post-Covid Period S9.5 Cong Zhang, Simai He, Bo Jiang and Yinyu Y: Privacy-Preserved Polynomial-Time Interior-Point Methods for Computing Market Equilibria

18.15 – 19.30 **Social event**

Social event with traditional Bled cream cake (Venue: Hotel Astoria)

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Friday

September 26, 2025

8.15 – Registration

9.00 – 10.00 **Plenary lecture** (Hall I, Chair: Lidija Zadnik Stirn)

ORP5 - Lidija Zadnik Stirn: Giving meaning to knowledge - a reflection on an interview with Drago Bokal

PL5 - **Drago Bokal**: Making sense of knowledge in the era of artificial intelligence

10.00 – 11.00 Parallel sessions:

(SS6) Chair: Alenka Brezavšek **Industry & Society 5.0: Optimization in Human and Industrial Environments** (Hall I)	**(S1)** Chair: Anita Čeh Časni **Econometric Models and Statistics** (Hall II)	
SS6.1 Elena Čokor, Nikola Čokor, Anja Šketa, Andraž Čevka, Dominik Jagodic and Drago Bokal: Data-Driven Optimization of Industrial Drying Process for Energy Usage and Emissions Reduction SS6.2 Drago Bokal, Špela Kajzer and Ernest Petrič: Making Sense of Rationality I: Pros and Cons. SS6.3 Špela Kajzer, Anja Šketa, Alenka Brezavšček and Drago Bokal: Making Sense of Rationality II: Emerging Concepts and Practices SS6.4 Janja Jerebic, Špela Kajzer and Alenka Brezavšček Foresight in the Danube Region: Awareness, Application and Training Needs	S1.11 Zoran Mihanović: Confirmatory Factor Analysis – Testing of a Model for Measuring Social Marketing Orientation of End Users in an Educational System S1.12 Ivana Mravak: Crypto Ownership and Financial Literacy in the European Union: A Spatial Econometric Analysis S1.13 Josip Visković, Slavko Šodan and Paško Burnać: Euro Adoption in Croatia: Assessing the Impact on Inflation S1.14 Josipa Višić and Petra Penga: Sensitivity of Profitability Determinants to the Choice of Indicators: A Panel Study of the Croatian Food Industry	

11.00 – 11.30 Coffee break

11.30 – 12.15 Parallel sessions:

SS6) Chair: Drago Bokal **Industry & Society 5.0: Optimization in Human and Industrial Environments** (Hall I)	**(SS7)** Chair: Goran Lesaja **Research and Application of MCDM Methods and Decision Systems in Cybersecurity** (Hall II)	
SS6.5 Polona Pavlovčič Prešeren, Staš Dolinšek, Boštjan Batagelj and Aljaž Blatnik: Detecting GNSS Interference Attacks with Fuzzy Logic Models SS6.6 Klemen Tršinar, Melani Potrč, Špela Kajzer, Špela Korpič, Simon Ravnič, Anja Šketa and Drago Bokal: BAT Inkubator: A Case Study of Applying the Maturity Model for Climate Neutrality and Business Process Optimization	SS7.1 Andrej Bregar: Systematization and Application of MCDM Methods in Cybersecurity SS7.2 Helena Gaspars-Wieloch and Kacper Zielak: Extended Sustainable Development Level Measuring – Case of Poland SS7.3 Mario Pepur, Goran Dedic and Lucija Skopljanac Macina: Consumer Sensitivity to Credit Card Misuse and Debt Accumulation: Insights from the Croatian Market	

12.15 – 13.00 Closing ceremony (Hall I)



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The 18th International Symposium on
Operational Research in Slovenia

SOR '25

Bled, SLOVENIA
September 24 - 26, 2025

Appendix

Sponsors' Notices

The 18th International Symposium on Operational Research in Slovenia - SOR '25
Bled, SLOVENIA, September 24 - 26, 2025

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Journal Introduction

Mathematics is an open access journal that has undergone rapid growth since its launch in 2013. It was included in the Emerging Sources Citation Index (ESCI) of the Web of Science in 2015, before being included in the Science Citation Index Expanded (SCIE) in 2018. The journal has been indexed in Scopus, beginning with volume 5 in May 2017. This journal received its first Impact Factor (IF) of 1.105 in June 2019, which increased to 1.747 in 2020, 2.258 in 2021, 2.592 in 2022, and 2.4 in 2023. In June 2025, it received its seventh IF of 2.2, securing a strong Q1 position in Clarivate's JCR. Additionally, the journal received a CiteScore of 1.1 in 2018, 1.0 in 2019, 1.4 in 2020, 2.2 in 2021, 2.9 in 2022, and 3.5 in 2023. It currently has a CiteScore (Scopus) of 4.6.

Mathematics (ISSN: 2227-7390) publishes high-quality, peer-reviewed papers that address both pure and applied mathematics. The journal collates articles focused on the mathematical study of questions arising in physics, chemistry, biology, statistics, finance, computer science, engineering, and sociology, particularly those that emphasize analytical and algebraic aspects and novel problems and their solutions. One of the journal's missions is to support mathematicians and scientists through the prompt publication of significant advances in any branch of science and technology and to provide a forum for the discussion of new scholarly developments.

Journal activities

In order to reward the academic community, especially young researchers, *Mathematics* has established the Young Investigator Award. This award celebrates the achievements of young investigators in the field of mathematical sciences.

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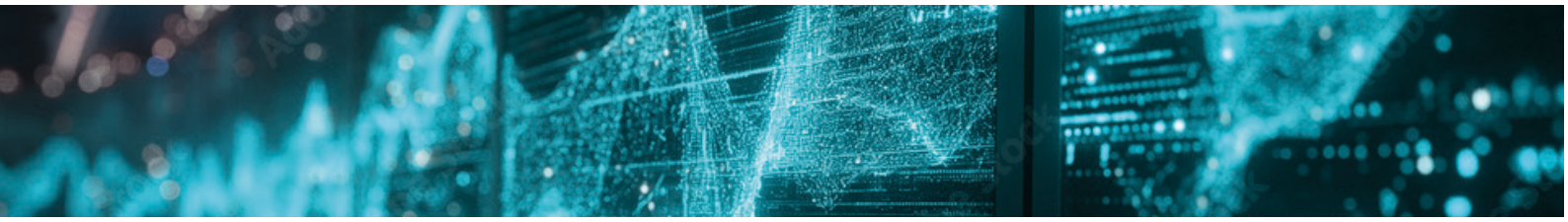
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Funded by the European Union. This work has received funding from the European High Performance Computing Joint Undertaking (JU) and Germany, Italy, Slovenia, Spain, Sweden, and France under grant agreement No 101092621.

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